

Contribution of the twist-3 fragmentation function to single transverse-spin asymmetry in SIDIS

Koichi Kanazawa^{*†}

Graduate School of Science and Technology, Niigata University, Ikarashi 2-8050, Niigata 950-2181, Japan

Department of Physics, Barton Hall, Temple University, Philadelphia, PA 19122, USA

E-mail: kanazawa@nt.sc.niigata-u.ac.jp

Yuji Koike[‡]

Department of Physics, Niigata University, Ikarashi 2-8050, Niigata 950-2181, Japan

E-mail: koike@nt.sc.niigata-u.ac.jp

We study the contribution of the twist-3 fragmentation function to the single transverse-spin asymmetry in SIDIS within the framework of the collinear factorization. Using the Ward-Takahashi identity in QCD, we establish the collinear twist-3 formalism in the Feynman gauge to calculate its *non-pole* contribution to the asymmetry.

*XXI International Workshop on Deep-Inelastic Scattering and Related Subject -DIS2013,
22-26 April 2013
Marseilles, France*

^{*}Speaker.

[†]K. K. is supported by JSPS Research Fellowships for Young Scientists (No.24.6959).

[‡]Y. K. is supported in part by the Grant-in- Aid for Scientific Research (No. 23540292) from the JSPS.

1. Introduction

The single-transverse spin asymmetry (SSA) in high-energy QCD reactions cannot be explained by the collinear parton model and its description requires some extensions of the framework for QCD hard processes. An extension is the twist-3 approach within the collinear factorization framework where twist-3 parton distribution and/or fragmentation functions are responsible for large SSAs in high- P_T particle productions.

So far, the collinear twist-3 approach has been mainly developed for the pole contribution of the twist-3 distributions. On the other hand, the study and knowledge of the twist-3 fragmentation has been scarce. For the SSA in the pp collision, a first calculation for the so-called derivative contribution has been performed in [1] while the complete formula has been derived in the light-cone gauge in [2]. A numerical estimate of its impact in [1] shows this effect could give a significant contribution to the SSA, which suggests it could be a possible origin of the observed remarkably large SSA for η -meson at RHIC [3]. The importance of the twist-3 fragmentation effect has also been pointed out in the context of the sign-mismatch problem between the Sivers function extracted from semi-inclusive DIS (SIDIS) and the quark-gluon correlation function determined in the pp process [4].

In this report, we study the contribution of the twist-3 fragmentation functions to the SSA in SIDIS. A characteristic property of the twist-3 fragmentation functions is that they could have complex phases to cause SSA in a nonperturbative way. Accordingly, for the calculation, we have to retain the *non-pole* part of the hard parts. So far, the color gauge-invariance of twist-3 contributions in a covariant gauge is only shown for pole contributions [5, 6] while for the non-pole contribution the proof is absent in the literature. Our main interest here is how the color gauge-invariance of the non-pole twist-3 contribution is realized in the Feynman gauge. For SIDIS, a first calculation has been made for the Collins azimuthal asymmetry [7] although the cross-section formula for the other azimuthal modulations has not been derived. Establishing the collinear twist-3 formalism, we derive a complete cross-section formula for the SSA in SIDIS. A comparison of its prediction with the future EIC experiment would shed new light on the origin of the SSA and multiparton correlations in hadrons.

2. Collinear twist-3 formalism for non-pole contribution

We first introduce the F-type twist-3 fragmentation functions for a spinless hadron h as [8]

$$\begin{aligned}\Delta_{Fij}^\alpha(z_1, z_2) &= \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2} - \frac{1}{z_1})} \langle 0 | \psi_i(0) | hX \rangle \langle hX | \bar{\psi}_j(\lambda w) g F^{\alpha\beta}(\mu w) w_\beta | 0 \rangle \\ &= \frac{M_N}{2z_2} (\gamma_5 \not{P}_h \gamma_\lambda)_{ij} \epsilon^{\lambda\alpha w P_h} \hat{E}_F(z_1, z_2) + \dots,\end{aligned}\quad (2.1)$$

