

Mass spectra and Radiative transitions of heavy quarkonia and $b\bar{c}$ mesons

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We compute the mass spectroscopy based of heavy mesons ($b\bar{b}$, $b\bar{c}$ and $c\bar{c}$) using various confinement potential indices. Using the parameters fitted to get ground state masses of vector and pseudoscalar bottomonium and charmonium, we compute the mass spectroscopy of $b\bar{c}$. The masses and wave functions thus obtained are utilized to do a parameter free computation of radiative transition widths of these systems. The numerical results are compared with available experimental as well as theoretical values from other groups.

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1. Introduction

The spectroscopy and decay rates of quarkonia are quite important to study due to huge amount of high precision data acquired using number of experimental facilities viz BES at the Beijing Electron Positron Collider (BEPC), E835 at Fermilab, CLEO at the Cornell Electron Storage Ring (CESR), the B-meson factories, BaBar at PEP-II, Belle at KEKB, the CDF and D0 experiments at Fermilab, the Selex experiment at Fermilab, ZEUS and H1 at DESY, PHENIX and STAR at RHIC, NA60 at CERN [1]. New states and production mechanisms, new decays and transitions have been identified and even larger data samples are expected to come from the BES-III upgraded experiment, while the B factories and the Fermilab Tevatron will continue to supply valuable data for few years. New facilities like the LHC experiments at CERN, PANDA at GSI etc. will offer greater challenges and opportunities in the field [2].

2. Theoretical framework

One of the tests for the success of any theoretical model for mesons is the correct prediction of their decay rates. Many phenomenological models predict the masses correctly but overestimate the decay rates [3, 4]. We have successfully employed phenomenological harmonic potential scheme to compute masses of bound states of heavy quarkonia and the resulting parameters and wave functions have been used to study various decay properties [5].

In relativistic harmonic confinement model (RHM) with scalar plus vector potential for the quark confinement, coloured quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic oscillator potential. The RHM has been extended to accommodate multi-quark states from lighter to heavier flavour sectors with unequal quark masses [6, 7]. The mass of a hadron having p number of quarks in this extended RHM (ERHM) is expressed as [5, 6, 7]

$$M_N(q_1 q_2 \dots) = \sum_{i=1}^p \epsilon_N(q_i, p)_{conf} + \sum_{i<j=1}^p \epsilon_N(q_i q_j)_{coul} + \sum_{i<j=1}^p \epsilon_N^J(q_i, q_j)_{SD} \quad (2.1)$$

First term is the total confined energies of the constituting quarks of the hadron; Second sum corresponds to the residual colour coulomb interaction energy between the confined quarks and Third sum is due to spin dependent terms. The coulombic part of the energy is computed using the residual coulomb potential given by $V_{coul}(q_i q_j) = \frac{\alpha_s(\mu)}{\omega_n r}$, here ω_n represents the colour dielectric “coefficient” which is found to be state dependent [6], so as to get consistent coulombic contribution to the excited states of the hadrons. It is a measure of the confinement strength through the non-perturbative contributions to the confinement scale at the respective threshold energy of the quark-antiquark excitations.

The wave functions for quarkonia are constructed here by retaining the nature of single particle wave function but with a two particle size parameter $\Omega_N(q_i q_j)$ instead of $\Omega_N(q)$ [5, 6, 7]. The spin average (center of weight) masses of the $c\bar{c}$ and $b\bar{b}$ ground states are obtained by choosing the model parameters $m_c = 1.428$ GeV, $m_b = 4.637$ GeV, $k = 0.19252$ and the confinement parameter $A = 0.0685$ GeV^{3/2} [5].

From the centre of weight masses, the pseudoscalar and vector mesonic masses are computed by incorporating the residual two body chromomagnetic interaction through the spin-dependent

Table 1: Masses of charmonia in GeV/c^2

State	ERHM [6, 7]	CPP _v [3]				[8]	[18]	[13]	[12]	[9]
		v=0.5	v=1.0	v=1.5	v=2.0	PDG				
1^1S_0	2.985	3.000	2.950	2.912	2.882	2.980	2.981	2.990	2.979	2.982
2^1S_0	3.626	3.352	3.522	3.636	3.852	3.638	3.625	3.627	3.588	3.619
3^1S_0	4.047	3.541	3.912	4.212	4.436	–	4.032	–	3.991	4.053
1^3S_1	3.096	3.092	3.112	3.129	3.144	3.097	3.089	3.097	3.096	3.097
2^3S_1	3.690	3.375	3.583	3.739	3.852	3.686	3.666	3.685	3.686	3.686
3^3S_1	4.082	3.553	3.950	4.285	4.547	4.040	4.060	4.050	4.088	4.102

Table 2: Masses of bottomonia in GeV/c^2

State	ERHM [6, 7]	CPP _v [3]				[8]	[10]	[11]	[12]
		v=0.5	v=1.0	v=1.5	v=2.0	PDG			
1^1S_0	9.425	9.426	9.411	9.399	9.389	9.300	9.457	9.421	9.400
2^1S_0	10.012	9.696	9.826	9.924	9.995	9.999[23]	10.018	10.004	9.993
3^1S_0	10.319	9.824	10.088	10.334	10.529	–	10.380	10.350	10.328
1^3S_1	9.461	9.463	9.468	9.472	9.475	9.460	9.460	9.460	9.460
2^3S_1	10.027	9.702	9.841	9.951	10.032	10.023	10.023	10.024	10.023
3^3S_1	10.329	9.827	10.097	10.334	10.529	10.355	10.385	10.366	10.355

