

COLLIER – A fortran-library for one-loop integrals

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We introduce the fortran-library COLLIER for the numerical evaluation of one-loop scalar and tensor integrals in perturbative relativistic quantum field theories. Important features are the implementation of dedicated methods to achieve numerical stability for 3- and 4-point tensor integrals, the support of complex masses for internal particles, and the possibility to choose between dimensional and mass regularization for infrared singularities. COLLIER supports one-loop N -point functions up to currently $N = 6$ and has been tested in various NLO QCD and EW calculations.

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1. Introduction

Next-to-leading order (NLO) predictions for processes induced by strong (QCD) and electroweak (EW) interactions are a basic ingredient for the analysis of high-energy collider experiments. In the past years many automatic tools based on different methods have been developed for the calculation of QCD corrections [1, 2], and an NLO generator for EW corrections has been constructed recently [3]. While in unitarity-based methods [4] a one-loop amplitude is directly expressed in terms of a set of basic scalar integrals, the traditional Feynman-diagrammatic approach as well as recently developed recursive methods [2, 3, 5] rely instead on tensor integrals. For the reduction of tensor integrals to scalar integrals various methods have been invented and refined over the past decades [6, 7, 8, 9], resulting in several libraries that are available for the calculation of one-loop scalar and tensor integrals [10]. In this article we introduce COLLIER, a Complex One-Loop Library in Extended Regularisations. Its particular strengths are the numerically stable calculation of 3- and 4-point tensor integrals owing to the implementation of sophisticated expansion methods for critical phase-space regions, the support of complex masses for internal particles, and the possibility to treat infrared singularities either via dimensional or via mass regularisation. Tensor integrals for 5-point and 6-point functions are reduced with methods that do not involve inverse Gram determinants. The library has already been applied successfully to many complex NLO QCD and EW calculations, among others to the processes [11, 12] $e^+e^- \rightarrow WW \rightarrow 4\text{fermions}$, $H \rightarrow 4\text{fermions}$, $pp \rightarrow t\bar{t}b\bar{b}$, $pp \rightarrow WWb\bar{b}$, $pp \rightarrow t\bar{t} + 2\text{jets}$, and $pp \rightarrow \ell\ell + 2\text{jets}$. It is integrated in the NLO generators OPENLOOPS [2] and RECOLA [3] and the publication of the code is in preparation [13].

2. Representation of tensor integrals

A one-loop N -point tensor integral of rank P has the general form

$$T^{N,\mu_1\dots\mu_P}(p_1,\dots,p_{N-1},m_0,\dots,m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^Dq \frac{q^{\mu_1}\dots q^{\mu_P}}{N_0N_1\dots N_{N-1}}. \quad (2.1)$$

The denominator factors are given by

$$N_k = (q + p_k)^2 - m_k^2 + i\delta, \quad k = 0, \dots, N-1, \quad p_0 = 0, \quad (2.2)$$

where p_k and m_k are the momentum and the mass of the particle in the corresponding loop-propagator and $i\delta$ ($\delta > 0$) is an infinitesimal imaginary part. While COLLIER accepts only real values for the four-momenta p_k , it permits complex values for the masses m_k . Thus, it can be applied to calculations in which propagators of unstable particles are regularised by a complex mass prescription [11, 14]. Lorentz covariance allows to decompose a tensor integral as

$$T^{N,\mu_1\dots\mu_P} = \sum_{n=0}^{\lfloor \frac{P}{2} \rfloor} \sum_{i_{2n+1}, \dots, i_P=1}^{N-1} \underbrace{\{g \dots g p \dots p\}}_n{}^{\mu_1\dots\mu_P}_{i_{2n+1}\dots i_P} T_{i_{2n+1}\dots i_P}^{N,0\dots 0}{}_{2n} \quad (2.3)$$

where $[P/2]$ is the largest integer number smaller or equal to $P/2$ and where the basic tensor structures are recursively defined according to

$$\begin{aligned} \{p \dots p\}_{i_1 \dots i_P}^{\mu_1 \dots \mu_P} &= p_{i_1}^{\mu_1} \dots p_{i_P}^{\mu_P}, \\ \underbrace{\{g \dots g p \dots p\}}_n^{\mu_1 \dots \mu_P}_{i_{2n+1} \dots i_P} &= \frac{1}{n} \sum_{\substack{k,l=1 \\ k < l}}^P g^{\mu_k \mu_l} \underbrace{\{g \dots g p \dots p\}}_{n-1}^{\mu_1 \dots \mu_{k-1} \mu_{k+1} \dots \mu_{l-1} \mu_{l+1} \dots \mu_P}. \end{aligned} \quad (2.4)$$

