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Light-by-light scattering, a new process for potential discoveries at the Large Hadron Collider

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The scheduled installation of forward proton detectors at the Large Hadron Collider (LHC) nearby the CMS and ATLAS experiments will provide an unprecedented opportunity to measure the light-by-light scattering with very good precision by taking advantage of the coherent photon flux emitted by the protons. The detection of the intact protons allows to reconstruct the full kinematics of the event, which is found to be very powerful to reject background. It is then possible to probe four photon anomalous couplings (4γ) with an excellent accuracy whereas very few constraints on those couplings exist at the moment. First, the Standard Model production is examined and then model-independent bounds on generic massive charged particles are derived based on the full statistics of proton-proton collisions expected at the LHC. We also discuss the sensitivity to new neutral particles by using an effective field theory approach and claim they could be discovered for masses in the multi-TeV range.

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1. Motivations for a light-by-light scattering measurement at the Large Hadron Collider

The Large Hadron Collider (LHC) is the most powerful proton-proton collider in operation. It is expected to deliver about 300 fb^{-1} of data at a center-of-mass energy $\sqrt{s} = 14$ TeV[1] between 2015 and 2019 [1]. In the following, we discuss a possible measurement of the four-photon couplings (4γ) from exclusive di-photon production via light-by-light scattering at the LHC. The leading-order diagrams of this process as predicted in the Standard Model (SM) are shown in Figure 1 (left). Possible additional contributions from anomalous 4γ couplings are illustrated by the diagram in Figure 1 (right). Very few constraint on anomalous 4γ couplings are reported in literature, making the study of this channel an interesting probe to search for new physics [2, 3].

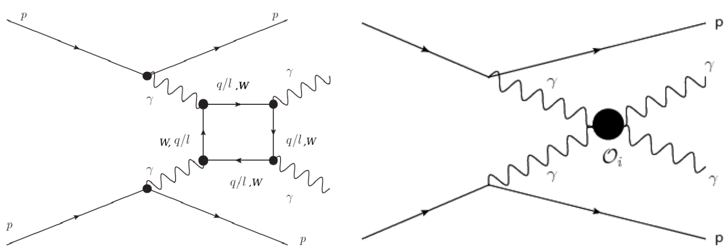

Figure 1: Exclusive di-photon production via light-by-light scattering as predicted in the Standard Model through quark, lepton or W boson loop (left) and possible contributions from higher dimension operators $\mathcal{O}_\mathrm{i}$ (right).

We show on purpose the peculiar case where the two initial-state photons are emitted coherently from the whole protons, so that the protons have high chances to stay intact after interaction [4, 5]. They can then be detected in dedicated forward proton detectors, planned to be installed in 2016-2017 on both sides of the two multi-purpose detectors at the LHC, *ie.* the ATLAS and CMS-TOTEM experiments [6, 7]. A sketch of one of the forward proton detectors and its future location with respect to the interaction point (IP) is shown in Figure 2 (left). A similar station will be located on the opposite side of the ATLAS and CMS IPs. The forward detectors will have a limited acceptance in terms of proton missing momentum fraction ξ (about $0.015 < \xi < 0.15$ is expected for nominal runs) because of the LHC beam size and optics. It will directly impact the acceptance of the proton missing mass $\sqrt{\xi_1\xi_2 s}$ which can be reconstructed in case of a detection on both side of the IP, as shown in Figure 2 (right).

Hence, the proposed channel gives the possibility to detect the two intact protons in addition to the two photons which are reconstructed in the central part of the ATLAS (or CMS) detector [8, 9]. This allows to reconstruct the full kinematics of the event and thus to reject background very efficiently, whereas it is usually the dominant source of uncertainty in photon measurements [10]. On the other hand, the production cross-section is reduced with respect to the inclusive channel due to the requirement of intact protons within the forward detector acceptance.

All the Standard Model contributions to exclusive di-photon production at the LHC, including interference and main background processes, have been implemented in the Forward Physics

[1]The center-of-mass energy of the proton-proton collisions at the LHC has been decreased to 13 TeV for nominal operation, due to technical reasons related to the magnets.

