

Flavored pions and kaons at next-to-leading order in mixed-action staggered chiral perturbation theory

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Different versions of improved staggered fermions can be used as valence quarks to reduce discretization effects in lattice QCD calculations while increasing statistics on existing staggered gauge ensembles. Such mixed-action simulations can be used to improve determinations of light quark masses, Gasser-Leutwyler couplings, decay constants, and other parameters relevant to particle phenomenology. We recall the generalization of ordinary, unmixed staggered chiral perturbation theory required to describe data from lattice calculations with a mixed action such as with HYP staggered valence quarks and asqtad sea quarks. We calculate the next-to-leading order loop diagrams contributing to the masses and decay constants of the flavored pseudo-Goldstone bosons of all tastes and here report results for the decay constants and valence-valence masses.

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1. Introduction

Staggered chiral perturbation theory (SChPT) has been used extensively to control extrapolations from unphysical light-quark simulation masses to physical masses and to remove dominant light quark and gluon discretization effects [1]. Mixed-action ChPT was developed for simulations performed with Ginsparg-Wilson valence quarks and (less computationally expensive) Wilson sea quarks [2]. The formalism for staggered sea quarks and Ginsparg-Wilson valence quarks was developed in Ref. [3]. Mixed-action ChPT for differently improved staggered fermions was introduced for calculations of the $K^0 - \bar{K}^0$ bag parameter [4] and the $K \rightarrow \pi \ell \nu$ vector form factor [5]. As a by-product, the pseudo-Goldstone boson (PGB) masses and propagators were calculated at tree level in mixed-action SChPT. While contributions to overall errors were very small, some of the low-energy couplings (LECs) of mixed-action SChPT were not well determined by the data [4, 5].

Here we present a calculation of the next-to-leading order (NLO) loop corrections to the masses and decay constants of the flavor-charged PGBs in all taste irreps. Analyses of corresponding spectrum data may improve our knowledge of LECs that are poorly determined by existing simulations. Our results can also be used to improve determinations of the light quark masses, Gasser-Leutwyler couplings, and pion and kaon decay constants. Data for such analyses could be generated with, *e.g.*, asqtad sea quarks and valence HYP, or with HISQ sea quarks and improved staggered valence quarks that would allow for simulating relativistic bottom quarks at lattice spacings $\gtrsim 0.03$ fm. Mixed-action SChPT results could facilitate future calculations of quantities such as $|V_{cb}|$ and the $K \rightarrow \pi\pi$ amplitudes.

2. Mixed-action staggered chiral perturbation theory

Mixed-action theories are generalizations of partially quenched theories with different valence and sea quark actions. The symmetries relating valence and sea quarks are broken, but with differently improved versions of the same action, the symmetries of the valence (sea) sector are the same as in the unmixed theory. As for ordinary, unmixed SChPT, mixed-action SChPT is constructed in two steps. First one builds the Symanzik effective theory (SET) for the (mixed-action) lattice theory. One then maps the operators of the SET into those of ChPT [6].

2.1 The leading order Lagrangian

Mapping the SET Lagrangian into the chiral theory through NLO, we have

$$S_{\text{eff}} = S_{\text{QCD}} + a^2 S_6 + \dots \longrightarrow \mathcal{L}_{\text{SChPT}} = \mathcal{L}_{\text{LO,ChPT}} + a^2 \mathcal{V} + \dots \quad (2.1)$$

Here S_{QCD} ($\mathcal{L}_{\text{LO,ChPT}}$) has the form of the QCD action (the leading order Lagrangian of continuum ChPT), but respects the doubler symmetry, taste $SU(4)$. As in continuum ChPT, the Lagrangian $\mathcal{L}_{\text{LO,ChPT}}$ contains kinetic energy, mass, and anomaly terms. The operators in S_6 break the continuum symmetries, including those relating valence and sea quarks, to those of the mixed-action lattice theory. A subset of four-fermion operators in S_6 respects $\Gamma_4 \rtimes SO(4) \subset SU(4)$. They map to the potential \mathcal{V} and can be obtained from those of the unmixed SET by introducing projection operators $P_{\nu,\sigma}$ onto the valence and sea sectors and allowing the LECs to differ in the valence and