Since the tensor $T^{N, \mu_1 \dots \mu_P}$ is totally symmetric, the Lorentz-invariant coefficients $T_{0 \dots 0 i_{2n+1} \dots i_P}^N$ are symmetric in i_{2n+1}, \dots, i_P .

Ultraviolet- (UV-) or infrared- (IR-) singular integrals are represented in dimensional regularisation, where $D = 4 - 2\epsilon$, as

$$T^N = T_{\text{fin}}^N(\mu_{\text{UV}}^2, \mu_{\text{IR}}^2) + a^{\text{UV}} \Delta_{\text{UV}} + a_2^{\text{IR}} \left(\Delta_{\text{IR}}^{(2)} + \Delta_{\text{IR}}^{(1)} \ln \mu_{\text{IR}}^2 \right) + a_1^{\text{IR}} \Delta_{\text{IR}}^{(1)}, \quad (2.5)$$

with

$$\Delta_{\text{UV}} = \frac{c(\epsilon_{\text{UV}})}{\epsilon_{\text{UV}}}, \quad \Delta_{\text{IR}}^{(2)} = \frac{c(\epsilon_{\text{IR}})}{\epsilon_{\text{IR}}^2}, \quad \Delta_{\text{IR}}^{(1)} = \frac{c(\epsilon_{\text{IR}})}{\epsilon_{\text{IR}}}. \quad (2.6)$$

Note that we distinguish between singularities resulting from the IR and from the UV domain and that we absorb a term $c(\epsilon) = \Gamma(1 + \epsilon)(4\pi)^\epsilon$ in the constants Δ_{UV} , $\Delta_{\text{IR}}^{(2)}$ and $\Delta_{\text{IR}}^{(1)}$. COLLIER provides numerical results for the complete integrals T^N , i.e. for the sum of the finite part $T_{\text{fin}}^N(\mu_{\text{UV}}^2, \mu_{\text{IR}}^2)$ and the a^{UV} -, a_2^{IR} - and a_1^{IR} -terms. The user can assign arbitrary values to the unphysical mass scales μ_{UV}^2 , μ_{IR}^2 as well as to the constants Δ_{UV} , $\Delta_{\text{IR}}^{(2)}$, and $\Delta_{\text{IR}}^{(1)}$, which have to drop out in UV- and IR-finite quantities. Varying these parameters allows to check numerically the cancellation of singularities.

UV- and IR-singular integrals are by default calculated in dimensional regularisation. Collinear singularities can also be regularised with small masses. To this end, masses must be declared *small* in the initialisation together with corresponding (not necessarily small) numerical values. The *small* masses are treated as infinitesimally small in the scalar and tensor functions, and only in mass-singular logarithms the finite values are kept.

A general one-loop amplitude $\delta\mathcal{M}$ can be written in terms of tensor integrals as

$$\delta\mathcal{M} = \sum_j \sum_{P_j} c_{\mu_1 \dots \mu_{P_j}}^j T_j^{N_j, \mu_1 \dots \mu_{P_j}} = \sum_j \sum_{P_j} \sum_{n=0}^{\lfloor \frac{P_j}{2} \rfloor} \sum_{i_{2n+1}, \dots, i_{P_j}=1}^{N-1} \underbrace{\tilde{c}_{0 \dots 0 i_{2n+1} \dots i_{P_j}}^j}_{2n} T_j^{N_j}_{j, 0 \dots 0 i_{2n+1} \dots i_{P_j}}, \quad (2.7)$$