$$\begin{aligned}\tilde{\Delta}_{Fij}^\alpha(z_1, z_2) &= \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} \frac{d\mu}{2\pi} e^{-i\frac{\lambda}{z_1}} e^{-i\mu(\frac{1}{z_2} - \frac{1}{z_1})} \langle 0 | \bar{\psi}_j(\lambda w) \psi_i(0) | hX \rangle \langle hX | g F^{\alpha\beta}(\mu w) w_\beta | 0 \rangle \\ &= \frac{M_N}{2z_2} (\gamma_5 \not{P}_h \gamma_\lambda)_{ij} \epsilon^{\lambda\alpha w P_h} \tilde{E}_F(z_1, z_2) + \dots,\end{aligned}\quad (2.2)$$

where ψ_i is the quark field with spinor index i while $F^{\alpha\beta} \equiv F^{\alpha\beta} T^a$ is the gluon's field strength with T^a being the color matrices. M_N is the nucleon mass, w^μ is a light-like vector satisfying

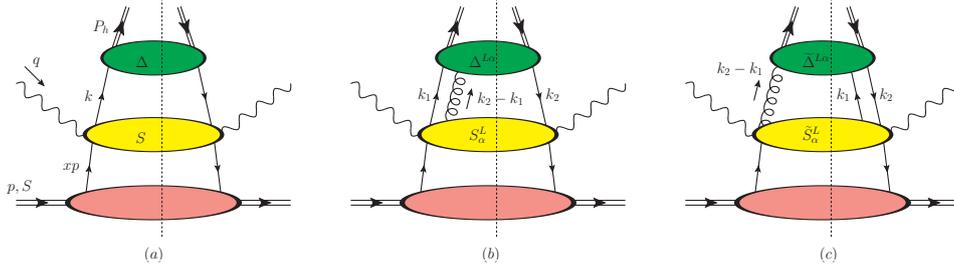


Figure 1: Generic diagrams giving rise to the twist-3 fragmentation function contribution to the asymmetry. The top blobs are the fragmentation matrix elements, the middle blobs are the hard parts, and the bottom blobs represent the quark transversity. The mirror diagrams of (b) and (c) also contribute.

$P_h \cdot w = 1$, $N = 3$ is the number of colors and $\varepsilon^{\lambda\alpha w P_h} \equiv \varepsilon^{\lambda\alpha\rho\sigma} w_\rho P_{h\sigma}$ with the Levi-Civita tensor being $\varepsilon_{0123} \equiv +1$. We have suppressed the gauge-link operators between the fields for simplicity. $\{\hat{E}_F, \tilde{E}_F\}$ is a complete set of the twist-3 quark-gluon correlation functions for the spinless hadron. These are chiral-odd functions and enter the single-spin dependent cross-section formula with the quark transversity distribution.

As is well-known, the F-type function $\hat{E}_F(z_1, z_2)$ is related with the D-type function $\hat{E}_D(z_1, z_2)$, which is defined with the covariant derivative D^α instead of $gF^{\alpha\beta} w_\beta$, as [8]

$$\hat{E}_D(z_1, z_2) = P \left(\frac{1}{1/z_1 - 1/z_2} \right) \hat{E}_F(z_1, z_2) + \delta \left(\frac{1}{z_1} - \frac{1}{z_2} \right) \tilde{e}(z_2), \quad (2.3)$$

where $\tilde{e}(z)$ is given by¹

$$\begin{aligned} \Delta_{\partial_{ij}}^\alpha(z) &= \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0 | [\infty w, 0] \psi_i(0) | hX \rangle \langle hX | \bar{\psi}_j(\lambda w) [\lambda w, \infty w] | 0 \rangle \overleftarrow{\partial}^\alpha \\ &= \frac{M_N}{2z} (\gamma_5 \not{P}_h \gamma_\lambda)_{ij} \varepsilon^{\lambda\alpha w P_h} \tilde{e}(z) + \dots \end{aligned} \quad (2.4)$$

Here we restore the gauge-link operators to emphasize the derivative $\overleftarrow{\partial}^\alpha$ hits both $\bar{\psi}(\lambda w)$ and $[\lambda w, \infty w]$. For the calculation, we also need the 2-parton correlator [11]

$$\begin{aligned} \Delta_{ij}(z) &= \frac{1}{N} \sum_X \int \frac{d\lambda}{2\pi} \langle 0 | \psi_i(0) | hX \rangle \langle hX | \bar{\psi}_j(\lambda w) | 0 \rangle \\ &= \frac{M_N}{2z} (\sigma_{\lambda\sigma} i\gamma_5)_{ij} \varepsilon^{\lambda\sigma w P_h} \hat{e}_1(z) + \dots \end{aligned} \quad (2.5)$$

This twist-3 function \hat{e}_1 is related to the imaginary part of the D-type function via the QCD equation of motion [2] and thus is expressed in terms of \hat{E}_F and \tilde{e} through Eq. (2.3).

