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Possible bound states in the T_{Qs} and $T_{QQ'}$ systems

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The heavy-strange tetraquarks ($T_{Qs} = ud\bar{s}\bar{Q}$) and the doubly heavy tetraquarks ($T_{QQ'} = ud\overline{QQ'}$) in the S-wave state are investigated by a quark model. Suppose the ud pair is in the spin 0 isospin 0 color $\bar{3}$ configuration, there will be an about 150 MeV attraction from the color magnetic interaction (CMI). The pair in the spin 1 isospin 0 color 6 configuration also gets a small attraction of about 25 MeV. We classified the systems according to the quantum numbers of the ud pair and evaluate the size of CMI in the short range region. There is a large attraction in the $ud\overline{QQ}$ $I(J^P)$=$0(1^+)$ (T_{cc}) and in the $ud\overline{QQ'}$ $0(0^+)$ and $0(1^+)$ configuration. A dynamical calculation is also performed for these states with a simple quark model by employing the resonating group method. It is found that there is a bound state in the $ud\overline{bb}$ $I(J^P)$=$0(1^+)$ (T_{bb}) state. The present model does not include the long range meson exchange interaction yet. There might be more bound states if their effects are introduced.

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1. Introduction

Recent experiments on the charmed systems have shown us a new stage of the hadron physics. For example, the $X(3872)$ has been found by first by Belle in the $J/\psi\pi\pi K$ observation from the B decay, and then confirmed by various experiments [1, 2]. The state is found to be $J^{PC} = 1^{++}$ [3], and it is very difficult to explain its features if one assumes a simple $c\bar{c}$ configuration. It is considered to have (at least) a large amount of the $qc\overline{qc}$ component.

One of the successful approaches to explain the $X(3872)$ behaviors is to consider the system as a two-meson state which couples to the $c\bar{c}$ configuration [4]. In this model, the $X(3872)$ is a superposition of a two-meson molecular state and the $c\bar{c}(2P)$ quarkonium. The model explains a rough production ratio of $X(3872)$ to the J/ψ in the $p\overline{p}$ collision, the absence of the charged partner of $X(3872)$ which decays to $J/\psi\rho^{\pm}$, a lack of the $\chi_{c1}(2P)$ peak above the open charm threshold predicted by the quark model, and the isospin symmetry breaking in the $X(3872)$ decay [5].

It has been pointed out that the T_{cc} ($cc\overline{ud}$ or $ud\overline{cc}$, $I(J^P) = 0(1^+)$) or similar other double charm or double bottom flavor mesons may exist as a bound state(s) [6, 7, 8, 9, 10]. They, however, do not agree on the point whether there really is a bound state. The corresponding energy region seems to be partly within the reach of the experiments. In this work, we extend the above approach for the $X(3872)$ to investigate the $qq\bar{s}\overline{Q}$ and $qq\overline{QQ}'$ systems hoping to contribute this still open problem.

2. Model

First we explain the quark model from which we extract the two meson interaction. They are defined as:

$$H_q = H_0 + V \quad \text{with} \quad H_0 = \sum_i^4 \sqrt{m_i^2 + p_i^2} \quad \text{and} \quad V = V_{\text{conf}} + V_{\text{coul}} + V_{\text{CMI}}, \tag{2.1}$$

where the m_i and p_i are the mass and the 3-momentum of the ith quark (or antiquark). We set the system is at rest ($p_G = 0$). As for the interaction between quarks, we employ the one used in the ref. [11]. In order to apply their interaction in the multiquark systems, we modified them to depend not to the momentum but to the flavors of the interacting quarks. The potential consists of the linear confinement term, V_{conf}, and the one-gluon exchange (OGE) term. The latter consists of the Coulomb term, V_{coul}, and the color magnetic interaction, V_{CMI}.

$$V_{\text{coul}} = \sum_{i<j} \frac{(\lambda_i \cdot \lambda_j)}{4} \frac{\alpha_{ij}}{r_{ij}} \tag{2.2}$$

$$V_{\text{conf}} = \sum_{i<j} \frac{(\lambda_i \cdot \lambda_j)}{4} \Big(-\frac{4}{3}\Big)^{-1} (v_b r_{ij} + v_c) \tag{2.3}$$

$$V_{\text{CMI}} = -\sum_{i<j} \frac{(\lambda_i \cdot \lambda_j)}{4} (\sigma_i \cdot \sigma_j) \frac{2\pi}{3} \alpha_{ij} \frac{\xi_{ij}}{m_i m_j} \delta^3(r_{ij}) \tag{2.4}$$

The values of the parameters are listed in Table 1.

