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Conformal dynamics in Nf=12 lattice QCD

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We study the infrared conformality of twelve-flavor QCD on the lattice, utilizing the Highly Improved Staggered Quark action, which realizes the simulations with minimal discretization error. The finite-size scaling test of the conformal hypothesis is performed for low-lying meson spectra. Our result is consistent with the conformal hypothesis for mass anomalous dimension $\gamma \sim 0.4-0.5$. Furthermore, the flavor-singlet scalar is found to be lighter than the pion, in sharp contrast to real-life QCD. This may be a hint to explain the Higgs boson mass as light as 125 GeV in walking technicolor.

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1. Introduction

Strongly coupled gauge theories near the conformal phase boundary are of great interest with regard to the walking technicolor model, having approximate scale invariance and a large anomalous dimension $\gamma \simeq 1$ [1]. The SU(3) gauge theory with N_f massless fermions in the fundamental representation (Large N_f QCD) is an attractive candidate of such a theory. In such a theory a light flavor singlet scalar could emerge as a pseudo-Nambu Goldstone boson of the spontaneously broken scale invariance, referred to as "technidilaton". Thus the new bosonic particle of mass $m_H = 125$ GeV discovered at the LHC [2, 3] could be explained in the context of the walking technicolor model. Large N_f QCD has been studied by many lattice groups with different lattice actions and methods to probe the conformal dynamics and search for a realistic walking technicolor model. For recent reviews, see Ref. [4] and references therein. We have been studying $N_f = 4, 8, 12$, and 16 QCD using lattice simulations with a common setup. In this report, we focus on the study of the conformal dynamics in $N_f = 12$ QCD.

If the theory is in the conformal phase, the hyperscaling in the mass-deformed conformal theory can be seen in various hadron spectra, such as the pseudoscalar mass M_π, its decay constant F_π, and the vector meson mass M_ρ. In Ref. [5] we found that $N_f = 12$ QCD has behavior consistent with hyperscaling, and the values of γ obtained from three quantities of M_π, F_π and M_ρ were reasonably consistent, with the exception of F_π at the finer lattice. This exception may be due to a sizable volume correction. In this report we provide a preliminary result with new data at a larger volume, $L = 36$, by which the consistency to the conformal hypothesis appears much clearer. As for the study of the flavor singlet scalar σ, we observe a mass lighter than the pseudoscalar in Ref. [6]. Although we regard the lightness of the scalar in $N_f = 12$ as due to the conformal dynamics, it is a promising signal for a walking theory, where similar conformal dynamics should be operative in such a way that the scale symmetry breaking originates from the dynamically generated fermion mass instead of the explicit breaking in the mass-deformed conformal theory.

In the followings, we explain the simulation setup and methods for a quantitative study of the (finite size) hyperscaling relations for the hadron spectra. We then examine a hyperscaling test of hadron spectra, including our preliminary result on larger volume. We also calculate the mass of the flavor singlet scalar state, from which we discuss an additional possible signal of the conformal dynamics.

2. Setup and primary result

The lattice gauge configurations are generated by the standard HMC algorithm using a tree-level Symanzik gauge action and the Highly Improved Staggered Quark action [7]. We simulate with various values for the fermion mass m_f on three volumes of $L = 24, 30, 36$ with fixed aspect ratio $T/L = 4/3$, at two lattice spacings of $\beta = 6/g^2 = 3.7$ and 4.0. Several hadron spectra such as the M_π, M_ρ and F_π are calculated. The flavor symmetry breaking effects in the staggered fermion are negligible for the theory in our simulation parameter region. For a primary study, dimensionless ratios composed of these measurements will be plotted against M_π. If the theory is in the conformal phase, the hadron mass M_p and its decay constant F_p in the infinite volume limit obey the conformal hyperscaling relations

$$M_p = c_p m_f^{1/(1+\gamma)}, \quad F_p = d_p m_f^{1/(1+\gamma)}, \tag{2.1}$$

where γ denotes the mass anomalous dimensions at the IR fixed point. Thus the spectra obtained in our lattice simulation will be tested against the conformal hypothesis.