The single-spin dependent cross-section in SIDIS may be written in terms of the leptonic and hadronic tensors, $L^{\mu\nu}$ and $W_{\mu\nu}$, as $\Delta\sigma \sim L^{\mu\nu} W_{\mu\nu}$ ². For the analysis of the twist-3 fragmentation effect, we factorize the quark transversity distribution $h(x)$ from the hadronic tensor,

$$W_{\mu\nu} = \int \frac{dx}{x} h(x) w_{\mu\nu}. \quad (2.6)$$

¹Note the F-type function vanishes at the soft-gluon pole point: $\hat{E}_F(z, z) = 0$ [9, 10].

²For the kinematics, see [12].

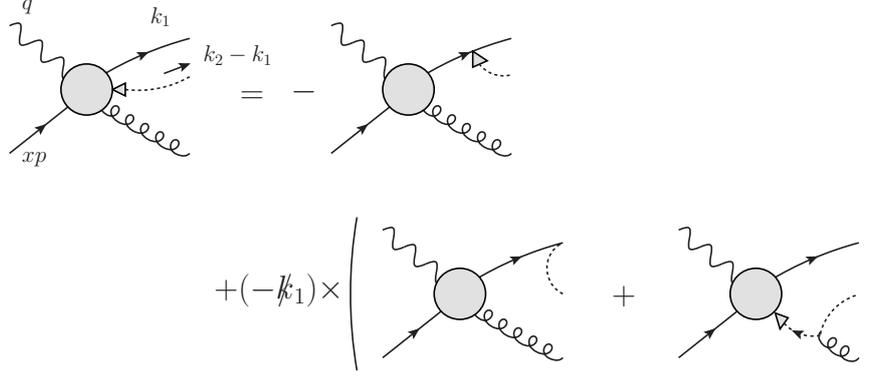


Figure 2: Ward-Takahashi identity for a coupling of the scalar-polarized gluon $(k_1 - k_2)^\sigma$ onto the amplitude in the left-hand-side of the final-state cut in $S_\sigma^{La}(k_1, k_2)$. The circle represents the sum of the s -channel and t -channel diagrams for the partonic subprocess $\gamma^* q \rightarrow qg$.

The generic diagrams which contain twist-3 fragmentation effects are shown in Fig. 1. We first consider the diagrams (a), (b) and the mirror diagram of (b). A standard and systematic way to extract twist-3 effects is the collinear expansion of the hard parts. Following the procedure in [5], it is easy to see the twist-3 contribution is expressed in terms of the gauge-invariant F-type matrix element (2.1) and several gauge-dependent ones. As is well-established, for pole contributions, the latter gauge-dependent terms vanish owing to the Ward identities for the hard parts which hold due to the on-shell conditions associated with the poles of the internal propagators [5, 6]. Thus the color gauge-invariance is ensured for the pole contribution at twist-3. For the non-pole contribution, however, such special on-shell conditions are lacking, so that it is necessary to develop the collinear twist-3 formalism without relying on those conditions.

In order to show the desired factorization property for the non-pole contribution, what we have to do is to prove all those gauge-dependent terms are combined into matrix elements for the gauge-invariant operators. To show this is the case, we note the hard part S_σ^L with an off-shell quark leg obeys a Ward-Takahashi identity, as shown in Fig. 2,

$$(k_2 - k_1)^\sigma S_\sigma^{La}(k_1, k_2) = T^a S(k_2). \quad (2.7)$$

Here we discarded the ghost-like term, the last diagram in the right-hand-side in Fig. 2. This can be done in the twist-3 accuracy because the hard part for the ghost-like term and its first derivatives with respect to $k_{1,2}$ vanish in the collinear limit. With the identity (2.7), we find the light-cone matrix elements produced in the course of the collinear expansions are eventually reorganized into the color gauge-invariant ones as³

