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Quark and gluon spin-2 form factors to two loops in QCD

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We present two-loop QCD corrections to the quark form factor from the amplitude $G^* \to q\bar{q}$ and the gluon form factor from $G^* \to gg$ in SU(N) gauge theory with n_f light flavours. We use d-dimensional regularisation and the results are presented to all orders in $\varepsilon = d - 4$. These form factors can be used to obtain next-to-next-to-leading order QCD corrections to hadronic scattering processes in large extra-dimension model where Kaluza-Klein graviton modes couple to Standard Model fields. We find that the form factors satisfy Sudakov integro-differential equation and the resulting cusp, collinear and soft anomalous dimensions are identical to those of electroweak vector boson and gluon form factors. We establish the universal behaviour of the infrared singularities in accordance with the proposal by Catani.

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1. Introduction

One of the solutions to the hierarchy problem in the Standard Model (SM) comes from models with extra dimensions. Two popular models are (1) the flat extra dimension model (ADD) [1] and (2) the warped extra dimension model (RS) with a large curvature [2]. The spin-2, Kaluza-Klein (KK) modes of the gravitons couple to the SM particles through the energy-momentum tensor of the SM. Such a coupling can change SM predictions and groups in both the ATLAS [3] and CMS [4] collaborations have been studying various processes such as di-lepton, di-photon, mono-jet, mono-photon production etc to constrain the parameters of these models. In the theoretical context, the scales resulting from renormalisation and factorisation in the hadronic cross-sections provide estimates of uncertainties of the uncalculated higher order corrections. Next-to-leading order (NLO) QCD calculations are known for di-lepton [5], di-photon [6] and di-electroweak gauge boson [7] production via virtual KK modes in addition to the SM contributions. In the aMC@NLO framework, the results to NLO+PS accuracy are also available [8]. In all these processes the factorisation scale dependence reduces substantially when we add NLO contribtuions but the renormalisation scale dependence starts only at NLO in QCD. Hence the next-to-next-to-leading order (NNLO) is warranted. Such a computation involves the knowledge of graviton-quark-antiquark, $G^* \to q\overline{q}$, and graviton-gluon-gluon, $G^* \to gg$, form factors up to the two-loop level in QCD as well as double real emission and one-loop single real emission scattering processes at the parton level.

The action that describes the interaction of SM fields with the KK modes of gravity is obtained coupling the graviton fields with the energy-momentum tensor of the SM. Here, we restrict ourselves to the QCD part of the energy-momentum tensor:

$$\mathscr{S} = \mathscr{S}_{SM} - \frac{\kappa}{2} \int d^4x \, T^{QCD}_{\mu\nu}(x) \, h^{\mu\nu}(x) \,, \tag{1.1}$$

where κ is the coupling and $T^{QCD}_{\mu\nu}$ is the energy-momentum tensor of QCD [5].

The bare form factors are evaluated using the truncated matrix elements $\hat{\mathscr{M}}_I$ of $T^{QCD}_{\mu\nu}$ between on-shell gluon ($I = g$) and quark/anti-quark ($I = q, \overline{q}$) states:

$$\hat{\mathscr{M}}_I = \hat{\mathscr{M}}_I^{(0)} + \hat{a}_s \left(\frac{Q^2}{\mu^2}\right)^{\frac{\varepsilon}{2}} S_\varepsilon \hat{\mathscr{M}}_I^{(1)} + \hat{a}_s^2 \left(\frac{Q^2}{\mu^2}\right)^{\varepsilon} S_\varepsilon^2 \hat{\mathscr{M}}_I^{(2)} + \mathscr{O}\left(\hat{a}_s^3\right) , I = g, q, \overline{q} \,, \tag{1.2}$$

where the unrenormalised coupling constant $\hat{a}_s = \hat{g}_s^2/16\pi^2$ and the space-time dimension is $d = 4+\varepsilon$. The scale $Q^2 = -q^2 - i\varepsilon$, where q is the momentum transfer. $\hat{a}_s$ is related to the renormalised one, $a_s(\mu_R^2)$, by the renormalisation constant $Z(\mu_R^2)$, with μ_R-the renormalisation scale. The form factors are defined as

$$\hat{F}_I^{T,(n)} = \frac{\hat{\mathscr{M}}_I^{(0)*} \cdot \hat{\mathscr{M}}_I^{(n)}}{\hat{\mathscr{M}}_I^{(0)*} \cdot \hat{\mathscr{M}}_I^{(0)}} \,, \tag{1.3}$$

and the symbol $\cdot$ takes care of the color and spin/polarisation sums.