sea sectors. Generically, we have

$$c \bar{\psi}(\gamma_s \otimes \xi_t) \psi \bar{\psi}(\gamma_s \otimes \xi_t) \psi \longrightarrow c_{vv} \bar{\psi}(\gamma_s \otimes \xi_t) P_v \psi \bar{\psi}(\gamma_s \otimes \xi_t) P_v \psi + (v \rightarrow \sigma) \quad (2.2)$$

$$+ 2c_{v\sigma} \bar{\psi}(\gamma_s \otimes \xi_t) P_v \psi \bar{\psi}(\gamma_s \otimes \xi_t) P_\sigma \psi$$

where γ_s (ξ_t) is a spin (taste) matrix. To construct the potential \mathcal{V} , the projection operators are conveniently included in spurions. The resulting potential is $\mathcal{V} = \mathcal{U} + \mathcal{U}' - C_{\text{mix}} \text{Tr}(\tau_3 \Sigma \tau_3 \Sigma^\dagger)$, where \mathcal{U} (\mathcal{U}') contains single-(double-)trace operators that are direct generalizations of those in unmixed SChPT, and the last term is a taste-singlet potential new in the mixed-action theory, with $\tau_3 \equiv P_\sigma - P_v$. The operators in $\mathcal{U}^{(l)}$ have independent LECs for the valence-valence, sea-sea, and valence-sea sectors. For example,

$$C_1 \text{Tr}(\xi_5 \Sigma \xi_5 \Sigma^\dagger) \longrightarrow C_1^{vv} \text{Tr}(\xi_5 P_v \Sigma \xi_5 P_v \Sigma^\dagger) + (v \rightarrow \sigma) + C_1^{v\sigma} [\text{Tr}(\xi_5 P_v \Sigma \xi_5 P_\sigma \Sigma^\dagger) + p.c.]. \quad (2.3)$$

where *p.c.* means parity conjugate. For the unmixed case, $C_1^{vv} = C_1^{\sigma\sigma} = C_1^{v\sigma} = C_1$, $C_{\text{mix}} = 0$, and the potential reduces to that of ordinary SChPT. The full expressions for the potentials $\mathcal{U}^{(l)}$ are somewhat lengthy, and we defer writing them down [7].

2.2 Tree-level propagators and flavored PGB masses

The potential \mathcal{V} contributes to the tree-level masses of the PGBs, which fall into irreps of $\Gamma_4 \rtimes \text{SO}(4)$. For a taste t PGB ϕ_{xy}^t composed of quarks with flavors x, y , $x \neq y$,

$$m_{xy,t}^2 = \mu(m_x + m_y) + a^2 \Delta_F^{xy}, \quad t \in F \in \{P, A, T, V, I\}, \quad (2.4)$$

where F labels the taste $\Gamma_4 \rtimes \text{SO}(4)$ irreps (pseudoscalar, axial, tensor, vector, or scalar), and μ is the condensate parameter. Δ_F^{xy} is the tree-level mass splitting, which depends on the LECs in \mathcal{U} and C_{mix} , as well as the sector (valence or sea) of the flavors x, y . We have

$$\Delta_a^{vv} = \frac{8}{f^2} \sum_{b \neq I} C_b^{vv} (1 - \theta^{ab} \theta^{b5}), \quad \Delta_a^{\sigma\sigma} = \frac{8}{f^2} \sum_{b \neq I} C_b^{\sigma\sigma} (1 - \theta^{ab} \theta^{b5}) \quad (2.5)$$

$$\Delta_a^{v\sigma} = \frac{16C_{\text{mix}}}{f^2} + \frac{8}{f^2} \sum_{b \neq I} \left[\frac{1}{2} (C_b^{vv} + C_b^{\sigma\sigma}) - C_b^{v\sigma} \theta^{ab} \theta^{b5} \right], \quad (2.6)$$