Table 3: Masses of B_c meson in GeV/c^2

State	ERHM [6, 7]	CPP _v [5]				[12]	[22]	[20]	[19]	[21]
		v=0.5	v=1.0	v=1.5	v=2.0					
1^1S_0	6.256	6.291	6.269	6.252	6.237	6.270	6.349	6.271	6.253	6.260
2^1S_0	6.929	6.582	6.743	6.860	6.935	6.835	6.821	6.855	6.867	6.850
3^1S_0	7.308	6.743	7.075	7.351	7.558	7.193	7.175	7.250	–	–
1^3S_1	6.314	6.330	6.337	6.344	6.348	6.332	6.373	6.338	6.317	6.340
2^3S_1	6.968	6.591	6.767	6.900	6.991	6.881	6.855	6.887	6.902	6.900
3^3S_1	7.326	6.747	7.089	7.379	7.601	7.235	7.213	7.272	–	–

term of the COGEP perturbatively as $\epsilon_N^J(q_i q_j)_{S.D.} = \langle NJ | V_{SD} | NJ \rangle$. The two body spin-hyperfine interaction contains two body spin-orbit interaction of the residual (effective) confined one gluon exchange potential (COGEP) used in our previous work [5, 6, 7]. The computed S -wave masses are given in Tables 1 – 3 in comparison with other theoretical outcomes and available experimental observations.

3. Radiative M1 Transitions

The CLEO-c experiment has measured the magnetic dipole (M1) transitions $J/\psi(1S) \rightarrow \gamma \eta_c(1S)$ and $J/\psi(2S) \rightarrow \gamma \eta_c(1S)$ using combination of inclusive and exclusive techniques rec-

Table 4: Radiative M1 transitions of bottomonia (eV)

Transition	ERHM[6, 7]	[16]	[17]	[11]	[12]	World Average[15]
$1^3S_1 \rightarrow 1^1S_0$	2.28 (36)	9.2	7.7 (59)	4.0	5.8 (60)	8.95
$2^3S_1 \rightarrow 2^1S_0$	0.168 (15)	0.6	0.53 (25)	0.5	1.40 (33)	1.51
$3^3S_1 \rightarrow 3^1S_0$	0.050 (10)	0.6	0.13 (16)	–	0.80 (27)	0.826
$2^3S_1 \rightarrow 1^1S_0$	212.9 (580)					

Table 5: Radiative M1 transitions of charmonia (keV)

Transition	ERHM[6, 7]	NR[18]	GI[18]	[15]	[8]
$1^3S_1 \rightarrow 1^1S_0$	2.41 (110)	2.90 (116)	2.4	1.5 ± 1.0	1.21 ± 0.37
$2^3S_1 \rightarrow 2^1S_0$	0.496 (62)	0.21 (48)	0.17		< 0.67
$3^3S_1 \rightarrow 3^1S_0$	0.085 (17)	0.046 (29)	–		–
$2^3S_1 \rightarrow 1^1S_0$	20.99 (654)				

onciling with theoretical calculations of lattice QCD and effective field theory techniques [14, 15]. M1 transition rates are normally weaker than E1 rates, but they are of more interest because they may allow access to spin-singlet states that are very difficult to produce otherwise. The spectroscopic parameters of extended harmonic confinement model which has been successful in prediction of masses quarkonia have been utilized for the present computations. In the non-relativistic limit, the M1 transition width is given by [15]

$$\Gamma_{n^3S_1 \rightarrow n'^1S_0\gamma} = \frac{4}{3} \frac{2J'+1}{2L+1} \delta_{LL'} \delta_{S,S'\pm 1} \alpha e_Q^2 \frac{k_\gamma^3}{m^2} \left| \int_0^\infty r^2 dr R_{n'0}(r) R_{n0}(r) j_0\left(\frac{k_\gamma r}{2}\right) \right|^2 \quad (3.1)$$

where e_Q is the fraction of electrical charge of the heavy quark ($e_b = -1/3$, $e_c = 2/3$), α is the fine structure constant and $R_{n0}(r)$ are the S -wave radial wave functions.

4. Conclusion

We have been able to do parameter free prediction of the radiative magnetic dipole transitions of heavy quarkonia and mass spectra of B_c meson. The photon energies depend on the model in most cases as their theoretical mass predictions are used for unknown states. For the low-energy favored M1 transitions, the photon energies are found to be nearly the same as the mass splittings. The computed mass of $\eta_b(2S)$ is found to be very close with recent experimental observation [23]. The wide variation in predicted hyperfine splittings leads to considerable uncertainty in predicted rates for these transitions. For the higher-energy hindered M1 transitions in bottomonia, the expected photon energies are not so sensitive to hyperfine splittings. The study of hindered transitions and relativistic corrections required for estimation of magnetic dipole radiative transitions for heavy and quarkonia is underway.

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