Table 1: The quark model parameters

m_u (MeV)	m_s (MeV)	m_c (MeV)	m_b (MeV)	v_b (MeVfm^{-1})	v_c (MeV)
220	419	1628	4977	912	−253

qq	uu	us	uc	ub	ss	sc	sb	cc	cb	bb
b_{qq}	0.4216	0.4030	0.3684	0.3580	0.3787	0.3390	0.3262	0.2619	0.2272	0.1665
α_{qq}	0.9737	0.9229	0.6920	0.5921	0.8506	0.6709	0.5794	0.5947	0.5231	0.4695
ξ_{qq}	0.1238	0.1688	0.2386	0.2529	0.2202	0.3696	0.3942	0.5883	0.6837	0.9838

Table 2: Matrix elements of the color and the color-spin operator for two light quarks in the (quark spin, color)$_{\text{Isospin}}$ states. The isospin for $q\bar{q}$ can be either 0 or 1.

	qq or $\overline{qq}$				$q\bar{q}$			
$(S,\text{color})_I$	$(0,\bar{\mathbf{3}})_0$	$(1,\bar{\mathbf{3}})_1$	$(0,\mathbf{6})_1$	$(1,\mathbf{6})_0$	$(0,\mathbf{1})$	$(1,\mathbf{1})$	$(0,\mathbf{8})$	$(1,\mathbf{8})$
$\langle\lambda\lambda\rangle$	$-8/3$	$-8/3$	$4/3$	$4/3$	$-16/3$	$-16/3$	$2/3$	$2/3$
$-\langle\lambda\lambda\sigma\sigma\rangle$	-8	$8/3$	4	$-4/3$	-16	$16/3$	2	$-2/3$

In order to see if there is an actual bound state, we perform a dynamical calculation. We explain the model with the $qq\overline{sc}$ $0(2^+)$ case in the following. The wave function is assumed as:

$$\begin{aligned}\Psi = &\sum_i c_i^1 \frac{(ud-du)\bar{s}\bar{b}}{\sqrt{2}} \chi_2^\sigma \Big(|(\mathbf{11})\mathbf{1}\rangle^{\text{color}}_{13;24}\, \chi_i(13;24) + |(\mathbf{11})\mathbf{1}\rangle^{\text{color}}_{14;23}\, \chi_i(14;23) \Big) \\ &+ \sum_i c_i^8 \frac{(ud-du)\bar{s}\bar{b}}{\sqrt{2}} \chi_2^\sigma \Big(|(\mathbf{88})\mathbf{1}\rangle^{\text{color}}_{13;24}\, \chi_i(13;24) + |(\mathbf{88})\mathbf{1}\rangle^{\text{color}}_{14;23}\, \chi_i(14;23) \Big) \end{aligned} \tag{2.5}$$

$$\chi_i(13;24) = \phi(b_{13},r_{13})\phi(b_{24},r_{24})\phi(b_i,R_{13}-R_{24}) \tag{2.6}$$

where $\phi(b,r)$ is the gaussian wave function with the size parameter b, $|(\mathbf{11})\mathbf{1}\rangle^{\text{color}}_{13;24}$ is the color part of the wave function in which each of the 1st-3rd quark pair and 2nd-4th quark pair is in the color singlet configuration. In this restricted model space, the equation of the motion (Resonating group method equation) can be obtained from the hamiltonian for quarks (eq. 2.1):

$$(\mathscr{K}+\mathscr{V}-E\mathscr{N})\bar{\psi} = 0 \qquad \text{with} \qquad \mathscr{O} = \int \Psi(R) O \Psi(R') \tag{2.7}$$

$$\Psi_f(R) = \frac{(ud-du)\bar{s}\bar{b}}{\sqrt{2}} \chi_2^\sigma \Big(|(\mathbf{ff})\mathbf{1}\rangle^{\text{color}}_{13;24}\, \chi(13;24,R) + |(\mathbf{ff})\mathbf{1}\rangle^{\text{color}}_{14;23}\, \chi(14;23,R) \Big) \tag{2.8}$$

$$\chi(13;24,R) = \phi(b_{13},r_{13})\phi(b_{24},r_{24})\delta(R_{13}-R_{24}-R) \tag{2.9}$$

By restricting the configuration as above, the van der Waals force from the confinement potential disappears, which is the model artifact appearing in the multiquark systems.

3. Results and discussion

In Table 2, we show the matrix elements of $(\lambda_1 \cdot \lambda_2)$ and $-(\lambda_1 \cdot \lambda_2)(\sigma_1 \cdot \sigma_2)$ for the qq, $\overline{qq}$ and $q\overline{q}$ pairs. It shows that CMI is attractive in the (S, color)$_I$ = (0, $\bar{\mathbf{3}}$)$_0$ and (1, $\mathbf{6}$)$_0$ qq or $\overline{qq}$ pairs. It is also attractive in the (S, color) = (0, $\mathbf{1}$) $q\overline{q}$ pairs, namely in the (pseudo)scalar mesons. Comparing that in the vector meson, (1, $\mathbf{1}$), CMI in the (1, $\bar{\mathbf{3}}$)$_1$ pair can be attractive.