where the splitting is $\Delta_a^{vv(\sigma\sigma)}$ if both quarks are valence (sea) quarks and $\Delta_a^{v\sigma}$ otherwise; sub(super)scripts a, b are taste indices labeling the generators of the fundamental irrep of $\text{U}(4)$; and $\theta^{ab} = +1(-1)$ if the generators for a and b (anti)commute. The LECs $C_b^{vv, \sigma\sigma, v\sigma}$ come from the potential \mathcal{U} and are defined in analogy with the unmixed case in Ref. [11]. The residual chiral symmetry in the valence-valence sector, as for the unmixed theory (equivalently, the sea-sea sector), implies $F = P$ particles are Goldstone bosons for $a \neq 0$, $m_q = 0$, and therefore $\Delta_p^{vv} = 0$. The same is not true for the taste pseudoscalar, valence-sea PGBs, and generically, $\Delta_p^{v\sigma} \neq 0$.

In the flavor-neutral sector, $x = y$, the PGBs mix in the taste singlet, vector, and axial irreps. The Lagrangian mixing terms (hairpin terms) are

$$\frac{1}{2} \delta \phi_{ii}^I \phi_{jj}^I + \frac{1}{2} \delta_V^{vv} \phi_{ii}^\mu \phi_{jj}^\mu + \frac{1}{2} \delta_V^{\sigma\sigma} \phi_{ii}^\mu \phi_{jj}^\mu + \delta_V^{v\sigma} \phi_{ii}^\mu \phi_{jj}^\mu + (V \rightarrow A, \mu \rightarrow \mu 5), \quad (2.7)$$

where i, j are flavor indices; μ ($\mu 5$) is a taste index in the vector (axial) irrep; and we use an overbar (underbar) to restrict summation to the valence (sea) sector. The δ -term comes from the anomaly contribution. In continuum ChPT, taking $\delta \rightarrow \infty$ at the end of the calculation decouples the η' [8]. In SChPT, taking $\delta \rightarrow \infty$ decouples the η'_l . The flavor-singlets in other taste irreps are PGBs and do not decouple [9]. The $\delta_{V,A}^{vv,\sigma\sigma,v\sigma}$ -terms are lattice artifacts from the potential $a^2\mathcal{U}'$, and the couplings $\delta_{V,A}^{vv,\sigma\sigma,v\sigma}$ depend linearly on its LECs.

Although the mass splittings and hairpin couplings are different in the three sectors, we find the tree-level propagator can be written in the same form as in the unmixed case. We have

$$G_{ij,kl}^{ab}(p^2) = \delta^{ab} \left(\frac{\delta_{il}\delta_{jk}}{p^2 + \mu(m_i + m_j) + a^2\Delta_a^{ij}} + \delta_{ij}\delta_{kl}D_{il}^a \right), \quad (2.8)$$

where the disconnected propagators vanish by definition in the pseudoscalar and tensor irreps, and for the singlet, vector, and axial irreps,

$$D_{ij}^a \equiv \frac{-1}{I_a J_a} \begin{cases} \frac{\delta_a^{ij}}{1 + \delta_a^{\sigma\sigma} \underline{\sigma}_a} & ij \notin vv \\ \left(\frac{(\delta_a^{v\sigma})^2 / \delta_a^{\sigma\sigma}}{1 + \delta_a^{\sigma\sigma} \underline{\sigma}_a} + \delta_a^{vv} - (\delta_a^{v\sigma})^2 / \delta_a^{\sigma\sigma} \right) & ij \in vv, \end{cases} \quad (2.9)$$

where $\delta_l^{ij} \equiv \delta$, $I_a \equiv p^2 + 2\mu m_i + a^2\Delta_a^{ii}$, $J_a \equiv p^2 + 2\mu m_j + a^2\Delta_a^{jj}$, and we used the replica method to quench the valence quarks [10] and root the sea quarks [9], so that

$$\underline{\sigma}_a \equiv \sum_{\bar{i}} \frac{1}{p^2 + 2\mu m_{\bar{i}} + a^2\Delta_a^{\sigma\sigma}} = \frac{1}{4} \sum_{\bar{i}'} \frac{1}{p^2 + 2\mu m_{\bar{i}'} + a^2\Delta_a^{\sigma\sigma}} \quad (2.10)$$

$$\bar{\sigma}_a \equiv \sum_{\bar{i}} \frac{1}{p^2 + 2\mu m_{\bar{i}} + a^2\Delta_a^{vv}} = 0. \quad (2.11)$$

The index \bar{i} runs over the replica flavors which include the taste degrees of freedom. The index \bar{i}' is summed over the physical sea quark flavors such as u, d, s .