The Feynman amplitudes are generated using the computer program, QGRAF [9]. We find 12 one-loop and 153 two-loop diagrams that contribute for the external gluon states while 4 one-loop and 54 two-loop diagrams for quark-antiquark states. Using in-house FORM [10] routines we have converted the output of QGRAF to a form suitable for further manipulations. Reduction of

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loop tensor integrals to a set of a few master integrals was achieved by FIRE [11], a Mathematica package, which extensively uses the IBP [12] and LI [13] identities implemented using Laporta's algorithm [14]. We used LiteRed [15] to cross-check our results obtained using FIRE. The one-loop and two-loop master integrals are now known to all orders in ε and are given in [16]. The results for $\hat{F}_I^{T,(n)}$ for $I = g,q; n = 1,2$ can be found in [17] expressed in d and also in power series expansions in $\varepsilon = d-4$.

2. Infrared divergence structure

In this section, we study the infrared pole structure of the form factors to establish the universal behaviour of these QCD amplitudes. Quark and gluon form factors have been studied through Sudakov integro-differential equations [18, 19, 20, 21, 22], see also [23, 24, 25, 26, 27]. We find that the unrenormalised form factors, $\hat{F}_I^T(\hat{a}_s, Q^2, \mu^2, \varepsilon)$, satisfy a similar integro-differential equation:

$$Q^2 \frac{d}{dQ^2} \ln \hat{F}_I^T\left(\hat{a}_s, Q^2, \mu^2, \varepsilon\right) = \frac{1}{2}\left[K^{T,I}\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) + G^{T,I}\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right)\right], \tag{2.1}$$

where the $K^{T,I}$ depend only on the poles in ε, and the $G^{T,I}$ are finite as ε becomes zero. From renormalisation group (RG) invariance,

$$\mu_R^2 \frac{d}{d\mu_R^2} K^{T,I}\left(\hat{a}_s, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) = -A^{T,I}(a_s(\mu_R^2)),$$

$$\mu_R^2 \frac{d}{d\mu_R^2} G^{T,I}\left(\hat{a}_s, \frac{Q^2}{\mu_R^2}, \frac{\mu_R^2}{\mu^2}, \varepsilon\right) = A^{T,I}(a_s(\mu_R^2)). \tag{2.2}$$

The quantities $A^{T,I}$ are the cusp anomalous dimensions. The solutions take the form

$$\ln \hat{F}_I^T(\hat{a}_s, Q^2, \mu^2, \varepsilon) = \sum_{i=1}^{\infty} \hat{a}_s^i \left(\frac{Q^2}{\mu^2}\right)^{i\frac{\varepsilon}{2}} S_\varepsilon^i \, \hat{\mathscr{L}}_{F^T}^{I(i)}(\varepsilon), \tag{2.3}$$

where

$$\hat{\mathscr{L}}_{F^T}^{I(1)} = \frac{1}{\varepsilon^2}\left(-2A_1^{T,I}\right) + \frac{1}{\varepsilon}\left(G_1^{T,I}(\varepsilon)\right),$$

$$\hat{\mathscr{L}}_{F^T}^{I(2)} = \frac{1}{\varepsilon^3}\left(\beta_0 A_1^{T,I}\right) + \frac{1}{\varepsilon^2}\left(-\frac{1}{2}A_2^{T,I} - \beta_0 G_1^{T,I}(\varepsilon)\right) + \frac{1}{2\varepsilon} G_2^{T,I}(\varepsilon). \tag{2.4}$$