$$w^{(a)} + w^{(b)} + w^{(b)*} = \int \frac{dz}{z^2} \text{Tr} [\Delta(z) S(z)] + \Omega_\beta^\alpha \int \frac{dz}{z^2} \text{ImTr} \left[\Delta_\partial^\beta(z) \frac{\partial S(k)}{\partial k^\alpha} \Big|_{k \rightarrow \frac{p_h}{z}} \right] - 2\Omega_\beta^\alpha \int \frac{dz_1}{z_1^2} \frac{dz_2}{z_2^2} P \left(\frac{1}{1/z_2 - 1/z_1} \right) \text{ImTr} \left[\Delta_F^\beta(z_1, z_2) S_\alpha^L(z_1, z_2) \right], \quad (2.8)$$

³We have suppressed the Lorentz indices μ and ν for simplicity.

with the projection operator $\Omega^{\alpha\beta} = g^{\alpha\beta} - P_h^\alpha w^\beta$. Note in this analysis we have recovered up to the $\mathcal{O}(gA)$ terms including the expansion of the gauge-link operators in Eq. (2.4).

For completeness, we have to take into account the contributions from the diagram (c) in Fig. 1 and its mirror diagram. In this case, the hard part satisfies a simple Ward identity,

$$(k_2 - k_1)^\sigma \tilde{S}_\sigma^L(k_1, k_2) = 0. \quad (2.9)$$

Since this is the same with the case in pole contribution, it is straightforward to show the factorization property and the color gauge-invariance. The resultant factorization formula is expressed in terms of only the gauge-invariant F-type matrix element (2.2) as

$$w^{(c)} + w^{(c)*} = -2\Omega_\beta^\alpha \int \frac{dz_1}{z_1^2} \frac{dz_2}{z_2^2} P\left(\frac{1}{1/z_2 - 1/z_1}\right) \text{ImTr} \left[\tilde{\Delta}_F^\beta(z_1, z_2) \tilde{S}_\alpha^L(z_1, z_2) \right]. \quad (2.10)$$

Equations (2.8) and (2.10) show all contributions arising from the twist-3 fragmentation functions have definite factorization property with manifest color gauge-invariance. The complete partonic cross-section formula can be found in [13].

3. Summary

We have discussed the contribution of the twist-3 fragmentation function to the SSA in SIDIS in the framework of the collinear factorization. We have established the collinear twist-3 formalism in the Feynman gauge to derive the gauge-invariant factorized cross-section formula. There, the relations among the hard parts by the Ward-Takahashi identity play a crucial role in reorganizing the twist-3 light-cone matrix elements into the color gauge-invariant ones. The single-spin dependent cross-section formula is expressed in terms of the gauge-invariant functions, \hat{E}_F, \tilde{E}_F , and \tilde{e} . Future EIC experiments would provide us with a unique opportunity to determine these functions and clarify the origin of large SSAs.

References

- [1] Z.-B. Kang, F. Yuan, and J. Zhou, Phys. Lett. B **691**, 243 (2010).
- [2] A. Metz and D. Pitonyak, Phys. Lett. B **723**, 365 (2013).
- [3] STAR Collaboration, L. Adamczyk *et al.*, Phys. Rev. D **86**, 051101 (2012).
- [4] Z.-B. Kang, J.-W. Qiu, W. Vogelsang, and F. Yuan, Phys. Rev. D **83**, 094001 (2011).
- [5] H. Eguchi, Y. Koike, and K. Tanaka, Nucl. Phys. B **763**, 198 (2007).
- [6] H. Beppu, Y. Koike, K. Tanaka, and S. Yoshida, Phys. Rev. D **82**, 054005 (2010).
- [7] F. Yuan and J. Zhou, Phys. Rev. Lett. **103**, 052001 (2009).
- [8] H. Eguchi, Y. Koike, and K. Tanaka, Nucl. Phys. B **752**, 1 (2006).
- [9] S. Meissner and A. Metz, Phys. Rev. Lett. **102**, 172003 (2009).
- [10] L. P. Gamberg, A. Mukherjee, and P. J. Mulders, Phys. Rev. D **83**, 071503 (2011).
- [11] X. Ji, Phys. Rev. D **49**, 114 (1994).
- [12] Y. Koike, K. Tanaka, and S. Yoshida, Phys. Rev. D **83**, 114014 (2011).
- [13] K. Kanazawa and Y. Koike, in preparation.