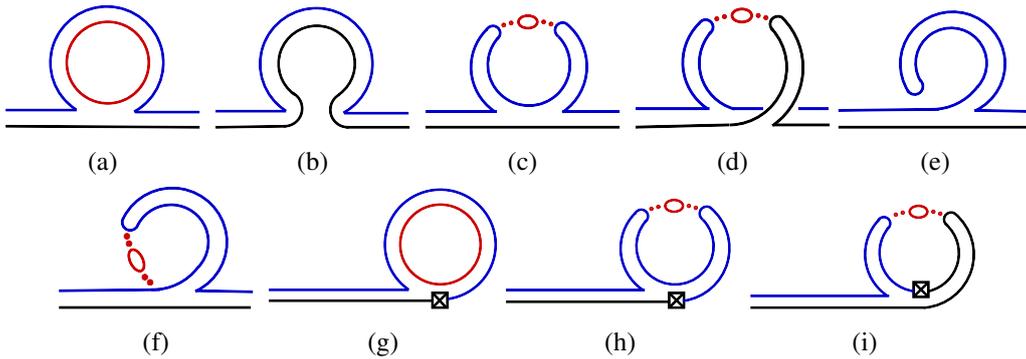


Figure 1: Quark flows for the NLO self-energy tadpoles (a-f) and current-vertex loops (g-i). The x and y quarks are represented by blue and black lines, sea quarks are represented by red lines, and current insertions are represented by crossed boxes.

3. NLO loop corrections to masses

For a taste t PGB ϕ_{xy}^t composed of quarks with flavors x, y , $x \neq y$, the mass is defined in terms of the self-energy, as in continuum ChPT. The NLO mass can be obtained by adding the NLO self-energy to the tree-level value,

$$M_{xy,t}^2 = m_{xy,t}^2 + \Sigma_{xy,t}(-m_{xy,t}^2). \quad (3.1)$$

$\Sigma_{xy,t}$ consists of connected and disconnected tadpole loops with vertices from the LO Lagrangian at $\mathcal{O}(\phi^4)$ and analytic terms with vertices from the NLO Lagrangian at $\mathcal{O}(\phi^2)$.

For a general $\Gamma_4 \rtimes \text{SO}(4)$ irrep, the calculation of the valence-valence, flavored PGB self-energies proceeds as for the unmixed case. Quark flows for the tadpoles are shown in graphs (a-f) of Fig. 1. The kinetic energy, mass, and \mathcal{U} vertices yield graphs of type (a), (c), and (d), and the taste-singlet potential vertices ($\propto C_{\text{mix}}$) yield graphs of type (a),

$$\frac{a^2 C_{\text{mix}}}{3f^2 (4\pi f)^2} \sum_{\vec{i}' a} \ell(m_{\vec{i}' a}^2), \quad (3.2)$$

where \vec{i}' is summed over x, y ; i' is summed over the physical sea quarks; and $\ell(m^2) \equiv m^2 \ln(m^2/\Lambda^2)$ is the chiral logarithm, with Λ the scale of dimensional regularization.

Vertices from \mathcal{U}' yield graphs of type (b), (e), and (f). The hairpin vertex graphs are of type (e) and (f). As in the unmixed case, they ((e) and (f)) can be combined and converted into the form of (d). In the mixed-action case, the necessary identity is

$$\frac{\delta_a^{vv}}{p^2 + 2\mu m_x + a^2 \Delta_a^{vv}} + \frac{\delta_a^{v\sigma}}{4} \sum_{i'} D_{xi'}^a = -(p^2 + 2\mu m_y + a^2 \Delta_a^{vv}) D_{xy}^a, \quad a \in V, A, \quad (3.3)$$

where x, y are valence quarks. This relation follows from Eq. (2.9). Graphs of type (b) come from vertices $\propto \omega_{V,A}^{vv}$; they have the same form as those in the unmixed case [11].