The cusp anomalous dimensions $A_i^{T,I}$ can be extracted from the computed form factors. We find that they coincide with those obtained in [28]. The $G_i^{T,I}(\varepsilon)$ are found to take the following form

$$G_1^{T,I}(\varepsilon) = 2\,B_1^{T,I} + f_1^{T,I} + \sum_{k=1}^{\infty} \varepsilon^k g_1^{T,I,k},$$

$$G_2^{T,I}(\varepsilon) = 2\,B_2^{T,I} + f_2^{T,I} - 2\beta_0 g_1^{T,I,1} + \sum_{k=1}^{\infty} \varepsilon^k g_2^{T,I,k}, \tag{2.5}$$

where again the collinear anomalous dimensions $B_i^{T,I}$ and soft anomalous dimensions $f_i^{T,I}$ are found to be identical to B_i^I and f_i^I obtained in [29, 23] for quark and gluon form factors. The constants $g_i^{T,I,k}$ are found to depend on the operator.

Since the form factors are obtained from the amplitudes, we can relate these form factors with the general structure of QCD amplitudes. This was a very successful proposal by Catani [30] (also see [31]) for one and two-loop QCD amplitudes in terms of the universal factors $\mathbf{I}_I^{(i)}(\varepsilon)$ and $\mathbf{H}_I^{(i)}$, $i=1,2$. For an all order generalisation of Catani's proposal, we refer to the works by Becher and Neubert [32] and also by Gardi and Magnea [33].

Following [30], the matrix elements can be expressed in terms of UV renormalised ones as

$$\hat{\mathscr{M}}_I = \mathbf{M}_I^{(0)} + a_s(\mu_R^2)\mathbf{M}_I^{(1)} + a_s^2(\mu_R^2)\mathbf{M}_I^{(2)} + \mathscr{O}\left(a_s^3(\mu_R^2)\right), \qquad I = g,q,\bar{q}. \tag{2.6}$$

In terms of the universal $\mathbf{I}_I(\varepsilon)$ obtained by Catani, the amplitudes take the form

$$\begin{aligned}
\mathbf{M}_I^{(1)} &= 2\mathbf{I}_I^{(1)}(\varepsilon)\mathbf{M}_I^{(0)}(\varepsilon) + \mathbf{M}_{I,fin}^{(1)}(\varepsilon),\\
\mathbf{M}_I^{(2)} &= 2\mathbf{I}_I^{(1)}(\varepsilon)\mathbf{M}_I^{(1)}(\varepsilon) + 4\mathbf{I}_I^{(2)}(\varepsilon)\mathbf{M}_I^{(0)}(\varepsilon) + \mathbf{M}_{I,fin}^{(2)}(\varepsilon).
\end{aligned} \tag{2.7}$$

In terms of these $\mathbf{M}_I^{(i)}$, we find

$$\begin{aligned}
\hat{F}_I^{T,(1)} &= 2\mu_R^{\varepsilon}\mathbf{I}_I^{(1)}(\varepsilon) + \hat{F}_{I,fin}^{T,(1)}(\varepsilon),\\
\hat{F}_I^{T,(2)} &= 4\mu_R^{2\varepsilon}\Bigg[\left(\mathbf{I}_I^{(1)}(\varepsilon)\right)^2 + \mathbf{I}_I^{(2)}(\varepsilon) - \frac{\beta_0}{\varepsilon}\left(\mathbf{I}_I^{(1)}(\varepsilon) + \frac{\mu_R^{-\varepsilon}}{2}\hat{F}_{I,fin}^{T,(1)}(\varepsilon)\right)\\
&\quad + \frac{1}{2}\mu_R^{-\varepsilon}\mathbf{I}_I^{(1)}(\varepsilon)\hat{F}_{I,fin}^{T,(1)}(\varepsilon)\Bigg] + \hat{F}_{I,fin}^{T,(2)}(\varepsilon),
\end{aligned} \tag{2.8}$$

where

$$\hat{F}_{I,fin}^{T,(i)}(\varepsilon) = \mu_R^{i\varepsilon}\frac{\mathbf{M}_I^{(0)*}\cdot\mathbf{M}_{I,fin}^{(i)}}{\mathbf{M}_I^{(0)*}\cdot\mathbf{M}_I^{(0)}}, \qquad i=1,2. \tag{2.9}$$