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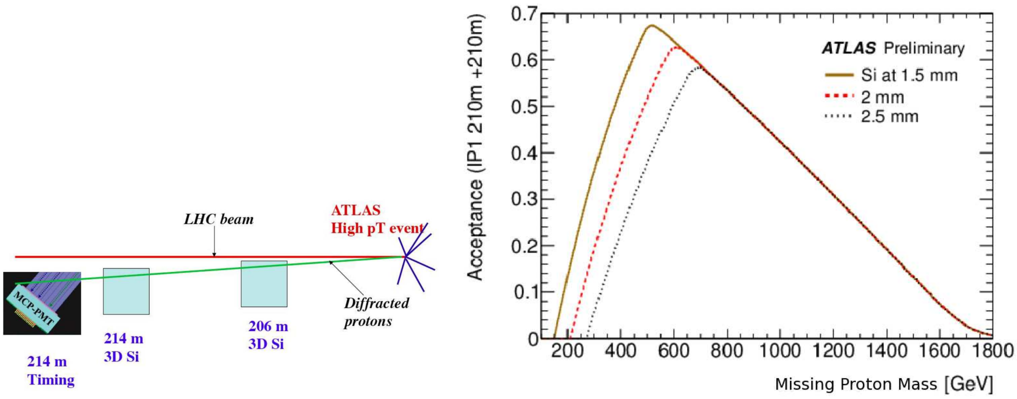

Figure 2: Sketch of one of the two stations planned to be installed on both sides of the ATLAS interaction point (left). Proton tracking and timing will be provided by the detection chain. The missing proton mass acceptance is shown on the right plot. The CT-PPS upgrade project (CMS-TOTEM) has similar characteristics.

Monte Carlo generator (FPMC), which is used for simulation [11]. The leading detector effects such as reconstruction efficiency, fake rates and energy and angular resolutions are emulated based on ECFA ATLAS studies [12]. In Section 2, we present the different contributions to exclusive di-photon production predicted by the SM and study the possibility to a measure them at the LHC.

In Section 3, potential additional contributions to the 4γ couplings from new physics are reviewed. First, an Effective Field Theory (EFT) approach via higher dimension operators parametrized by exotic coupling factors is used to compute beyond SM effects. Then, a full amplitude calculation valid for any new charged particle mass is presented. Both calculations have been implemented in FPMC. New physics signals are generated and compared to the background predictions for several scenarios in Section 4, including new neutral particle contributions at tree-level and new charged particles via loops. The corresponding sensitivities are shown in the different cases. Finally, conclusions and prospects are given in Section 5.

2. Exclusive di-photon production in the Standard Model

The Standard Model predicts exclusive di-photon production with two intact protons through various processes, which can be decomposed in QED production via photon exchange (see Figure 1, left) and QCD production via gluon exchange (see Figure 3, left) also called KMR production [13]. Those processes constitue the irreducible background for new physics searches. In Figure 3 (right), the total integrated cross-section is shown for the different SM contributions. For masses above a few hundred GeV, the SM predicts a cross-section below 0.01 fb, negligible with respect to the expected statistics (300 fb^{-1}). Consequently, the main background for new physics searches will come from other sources, as it will be discussed in Section 3.

To perform a SM cross-section measurement of exclusive di-photon production using nominal LHC run data is not realistic because of the too high threshold of the triggers and the too small cross-section. However, a low mass measurement of the QCD contribution may be possible in the case of the special runs at low luminosity and low pileup ($<\mu>\simeq$ 1–2) with modified optics,