Adding the graphs and evaluating the result at $p^2 = -m_{xy,t}^2 = -\mu(m_x + m_y) - a^2 \Delta_t^{vv}$, we have the NLO, one-loop contributions to the self-energies of the valence-valence, flavored PGBs,

$$-\Sigma_{xy,t}^{\text{NLO loop}}(-m_{xy,t}^2) = \frac{a^2}{48(4\pi f)^2} \sum_a \left[\left(\Delta_{at}^{\text{mix}} - \Delta_t^{vv} - \Delta_a^{v\sigma} + \frac{16C_{\text{mix}}}{f^2} \right) \sum_{\vec{i}' a} \ell(m_{\vec{i}' a}^2) \right] \quad (3.4)$$

$$+ \frac{3}{2} \left(\sum_{b \in V, A} \omega_b^{vv} \tau_{abt} \tau_{abt} (1 + \theta^{ab}) \right) \ell(m_{xy,a}^2) \quad (3.5)$$

$$+ \frac{1}{12(4\pi f)^2} \sum_a \left[a^2 (\Delta_{at}^{vv} - \Delta_t^{vv} - \Delta_a^{vv}) \int \frac{d^4 q}{\pi^2} (D_{xx}^a + D_{yy}^a) \right] \quad (3.6)$$

$$+ \int \frac{d^4 q}{\pi^2} \left[(2(1 - \theta^{at}) + \rho^{at}) q^2 + (2(1 + 2\theta^{at}) + \rho^{at}) m_{xy,5}^2 \right] \quad (3.7)$$

$$+ 2a^2 \Delta_{at}^{vv} + a^2 (2\theta^{at} \Delta_t^{vv} + (2 + \rho^{at}) \Delta_a^{vv}) \left] D_{xy}^a, \quad (3.8)$$

where $\rho^{at} \equiv -4(2 + \theta^{at})$ unless $a = I$, when it vanishes, $\tau_{abt} \equiv \text{Tr}(T^a T^b T^t)$ is a trace over (a product of) generators of $U(4)$, ω_b^{vv} depends on the LECs of \mathcal{U}' , and

$$\Delta_{at}^{vv} \equiv \frac{8}{f^2} \sum_{b \neq I} C_b^{vv} (5 + 3\theta^{ab}\theta^{bt} - 4\theta^{5b}\theta^{bt} - 4\theta^{ab}\theta^{b5}), \quad (3.9)$$

$$\Delta_{at}^{lvv} \equiv \frac{8\theta^{at}}{f^2} \sum_{b \neq I} C_b^{vv} (1 + 3\theta^{ab}\theta^{bt} - 2\theta^{5b}\theta^{bt} - 2\theta^{ab}\theta^{b5}), \quad (3.10)$$

$$\Delta_{at}^{\text{mix}} \equiv \frac{8}{f^2} \sum_{b \neq I} \left[\frac{1}{2}(9C_b^{vv} + C_b^{\sigma\sigma}) + C_b^{\sigma\sigma} (3\theta^{ab}\theta^{bt} - 4\theta^{ab}\theta^{b5}) - 4C_b^{vv}\theta^{5b}\theta^{bt} \right]. \quad (3.11)$$

The form of Eqs. (3.4)-(3.8) is the same as that in ordinary SchPT [11]; the differences are in the definition of the disconnected propagators and the LECs of the effective field theory.

The valence-valence, taste-pseudoscalar PGBs are true Goldstone bosons in the chiral limit, $m_x, m_y \rightarrow 0$, $a \neq 0$. Setting $t = 5$ in Eqs. (3.4) through (3.8) gives

$$-\Sigma_{xy,5}^{\text{NLO loop}}(-m_{xy,5}^2) = \frac{\mu(m_x + m_y)}{2(4\pi f)^2} \sum_a \theta^{a5} \int \frac{d^4 q}{\pi^2} D_{xy}^a, \quad (3.12)$$

which is the generalization of the results of Ref. [9] to the mixed-action case. Only graphs of type (d) contribute.