The functions $\mathbf{I}_I^{(i)}$ are given by

$$\begin{aligned}
\mathbf{I}_q^{(1)}(\varepsilon) &= -\frac{e^{-\varepsilon\gamma_E/2}}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\left(\frac{Q^2}{\mu_R^2}\right)^{\frac{\varepsilon}{2}}\left(4\frac{C_F}{\varepsilon^2} - 3\frac{C_F}{\varepsilon}\right),\\
\mathbf{I}_g^{(1)}(\varepsilon) &= -\frac{e^{-\varepsilon\gamma_E/2}}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\left(\frac{Q^2}{\mu_R^2}\right)^{\frac{\varepsilon}{2}}\left(4\frac{C_A}{\varepsilon^2} - \frac{\beta_0}{\varepsilon}\right),
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
\mathbf{I}_I^{(2)}(\varepsilon) &= -\frac{1}{2}\left(\mathbf{I}_I^{(1)}(\varepsilon)\right)^2 + \frac{\beta_0}{\varepsilon}\mathbf{I}_I^{(1)}(\varepsilon)\\
&\quad + \frac{e^{\frac{\varepsilon\gamma_E}{2}}\Gamma(1+\varepsilon)}{\Gamma\left(1+\frac{\varepsilon}{2}\right)}\left(-\frac{\beta_0}{\varepsilon} + K\right)\mathbf{I}_I^{(1)}(2\varepsilon) + \mathbf{H}_I^{(2)}\frac{1}{\varepsilon},
\end{aligned} \tag{2.11}$$

and

$$K = \left(\frac{67}{18} - \zeta_2\right) C_A - \frac{10}{9} T_F n_f \,. \tag{2.12}$$

Using our results for $\hat{F}^{T,(i)}$ and the results for $\mathbf{I}_I^{(i)}$ given in [30], we obtain $\mathbf{H}_I^{(2)}$:

$$\begin{aligned}
\mathbf{H}_g^{(2)} &= C_A^2\left(-\frac{5}{12} - \frac{11}{24}\zeta_2 - \frac{1}{2}\zeta_3\right) + C_A n_f\left(\frac{29}{27} + \frac{1}{12}\zeta_2\right) + C_F n_f\left(-\frac{1}{2}\right) + n_f^2\left(-\frac{5}{27}\right), \\
\mathbf{H}_q^{(2)} &= C_F^2\left(\frac{3}{8} - 3\zeta_2 + 6\zeta_3\right) + C_A C_F\left(-\frac{245}{216} + \frac{23}{8}\zeta_2 - \frac{13}{2}\zeta_3\right) + C_F n_f\left(\frac{25}{108} - \frac{1}{4}\zeta_2\right).
\end{aligned} \tag{2.13}$$

These coefficients are found to agree with the color diagonal part of eqn.(12) of [32] (see also eqn.(4.21) of [23] for quark and gluon form factors and [34, 35, 36, 37] for four parton amplitudes). This establishes the proposal by Catani on IR universality of QCD amplitudes with $T_{\mu\nu}$ insertion.

3. Conclusions

In this article, we have computed quark and gluon form factors of the energy-momentum tensor of the SM up to two loop level in perturbative QCD. This is a first step towards the full NNLO QCD correction to graviton mediated hadronic scattering processes. The computation uses dimensional regularisation and the results are presented for $SU(N)$ gauge theory with n_f light flavours. The computation involves use of IBP identities and LI identities. Due to the availability of master integrals with full d dependence for the present kinematics, we could obtain the form factors with d dependence. This will be useful for the ultraviolet renormalisation of these amplitudes at three-loop level. We have also studied the infrared structure of the amplitudes at two loop level. We showed that these form factors satisfy Sudakov integro-differential equations with the same cusp A_I, collinear B^I and soft f^I anomalous dimensions that appear in electroweak vector boson and gluon form factors. In addition, we establish the universal behaviour of the infrared poles in ε in accordance with the proposal by Catani.

Our results on spin-2 form factors will be useful for the study of spin-2 resonance productions in the context of the Higgs [38] and BSM models [39]. For the impact of our results on predicting threshold corrections to di-lepton production up to NNLO level QCD in the TeV scale gravity models we refer to [40].

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