where j runs over all appearing tensor integrals with rank P_j and N_j propagators. Traditional calculations rely on the representation of $\delta\mathcal{M}$ in terms of the $T_j^{N_j}_{j, i_1 \dots i_{P_j}}$ and perform algebraic manipulations of the corresponding coefficients $\tilde{c}_{i_1 \dots i_{P_j}}^j$ in D dimensions. New methods inspired by Ref. [5] and implemented in the automatic NLO generators OPENLOOPS [2] and RECOLA [3], on the other hand, make use of the representation in terms of the full tensors $T_j^{N_j, \mu_1 \dots \mu_{P_j}}$ and perform a recursive numerical calculation of the respective coefficients $c_{\mu_1 \dots \mu_{P_j}}^j$. COLLIER can be used in either of these approaches as it provides the Lorentz-covariant coefficients $T_j^{N_j}_{j, i_1 \dots i_{P_j}}$ as well as the full tensors $T_j^{N_j, \mu_1 \dots \mu_{P_j}}$.

3. Implemented methods

The method used to evaluate a tensor integral depends on the number N of its propagators. For $N = 1, 2$, explicit numerically stable expressions are employed [6, 9].

For $N = 3, 4$, scalar integrals are calculated using analytical expressions as given in Ref. [15], while tensor integrals $T^{N,P}$ of higher rank P by default are numerically reduced to integrals of lower rank $T^{N,P-1}$, $T^{N,P-2}$ and to integrals with a lower number of propagators T^{N-1} via standard Passarino–Veltman reduction. Schematically this can be written as

$$\Delta T^{N,P} = [T^{N,P-1}, T^{N,P-2}, T^{N-1}], \quad (3.1)$$

where [...] denotes a linear combination of the corresponding terms and the determinant $\Delta = \det(Z)$ of the Gram matrix $Z_{ij} = 2p_i p_j$ has been made explicit on the left-hand side. In certain regions of the phase-space the Gram determinant Δ can become small, so that the numerical solution of (3.1) gets unstable. This problem reflects the ambiguity of the representation of $T^{N,P}$ in terms of the integrals on the right-hand side which tend to become linearly dependent in this case. Since even the scalar integrals become dependent, this problem is intrinsic to all reduction methods relying on the full set of basic scalar integrals, i.e. it affects unitarity-based approaches as well. In the tensor reduction method, on the other hand, spurious Gram singularities can be avoided for delicate phase-space points by adjusting the strategy of solving the system of linear equations obtained from the Passarino–Veltman algorithm. Consider to this end (3.1) for $P \rightarrow P+1$,

$$\Delta T^{N,P+1} = [T^{N,P}, T^{N,P-1}, T^{N-1}], \quad (3.2)$$

in which the integral of interest, $T^{N,P}$, now appears on the right-hand side. Neglecting in first approximation terms of order $\mathcal{O}(\Delta)$, the integrals $T^{N,P}$ can be calculated recursively from integrals of lower rank $T^{N,P-1}$ and from integrals with a lower number of propagators T^{N-1} . In this way tensor integrals of arbitrary rank can be determined at zeroth order in the small parameter Δ . Inserting afterwards the so-determined higher-rank tensor integral $T^{N,P+1}$ into the left-hand side of (3.2) allows to calculate also terms of order $\mathcal{O}(\Delta)$ for $T^{N,P}$. Proceeding systematically in this way one obtains $T^{N,P}$ as a series expansion in the parameter Δ , where higher precision in the form of $\mathcal{O}(\Delta^k)$ terms is achieved at the prize of calculating higher-rank tensor integrals $T^{N,P+k}$.

Based on the described strategy, various expansion methods have been suggested in Ref. [9] with the respective expansion parameter(s) depending on the region in phase space. All these methods have been implemented in COLLIER to arbitrary order in the expansion parameter. In order to decide which method to use for a certain phase-space point, an a priori error estimate is performed for the different methods considering a simplified propagation of errors from scalar integrals and neglected higher-order terms into the tensor integrals of highest rank. During the actual calculation of an expansion the precision is further checked by analysing the correction of the last iteration. In single cases where the a priori error estimate turns out as having been too optimistic, other expansions are tried in addition. In this way stable results are obtained for almost all phase-space points ensuring reliable Monte Carlo integrations.

For $N = 5, 6$, tensor integrals are directly reduced to integrals with lower rank and lower N following Refs. [8, 9], i.e., without using inverse Gram determinants. The methods summarised there can be extended to the case of $N \geq 7$ in a straightforward way.