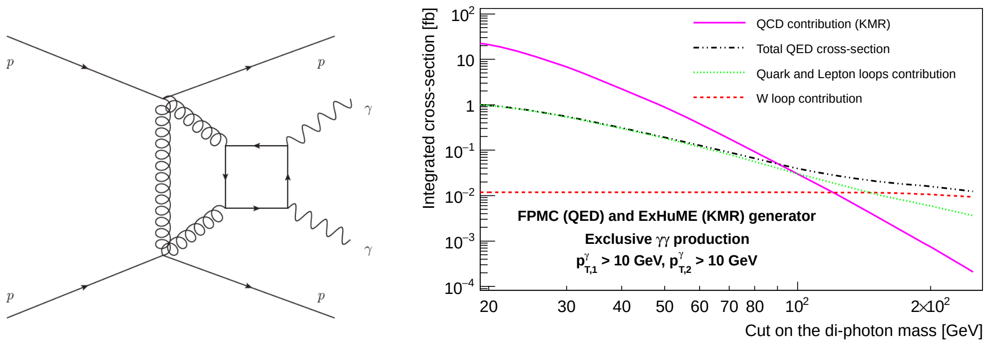

Figure 3: Leading-order diagram of the QCD exclusive di-photon production (left) and integrated cross-section of the various SM processes contributing to exclusive di-photon production according to the cut on the di-photon mass (right). The QCD contribution has been generated with the ExHuME MC generator [14].

currently being discussed among the different LHC experiments [15]. Indeed, thanks to the low instantaneous luminosity of those special runs one should be able to implement a dedicated di-photon trigger with photon transverse momentum thresholds as low as 5 GeV. This would allow to reach an integrated production cross-section of about 370 fb for the QCD production with about 0.1 fb^{-1} of data expected from the low luminosity runs, which gives about 37 expected events. This measurement has already been done by the CDF experiment at the TEVATRON and results compatible to the SM were found [16].

On the other hand, the QED contributions seems out of reach in proton-proton collisions, with an expected integrated cross-section below 6 fb for special runs. However, they may be measured from heavy ion collisions at the LHC, as suggested in a recent publication [17].

3. Potential beyond Standard Model contributions to the 4γ couplings

3.1 Effect Field Theory (EFT)

In the assumption of a new physics mass scale Λ heavier than the experimentally accessible energy E, all new physics manifestations can be described using an effective Lagrangian valid for $\Lambda >> E$. Among these operators, the two pure photon dimension-eight operators (see Equation 3.1) can enhance the 4γ couplings [2, 18, 19]. They are parametrized in the following by two exotic couplings ζ_1, ζ_2 in GeV^{-4}.

$$\mathscr{L}_{4\gamma} = \zeta_1^{\gamma} F_{\mu\nu}F^{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + \zeta_2^{\gamma} F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} \tag{3.1}$$

This parametrization is useful for the study of a broad class of models. At leading order, two kinds of new contributions can enhance the 4γ couplings : any new electrically charged particles through a loop (Figure 4, left) or new neutral particles at tree-level (Figure 4, right). In case of electrically charged particles (as for the SM contributions shown in Figure 1, left), the amplitude is characterized only by the electric charge, spin and mass, whereas for neutral contributions a non-renormalizable coupling f^{-1} needs to be introduced. One can show that the exotic couplings ζ_1, ζ_2 are related to the quantum numbers and the mass of the new particles in a simple form for both cases :

- For heavy charged particles, $\zeta_i^\gamma = \alpha_{\rm em}^2 Q^4 m^{-4} N c_{i,s}$, where $c_{i,s}$ is a constant increasing with the spin s of the heavy particle of mass m and electric charge Q running in the loop. In addition, the new particles have also in general a multiplicity $N \neq 1$ with respect to electromagnetism[2]. For convenience, we define the effective charge $Q_{\rm eff}^4 = Q^4 N$.

- For neutral resonances, $\zeta_i^\gamma = (f_s m)^{-2} d_{i,s}$, where $d_{i,s}$ and f_s are respectively a constant and the non-renormalizable $\gamma\gamma X$ coupling. They depend on the spin and parity of the heavy neutral particle of mass m.

Typical examples of new heavy charged particles are predicted for instance in the minimal composite Higgs models [20] and new neutral resonances coupled to photons arise in many strongly-coupled conformal extensions of the SM, such as the dilaton (scalar) or Kaluza-Klein gravitons (spin 2) [3]. In all cases, ζ_i^γ can easily exceed 10^{-14}-10^{-13} GeV^{-4} is certain regions of parameter space.

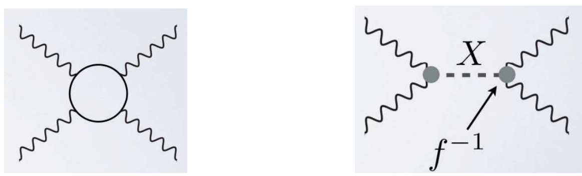

Figure 4: Typical contributions to light-by-light scattering from new electrically charged particles (left) or new neutral particles X (right). For neutral contributions, a non-renormalizable $\gamma\gamma X$ coupling f^{-1} is introduced.