In Table 3, we show the expectation values of CMI assuming that every size parameter of two quarks in the orbital part of the wave function is the same as that in the mesons with corresponding flavors (see b_{qq} in Table 1). For each quantum number, we show the possible spin-color configuration of qq and $\overline{QQ}$ pairs together with the diagonal CMI matrix elements. We also show the lowest eigenvalue of the CMI obtained when considering the mixing of those configurations. There are four configurations whose CMI is more attractive than that of the threshold in this very simple estimate: $qq\overline{cc}$ $I(J^P) = 0(1^+)$, $qq\overline{sc}$ $0(0^+)$, $0(1^+)$, and $0(2^+)$. Note that all of these states are iso-singlet. Note also that, since the $\lambda\lambda$ is positive for the color $\mathbf{6}$ configuration, the color Coulomb term will prevents the two quarks to come close to each other. So, the (s, $\mathbf{6}$)(s' $\mathbf{6}$) contribution should be considered as the mixing of the two meson states, (s_m, $\mathbf{1}$)(s'_m $\mathbf{1}$), which also has the color $\mathbf{6}$ components, rather than the (s, $\mathbf{6}$)(s' $\mathbf{6}$) cluster states.

As a dynamical calculation, we solve the RGM equation (2.7). There is a bound state with the binding energy 30 MeV in the $ud\overline{bb}$ $I(J^P)$=0(1$^+$) channel. In other channels we find no bound states. This calculation should be regarded as an estimate which may have a large model dependence. To obtain more reliable results, one should investigate at least the following four points. (a) In this model, the meson exchange between the u and d quarks is neglected. Namely, the situation corresponds to, for example, the NN system without the long-range interaction. (b) The kinetic term should be modified so that the clusters move with the observed meson masses. (c) The effects of the momentum dependence in the original quark interaction should be re-introduced because it will change the interaction range effectively. (d) To calibrate the interaction between the qq or $\overline{QQ}'$ quarks, it should be investigated whether this simplified quark model can also give the features of the uuQ or usQ baryons, which were reproduced by the original model [11].

Our approach to extract a hadron potential from the quark model enables us to investigate the resonances as well as the bound state. Though the system is in the S-wave, there is a possibility that the resonances appear if they are below the upper thresholds such as D^*D^*.

4. Summary

We have investigated the heavy-heavy and the heavy-strange tetraquark systems by a quark model. The eigenvalues of the color magnetic interaction can be very attractive. In a dynamical calculation, we find a bound state in the $ud\overline{bb}$ $I(J^P)$=0(1$^+$) channel even though our calculation is performed without the long-range meson exchange force. The two-meson potential can be constructed from the resonating group model equation, which enables us to investigate the scattering states.

References

[1] J. Beringer, *et al.* [Particle Data Group] Phys. Rev. D **86**, 010001 (2012).

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Table 3: Possible four-quark orbital states and their color-spin configurations. The numbers $a[b]$ or $a[b/c]$ for $\langle V_{\rm CMI}\rangle$ are the lowest eigenvalues of CMI for $ud\overline{cc}[ud\overline{bb}]$ or $ud\overline{sc}[ud\bar{s}\bar{b}/ud\bar{c}\bar{b}]$ in MeV. c_{int} is $\sum c_{ij} - c_{12} - c_{34}$ with $\zeta_{sQ} = \xi_{sQ}/\xi_{ud}$, $\zeta_Q = \xi_{uQ}/\xi_{ud}$, *etc.*