	$P=0$	$P=1$	$P=2$	$P=3$	$P=4$	$P=5$	$P=6$
$N=3$	1	2	4	6	9	12	16
$N=4$	1	3	7	13	22	34	50
$N=5$	1	4	11	24	46	80	130
$N=6$	1	5	16	40	86	166	296
$N=7$	1	6	22	62	148	314	610
tensor	1	4	10	20	35	56	84

Table 1: Number of invariant coefficients $T_{i_1\dots i_p}^N$ for $N=3, \dots, 7$ and rank $P=0, \dots, 6$ (rows 2-6) and number of tensor elements $T^{N, \mu_1 \dots \mu_p}$ for rank $P=0, \dots, 6$ (last row)

While the methods described so far are formulated in the literature in terms of the Lorentz-invariant coefficients $T_{i_1\dots i_p}^N$, a new generation of NLO generators, such as OPENLOOPS and RECOLA, needs the elements of the full tensors $T^{N, \mu_1 \dots \mu_p}$. To this end, an efficient algorithm has been implemented in COLLIER to construct the tensors $T^{N, \mu_1 \dots \mu_p}$ from the coefficients $T_{i_1\dots i_p}^N$. It performs a recursive calculation of those tensor structures (2.4) that are built exclusively from momentum vectors. Non-vanishing elements of other tensor structures involving metric tensors are then obtained by adding pairwise equal Lorentz indices, and their value differs from the corresponding value of the pure momentum tensor only by a combinatorial factor and a potential minus-sign induced by the metric tensors. The relevant combinatorial factors are calculated and tabulated during the initialisation of COLLIER.

The numbers of invariant coefficients $T_{i_1\dots i_p}^N$ and tensor elements $T^{N, \mu_1 \dots \mu_p}$ are compared in Table 3. For $N \leq 4$ the number of invariant coefficients is smaller than the number of tensor elements, and this fact constitutes a basic precondition of the Passarino–Veltman reduction method. For $N \geq 5$, on the other hand, there are less tensor elements than coefficients and the reduction method for $N \geq 6$ presented in (7.7) of Ref. [9] has been actually derived in terms of full tensors. Its translation to tensor coefficients requires an additional symmetrisation and the resulting coefficients are not unique because of the overdefined number of tensor structures. Therefore for the calculation of the tensors $T^{N, \mu_1 \dots \mu_p}$ the reduction for $N \geq 6$ has been implemented in COLLIER also directly at the tensor level without resorting to a covariant decomposition.

4. Structure of the library

The structure of the library COLLIER is illustrated schematically in Figure 4. The core of the library is formed by the building blocks `Coli` and `DD`. They constitute two independent implementations of the scalar integrals T_0^N and the Lorentz-invariant coefficients $T_{i_1\dots i_p}^N$ employing the methods described in the previous section. The module `tensors` provides routines for the construction of the tensors $T^{N, \mu_1 \dots \mu_p}$ from the coefficients $T_{i_1\dots i_p}^N$ as well as for a direct reduction of 6-point integrals at the tensor level. The user interacts with the basic routines of `Coli`, `DD` and `tensors` via the global interface of COLLIER. It provides routines to set or extract numerical values of the parameters in `Coli` and `DD` as well as routines to call the calculation of tensor coeffi-