The left panel of Fig.1 shows the ratio F_π/M_π at two bare gauge couplings $\beta = 3.7$ and 4.0. The ratio on the larger volumes becomes flat towards the smaller M_π region for both values of β. The M_π dependence of the ratio F_π/M_π at larger mass can be understood as a correction to the hyperscaling, which may be different from one quantity to another. Thus, the scaling region can only be seen in smaller mass and larger volume. In addition, the flat region at $\beta = 3.7$ seems to be wider than the one at $\beta = 4.0$, which can be made possible if the lattice spacing decreases as β increases, since in such a case the physical mass M_π could be lighter for $\beta = 3.7$ than $\beta = 4.0$. In fact, a crude analysis for the matching of two lattice spacings suggests the lattice spacing $a(\beta = 3.7)$ is larger than $a(\beta = 4.0)$, which is consistent with being in the asymptotically free domain. Similar observations can be made in the ratio M_ρ/M_π shown in the right panel of Fig.1. The flattening region is observed for both values of β, but the range is wider than for F_π/M_π. These results suggest that F_π has larger mass and volume corrections to the hyperscaling, which will be discussed in the next section.

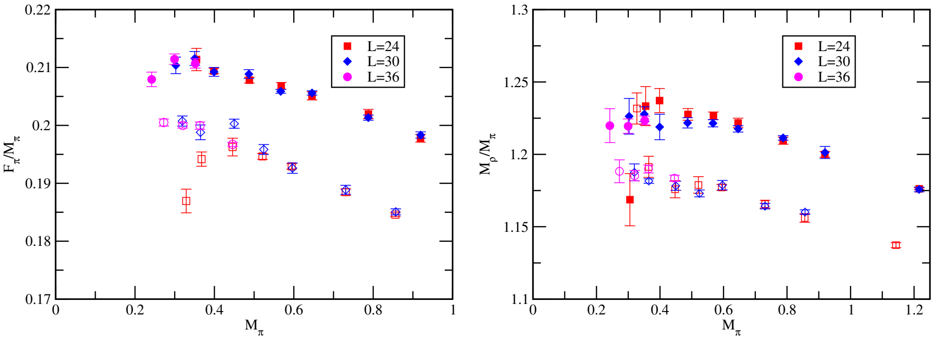

Figure 1: Dimensionless ratios F_π/M_π and M_ρ/M_π as functions of M_π for $N_f = 12$ at $\beta = 3.7$ (filled symbols) and 4.0 (open symbols).

3. Finite size hyperscaling analysis

In the case of a finite volume, the renormalization group tells us the scaling behavior for low-energy spectra: they should obey the universal scaling relations as

$$LM_p \equiv \xi_p = f_p(x), \quad LF_\pi \equiv \xi_F = f_F(x), \tag{3.1}$$

where $f_p(x)$ and $f_F(x)$ are functions of the scaling variable $x = Lm_f^{1/(1+\gamma)}$, and these functions are unknown in general[1]. Here we introduce an evaluation function $P(\gamma)$ to quantify how much the data can be matched to the function of x. Take ξ^j be a data for a measured observable p at $x_j = L_j m_j^{1/(1+\gamma)}$ and $\delta\xi^j$ to be the error of ξ^j. The label j identifies the set of parameters (L, m_f). Now let K be a subset of data points $\{(x_k, \xi_k)\}$, from which we construct a function $f^{(K)}(x)$ that represents the subset of data (K). The evaluation function is defined as

$$P(\gamma) = \frac{1}{\mathcal{N}} \sum_L \sum_{j \notin K_L} \frac{\left|\xi^j - f^{(K_L)}(x_j)\right|^2}{|\delta\xi^j|^2}, \tag{3.2}$$