3.2 Exact amplitudes

The effective field theory analysis has the advantage of being very simple. However it is only valid as long as the center-of-mass energy is small with respect to the threshold of pair-production of real particles in case of new loop-induced production, $s << 4m^2$. Since the maximum proton missing mass which can be detected (which is equal to the final-state di-photon invariant mass) is of the order of $\simeq 2$ TeV (see Figure 2, right), the effective field theory breaks down for particles lighter than $\simeq 1$ TeV and cannot be trusted anymore. In the case of neutral particles, $s < m^2$ is usually the strongest condition of EFT validity (so $m < 2$ TeV) unless the underlying new physics model is very strongly coupled so that unitarity bound starts playing a sizable role [3].

In literature, EFTs are usually corrected using ad-hoc form factors to extend their validity. A more correct approach is to take into account the full momentum dependence of the 4γ amplitudes instead. Generic fermion and boson loop contributions to light-by-light scattering have been calculated respectively in [21] and [22], and next-to-leading order corrections have been found to be negligible in [23]. They are implemented together with the EFT in FPMC and used to produce new physics signals. The full amplitude predictions are cross-checked with the EFT approach at high mass and found to be in good agreement (see Section 4.3). The implementation of the full amplitudes in case of neutral resonances, taking into account the $\sqrt{s}$ width dependence, in is progress and should be available in the coming year.

[2]For instance, the multiplicity is three if the particles are colored.

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Hence, for completeness, we quote in this document results from the EFT approach with and without form factors applied at amplitude-level ($1/(1+(m_{\gamma\gamma}^2/\Lambda')^2)$, Λ' = 1 TeV), and results from the full amplitude calculation for charged particles.

4. Sensitivity of the 4γ coupling to new physics

4.1 Background simulation

As mentioned in Section 2, the irreducible background at high mass, which is the most relevant region for new physics searches, is negligible. On the other hand, many other processes occurring at the LHC can mimic the signal : electrons and jets can fake photons in the detector and non-exclusive diffractive processes can give the good final-state $p\gamma\gamma p$ (+ X). Moreover, intact protons from pileup interactions ($<\mu>$ is assumed to be 50 in the following) in association to inclusive $\gamma\gamma$, di-jet or Drell-Yan production can also emulate the signal.

A broad range of possible background processes was generated with the FPMC generator and combined with PYTHIA8 [24] minimum bias events to simulate intact protons from pileup interactions. The number of expected event for each process is shown in Figure 5 after a minimal selection : photon individual transverse momentum above 200 (100) GeV for the first (second) photon candidate, pseudorapidity $|\eta^\gamma| < 2.37$ and two intact protons in the forward detector acceptance. "Exclusive background" includes irreducible background and exclusive di-electron production whereas "DPE background" includes Double Pomeron Exchange backgrounds (DPE $\gamma\gamma$, di-jet), which are not exclusive processes due to the Pomeron remnants. One can see that requiring a di-photon mass above 600 GeV suppresses all the backgrounds but the inclusive $\gamma\gamma$ production + intact protons from pileup, while keeping most of the signal events. This cut is applied in the following and the rest of the selection is discussed in the next Section.

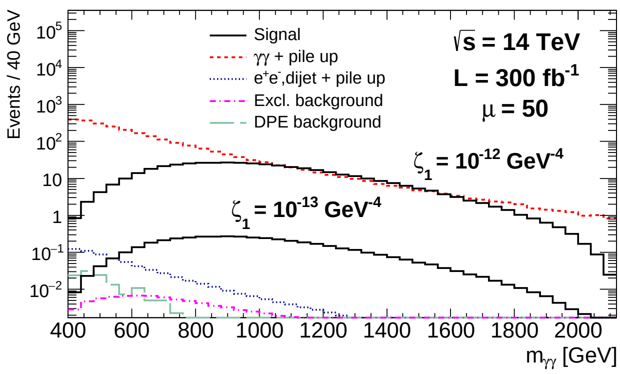

Figure 5: Number of expected events with 300 fb^{-1} of data at $\sqrt{s}$ =14 TeV for the EFT signal ($\zeta_1 \neq 0$, $\zeta_2 = 0$) and various background processes with respect to the di-photon mass. Only events with two intact protons tagged in the forward detectors and two photons with transverse momentum above 200 (100) GeV and pseudorapidity $|\eta^\gamma| < 2.37$ are considered.