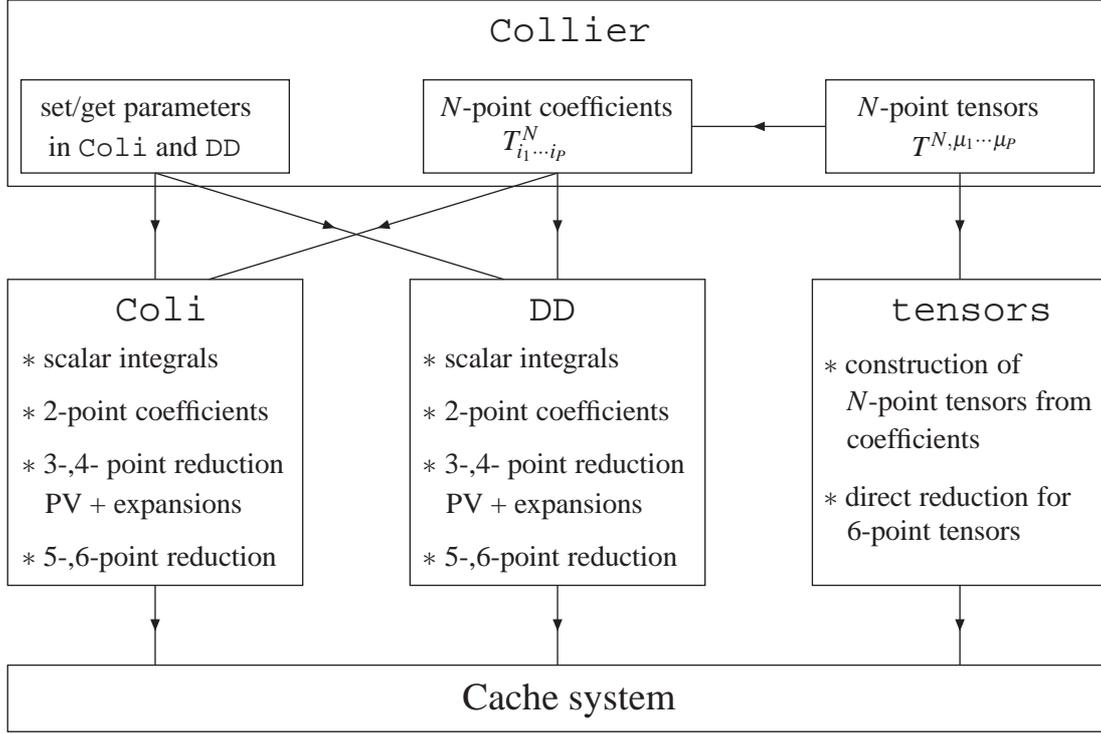


Figure 1: Structure of the library COLLIER.

coefficients $T_{i_1 \dots i_p}^N$ or tensor elements $T^{N, \mu_1 \dots \mu_p}$. The user can choose whether the `Coli`- or the `DD`-branch shall be used for the calculation of the integrals. It is also possible to calculate each integral with both branches for the purpose of comparison.

In the evaluation of a one-loop matrix element the same tensor integral is called various times: On the one hand, a single user call of an N -point integral leads to recursive internal calls of lower N' -point integrals and for $N' \leq N - 2$ the same integral is reached through more than one path in the reduction tree. On the other hand, different user calls and their reductions typically involve identical tensor integrals. In order to avoid multiple calculations of the same integral the sublibraries of COLLIER are linked to a global cache system which works as follows: A parameter N_{ext} numerates external integral calls, while for the book-keeping of internal calls a binary identifier id is propagated during the reduction. A pointer is assigned to each index pair (N_{ext}, id) . During the evaluation of the first phase-space points the arguments of the corresponding function calls are compared and pairs (N_{ext}, id) with identical arguments are pointed to the same address in the cache. For later phase-space points the result of the first call of an integral is written to the cache and read out in subsequent calls pointing to the same address.

5. Conclusions

We have introduced the fortran-based Complex One-Loop Library in Extended Regularizations COLLIER. It provides the complete set of basic scalar integrals as well as tensor integrals of arbitrary rank for up to $N = 6$ external particles (an implementation for $N \geq 7$ is in progress).

In order to ensure numerical stability the expansion methods for 3- and 4-point integrals of Ref. [9] have been implemented to arbitrary order in the corresponding expansion parameter. UV singularities are regularised dimensionally, IR singularities integrals can be regularised dimensionally or alternatively by introducing small masses. Complex values are supported for the masses of internal particles in loop propagators, permitting thus the application of COLLIER to processes involving unstable particles. As output the user obtains either the coefficients $T_{i_1 \dots i_p}^N$ of the covariant decomposition of the respective tensor integral or the elements of the tensor $T^{N, \mu_1 \dots \mu_p}$ themselves. A recalculation of identical integrals is avoided by an efficient built-in cache system. The fundamental building blocks of the library are provided in two implementations that allow for an independent calculation of each integral and for direct numerical cross-checks.

COLLIER has already been successfully applied to a large number of calculations of QCD and EW corrections and is integrated in the NLO generators OPENLOOPS and RECOLA. Publication of the code facilitating its use by other generators and other groups is in preparation.

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