[1] For a review of hyperscaling behavior, see, e.g., Refs. [8, 9].

where L runs through all the lattice sizes we have, the sum over j is taken for a set of data points which do not belong to K_L, which includes all the data obtained on the lattice with size L. $\mathscr{N}$ denotes the total number of summations, the combinatorial number of L and $j \notin K_L$. Here, we choose for the function $f^{(K_L)}$ a linear interpolation of the data points of the fixed lattice size L. In the evaluation function in Eq. (3.2), the data points need to be taken for a range of $x = L \cdot m_f^{1/(1+\gamma)}$ in which there is an overlap of available data for all volumes within the x-range.

We calculate the evaluation function $P(\gamma)$ for the three observables M_π, F_π, and M_ρ, at two lattice spacings $\beta = 3.7$, and 4.0 on three volumes $L = 24, 30, 36$. We take the value of $x_{\rm min}$ ($x_{\rm max}$) as the smallest (largest) m_f for the largest volume $L = 36$, that is, the minimum (maximum) value of m_f is 0.03 (0.05) for $\beta = 3.7$ and 0.04 (0.08) for $\beta = 4.0$. The left panel of Fig. 2 shows the result of $P(\gamma)$ for the three observables at $\beta = 3.7$. We observe a minimal at which the optimal alignment of the data is achieved. In order to systematically study how the range of fermion mass affects the result, we consider three windows of the data for the combination of two data sets among $L = 24, 30$, and 36. The results with all the errors added in quadrature are summarized in the right panel of Fig. 2. In case of F_π, we have a large error in both $\beta = 3.7$ and 4.0 due to finite volume (and mass) effects. If one restricts the volume range to the larger side, then the value of $\gamma(F_\pi)$ becomes closer to that for the other observables. We also observe a β dependence of $\gamma(F_\pi)$, where the value for $\beta = 3.7$ is closer to the one for other observables, while the values for M_π and M_ρ are stable against changes of β and volume. These results may be understood from the observation in previous section, that the scaling, in particular for F_π/M_π, is observed only in the small mass and larger volume region. As a result, all the values are consistent with each other within 2σ. From these analyses, we conclude that our data for the $N_f = 12$ theory are reasonably consistent with the finite size hyperscaling. The resulting γ from different quantities and two lattice spacings is also reasonably consistent.

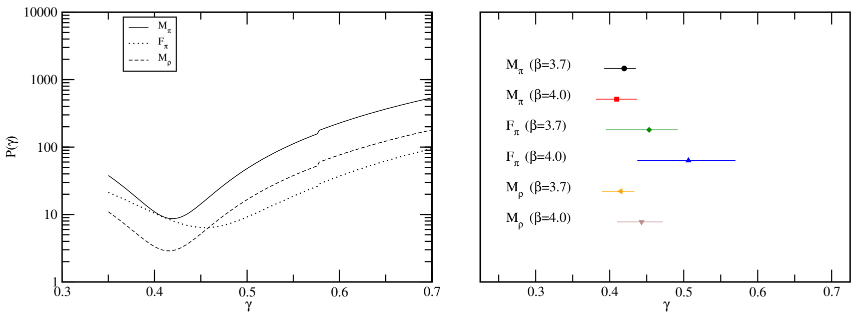

Figure 2: (Left): The γ dependence of the evaluation function P for M_π, F_π, and M_ρ at $\beta = 3.7$. The vertical axis shows the central values of P using the data for the three volumes. (Right): The values of γ for three observables at two β values are summarized, where the statistical and systematic errors are added in quadrature. All the results are consistent with each other within 2σ.

