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Fuzzy Topology, Quantization and Gauge Fields

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It was argued earlier that Dodson-Zeeman fuzzy topology (FT)[1] represents the possible mathematical basis for quantum space-time structure[2]. Here the quantization formalism related to it will be described[3,4].

As the example, the quantization of massive particles is considered, it's shown that the coordinate uncertainty is generic in FT. FT fundamental set D is Poset[3,4], so that some its element pairs in place of standard ordering relation $d_j \leq d_k$, can obey to incomparability relation: $d_l \sim d_m$. For illustration, consider discrete Poset $D = A^p \cup B$, which includes the subset of incomparable elements $A^p = \{a_j\}$, and the subset $B = \{b_i\}$ which is maximal totally ordered D subset. B indexes grow correspondingly to their ordering, *i.e.* $\forall i$, $b_i \leq b_{i+1}$. Suppose that for some a_j and B interval $\{b_l, b_n\}$, $a_j \sim b_i; \forall i : l \leq i \leq n$. In this case a_j is "smeared" over $\{b_l, b_n\}$ interval, which is analogue of a_j coordinate uncertainty, if to regard B as D "coordinate axe". Analogously to it, 1-dimensional model Universe corresponds to Poset $U = A^p \cup X$ where A^p is the massive particle subset, X-continuous ordered subset R^1, which describes 1-dimensional euclidian geometry. If for some a_j and X interval $\{x_c, x_d\}$ the relation $a_j \sim x_b$ holds for all $x_b \in \{x_c, x_d\}$, then a_j possess x-coordinate uncertainty of the order $|x_d - x_c|$. To detalize a_j characteristics, the corresponding fuzzy weight $w_j(x) \geq 0$ introduced with the norm $\|w_j\| = 1$, so that $w_j(x)$ value indicates where on X axe a_j is mainly concentrated[3]. In this framework a_j corresponds to the formal definition of fuzzy point and U of fuzzy set[3].

In such approach massive particle m can be described as the evolving fuzzy point $a_i(t)$ of U. It's shown then that the corresponding normalized m density $w(x,t)$ evolves according to the flow continuity equation: $\frac{\partial w}{\partial t} = -\frac{\partial (wv)}{\partial x}$ where $v(x,t)$ is w flow local velocity. The independent m parameters $w(x), v(x)$, which characterize m state, can be unambiguously mapped to normalized complex function $\varphi(x)$. Assuming space-time shift invariance, it's proved that $\varphi(x,t)$ evolution obeys to Schrödinger equation for arbitrary m mass μ, such theory can be also extended for 3-dimensional case[4]. It's proved also that in relativistic case m evolution described by Dirac equation for spin $\frac{1}{2}$. Particle's interactions on such fuzzy manifold are shown to be gauge invariant, the interactions of fermion muliplets are performed by Yang-Mills fields[5].

References

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