			$c_{ij} = -\langle\lambda_i\lambda_j\sigma_i\sigma_j\rangle\zeta_{ij}$			$\langle V_{\rm CMI}\rangle$	Threshold	$\langle V_{\rm CMI}\rangle$
$qq\overline{QQ}$	$I(J^P)$	$qq\ \overline{QQ}$	c_{12}	c_{34}	c_{int}	lowest	2 mesons	
$qq\overline{QQ}$	$0(0^+)$	-						
	$0(1^+)$	$(0,\bar{\mathbf{3}})_0(1,\mathbf{\bar{3}})$	-8	$^{8}/_{3}\zeta_{cc}$	0	$-207[-188]$	$\overline{D}\overline{D}^*$	$-71[-23]$
		$(1,\mathbf{6})_0(0,\mathbf{6})$	$-^{4}/_{3}$	$4\zeta_{cc}$	0			
	$0(2^+)$	-					-	
	$1(0^+)$	$(1,\bar{\mathbf{3}})_1(1,\bar{\mathbf{3}})$	$^{8}/_{3}$	$^{8}/_{3}\zeta_{cc}$	$-^{32}/_{3}\zeta_c$	$-80[-27]$	$\overline{D}\overline{D}$	$-213[-69]$
		$(0,\mathbf{6})_1(0,\mathbf{6})$	4	$4\zeta_{cc}$	0			
	$1(1^+)$	$(1,\bar{\mathbf{3}})_1(1,\bar{\mathbf{3}})$	$^{8}/_{3}$	$^{8}/_{3}\zeta_{cc}$	$-^{16}/_{3}\zeta_c$	43[60]	$\overline{D}\overline{D}^*$	$-71[-23]$
	$1(2^+)$	$(1,\bar{\mathbf{3}})_1(1,\bar{\mathbf{3}})$	$^{8}/_{3}$	$^{8}/_{3}\zeta_{cc}$	$^{16}/_{3}\zeta_c$	114[83]	$\overline{D}^*\overline{D}^*$	71[23]
$qq\overline{QQ}'$	$0(0^+)$	$(0,\bar{\mathbf{3}})_0(0,\bar{\mathbf{3}})$	-8	$-8\zeta_{sc}$	0	$-569[-472/-275]$	$K\overline{D}$	$-405[-333/-141]$
		$(1,\mathbf{6})_0(1,\mathbf{6})$	$-^{4}/_{3}$	$-^{4}/_{3}\zeta_{sc}$	$-^{40}/_{3}(\zeta_s+\zeta_c)$			
	$0(1^+)$	$(0,\bar{\mathbf{3}})_0(1,\bar{\mathbf{3}})$	-8	$^{8}/_{3}\zeta_{sc}$	0	$-344[-363/-217]$	$K\overline{D}^*$	$-263[-287/-95]$
		$(1,\mathbf{6})_0(0,\mathbf{6})$	$-^{4}/_{3}$	$4\zeta_{sc}$	0			
		$(1,\mathbf{6})_0(1,\mathbf{6})$	$-^{4}/_{3}$	$-^{4}/_{3}\zeta_{sc}$	$-^{20}/_{3}(\zeta_s+\zeta_c)$			
	$0(2^+)$	$(1,\mathbf{6})_0(1,\mathbf{6})$	$-^{4}/_{3}$	$-^{4}/_{3}\zeta_{sc}$	$^{20}/_{3}(\zeta_s+\zeta_c)$	128[104/23]	$K^*\overline{D}^*$	135[111/47]
	$1(0^+)$	$(1,\bar{\mathbf{3}})_1(1,\bar{\mathbf{3}})$	$^{8}/_{3}$	$^{8}/_{3}\zeta_{sc}$	$-^{16}/_{3}(\zeta_s+\zeta_c)$	$-228[-184/-30]$	$K\overline{D}$	$-405[-333/-141]$
		$(0,\mathbf{6})_1(0,\mathbf{6})$	4	$4\zeta_{sc}$	0			
	$1(1^+)$	$(1,\bar{\mathbf{3}})_1(0,\bar{\mathbf{3}})$	$^{8}/_{3}$	$-8\zeta_{sc}$	0	$-155[-158/-9]$	$K\overline{D}^*$	$-263[-287/-95]$
		$(1,\bar{\mathbf{3}})_1(1,\bar{\mathbf{3}})$	$^{8}/_{3}$	$^{8}/_{3}\zeta_{sc}$	$-^{8}/_{3}(\zeta_s+\zeta_c)$			
		$(0,\mathbf{6})_1(1,\mathbf{6})$	4	$-^{4}/_{3}\zeta_{sc}$	0			
	$1(2^+)$	$(1,\bar{\mathbf{3}})_1(1,\bar{\mathbf{3}})$	$^{8}/_{3}$	$^{8}/_{3}\zeta_{sc}$	$^{8}/_{3}(\zeta_s+\zeta_c)$	150[126/95]	$K^*\overline{D}^*$	135[111/47]

[2] N. Brambilla, *et al.*, Eur. Phys. J. **C71**, 1534 (2011).

[3] R. Aaij *et al.* [LHCb Collaboration] Phys. Rev. Lett. **110** 222001, (2013).

[4] M. Takizawa, and S. Takeuchi, Prog. Theor. Exp. Phys. **2013** 0903D01, (2013).

[5] S. Takeuchi, M. Takizawa, and K. Shimizu, Few-Body Systems, Online First, DOI: 10.1007/s00601-013-0784-0.

[6] H. J. Lipkin, Phys. Lett. B **172**, 242 (1986).

[7] S. Zouzou, B. Silvestre-Brac, C. Gignoux and J. M. Richard, Z. Phys. C **30**, 457 (1986).

[8] S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh and A. Hosaka, Phys. Rev. D **86**, 034019 (2012).

[9] S. H. Lee and S. Yasui, Eur. Phys. J. C **64**, 283 (2009).

[10] Y. Ikeda, *et al.*, arXiv:1311.6214 [hep-lat].

[11] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).