4. Flavor singlet scalar mass

Here we explain the mass of the flavor singlet scalar obtained from fermionic bilinear operators in detail. We use the local staggered fermion bilinear operator $O_S(t) = \sum_{i=1}^{3} \sum_x \bar{\chi}_i(x,t)\chi_i(x,t)$, where i labels the staggered species. Using this operator we measure the two-point correlation function $\langle O_S(t)O_S(0)\rangle \propto 3D(t)-C(t)$, where $C(t)$ and $D(t)$ are connected and the vacuum subtracted disconnected corelators, respectively. The disconnected correlator $D(t)$ can be calculated by inverting the staggered Dirac operator at each space-time point. The computational cost of $D(t)$ is mitigated by using a stochastic noise method with a variance reduction technique which has previously been employed for other measurements [10, 11, 12, 13]. To extract the mass of the flavor singlet scalar M_σ, we fit $D(t)$ in the range $t = 4$ to 8 for all the parameters. A systematic error is estimated by the difference of central values obtained with several fit ranges. We carry out the measurement of the scalar correlation functions at $\beta = 4.0$ on three physical volumes $L = 24, 30, 36$ at four different masses $m_f = 0.05, 0.06, 0.08$, and 0.10. For all the simulation parameters we have used more than 4,000 configurations. The details of our results can be found in Ref. [6].

The observed m_f dependence of M_σ is shown in Fig. 3. For M_σ on the largest two volumes at each m_f finite size effects are negligible in our statistics. For a check of consistency with the hyperscaling, we fit M_σ on the largest volume data at each m_f using the hyperscaling form in the infinite volume limit with a fixed $\gamma = 0.414$ which is consistent with the one obtained from M_π. The fit result is also shown in Fig. 3, and gives a reasonable value of $\chi^2/\mathrm{DOF} = 0.12$. This is consistent with the theory having infrared conformality. It is also found that the ratio M_σ/M_π for each m_f is smaller than unity. Such a light state can also be seen in the gluonic operators at small fermion mass, as shown in Fig. 3. We regard the light scalar state observed for $N_f = 12$ as a reflection of the dilatonic nature of the conformal dynamics, since otherwise the p-wave bound state (scalar) is expected to be heavier than the s-wave one (pseudoscalar). The scalar mass in $N_f = 12$ QCD sharply contrasts with real-life QCD.

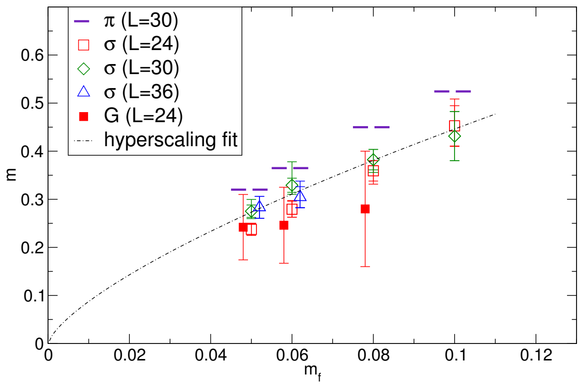

Figure 3: Mass of the flavor singlet scalar meson σ compared to the mass of the pseudoscalar and the mass from gluonic operators G. Errors are statistical and systematics added in quadrature. The hyperscaling curve is described in the text. The triangle and filled square symbols are slightly shifted for clarity.

5. Summary

We have studied the conformal dynamics in $N_f = 12$ QCD on the lattice. Our present data for M_π, F_π and M_ρ are consistent with the conformal hypothesis. The mass anomalous dimension was estimated through the finite size hyperscaling analysis; Our result, $\gamma \sim 0.4-0.5$, is not as large as $\gamma \sim 1$, which is required for a realistic technicolor model. We have also performed the first study of the flavor singlet scalar state. The most striking feature of the measured scalar spectrum is the appearance of a state lighter than the pseudoscalar state. We infer that the lightness of the scalar state is due to IR conformality. This result sheds some light on the possibility of a light composite Higgs boson in walking technicolor theories. It is thus interesting to investigate the scalar in $N_f = 8$ QCD, which was shown to be a good candidate for a walking technicolor model [14]. An indication of such a light scalar in $N_f = 8$ QCD has already been observed [15].

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