For the valence-sea, flavored PGB self-energies, the calculation proceeds similarly, but there is no symmetry under $x \leftrightarrow y$, and the taste pseudoscalars are not Goldstone bosons in the chiral limit. Graphs of type (a) contribute chiral logarithms from valence-sea and sea-sea PGBs in the loop, and graphs of type (b) enter with valence-sea PGBs in the loop. The elimination of contributions of type (e) and (f) in favor of contributions of type (d) leaves leftover chiral logarithms of the valence-valence PGBs, $\ell(m_{xx,a}^2)$, multiplied by combinations of hairpin couplings that vanish in the unmixed limit. For $a \neq 0$, $m_x, m_y \rightarrow 0$, graphs of type (a), (b), (c), and (e) contribute; the valence-sea, taste-pseudoscalars PGBs are not true Goldstone bosons in the chiral limit. However, no new loop integrals arise. The details of the calculation and the results will be presented in Ref. [7].

4. NLO loop corrections to decay constants

The decay constants are defined in terms of the matrix elements of the axial currents,

$$-if_{xy,t} p_\mu = \langle 0 | j_{xy,t}^{\mu 5} | \phi_{xy}^t(p) \rangle. \quad (4.1)$$

The NLO corrections come from wave function renormalization [graphs (a), (c), and (d) of Fig. 1], insertions of the $\mathcal{O}(\phi^3)$ -terms of the LO current [graphs (g), (h), and (i) of Fig. 1], and NLO analytic terms [12]. The LO current is determined by the kinetic energy vertices of the LO Lagrangian, and is therefore the same as in unmixed SchPT. Likewise, the NLO wave function renormalization corrections are determined by self-energy contributions from tadpoles with kinetic energy vertices from the LO Lagrangian. To generalize the results of the unmixed case, we have only to replace the propagators with those of the mixed-action theory; nothing in the calculation of the relevant part of the self-energies or the current-vertex loops is sensitive to the sector of the external quarks. We

have

$$\frac{f_{xy,t}^{\text{NLO loop}}}{f} = 1 - \frac{1}{8(4\pi f)^2} \sum_a \left[\frac{1}{4} \sum_{i',j'} \ell(m_{i',j',a}^2) + \int \frac{d^4 q}{\pi^2} (D_{xx}^a + D_{yy}^a - 2\theta^{at} D_{xy}^a) \right]. \quad (4.2)$$

where x, y can take either valence (\bar{x}, \bar{y}) or sea ($\underline{x}, \underline{y}$) indices. This result holds for valence-valence ($\bar{x} \neq \bar{y}$), sea-sea ($\underline{x} \neq \underline{y}$), and valence-sea (\bar{x} and \underline{y}) PGBs, and has the same form as in the unmixed theory [13].

5. Summary

Using mixed-action SChPT, we have calculated the NLO loop corrections to the valence-valence and valence-sea masses and decay constants of flavored PGBs in all taste irreps. The results have been cross-checked by performing two independent calculations, and we have verified the results reduce correctly when valence and sea quark actions are the same. In the valence-valence sector, the taste pseudoscalars are Goldstone bosons in the chiral limit, at nonzero lattice spacing. The NLO analytic terms arise from tree-level contributions of the (NLO) Gasser-Leutwyler and generalized Sharpe-Van de Water Lagrangians. They have the same form as in the unmixed case, with different couplings in the valence-valence, sea-sea, and valence-sea sectors.

The NLO loop corrections to the self-energies of the valence-valence, flavored PGBs are given in Eqs. (3.4)-(3.8); those for the valence-valence and valence-sea decay constants are given in Eq. (4.2). They have the same form as the results in ordinary, unmixed SChPT. The results for the valence-sea self-energies will be included in Ref. [7].

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