4.2 Selection cut

In order to get rid of the remaining background coming from inclusive $\gamma\gamma$ production + intact protons from pileup, we take advantage of the intact proton ξ measurement in the forward stations.

Indeed, since all final-state particles are detected for the signal, it is possible to perform a rapidity and invariant mass measurement from the central detector via the di-photon system and from the forward detectors via the two intact protons, *ie.* $m_{\gamma\gamma} = \sqrt{\xi_1 \xi_2 s}$ and $y_{\gamma\gamma} = 0.5 \times \ln(\xi_1/\xi_2)$ in case of signal. Matching those quantities is very efficient and essential to get rid of the background from pileup, where the two protons do not share the kinematics of the di-photon (see Figure 6).

In order to further reject background, we also require the two photons to be balanced in transverse momenta and back-to-back in terms of azimuthal angle (exclusive production). The cutflow for a signal corresponding to an exotic charged boson of $Q_{\text{eff}} = 4$ and m = 340 GeV is given in Table 1. The full amplitude signal predictions lie in between the predictions from the EFT with and without form factors, which is expected for this medium mass range ($m < 1$ TeV). After selection, no background events remains so that a single detection reaches a very high significance. A similar study at a higher pile-up of 200 was performed and led to an expected number of background events below 5 events, showing the robustness of the study.

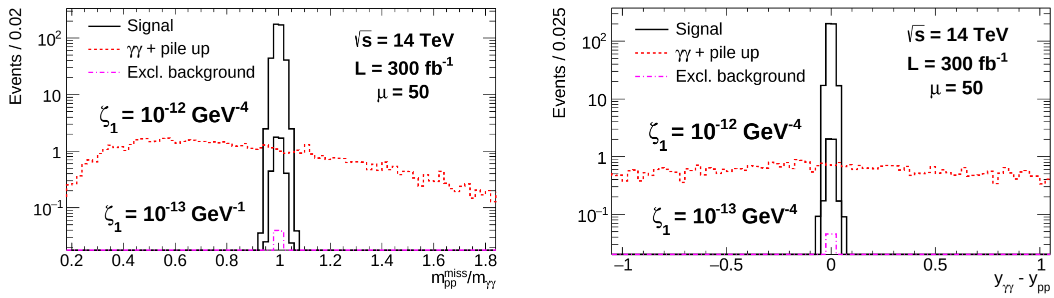

Figure 6: Comparison between the invariant mass (rapidity) of the di-photon system with respect to the di-proton system for signal and the remaining background, left (right). The two quantities do not match exactly for the signal because of the smearing applied to mimic the limited resolution of the detectors.

Cut / Process	Signal (full)	Signal with (without) f.f (EFT)	Excl.	DPE	DY, di-jet + pile up	$\gamma\gamma$ + pile up
$[0.015 < \xi_{1,2} < 0.15$, $p_{\text{T1,(2)}} > 200, (100)$ GeV]	65	18 (187)	0.13	0.2	1.6	2968
$m_{\gamma\gamma} > 600$ GeV	64	17 (186)	0.10	0	0.2	1023
$[p_{\text{T2}}/p_{\text{T1}} > 0.95$, $\lvert\Delta\phi\rvert > \pi - 0.01]$	64	17 (186)	0.10	0	0	80.2
$\sqrt{\xi_1\xi_2 s} = m_{\gamma\gamma} \pm 3\%$	61	12 (175)	0.9	0	0	2.8
$\lvert y_{\gamma\gamma} - y_{pp}\rvert < 0.03$	60	16 (169)	0.9	0	0	0

Table 1: Expected number of signal and background events after the various selection criteria in the case of 300 fb^{-1} of data at the 14 TeV LHC. The signal corresponds to an exotic charged boson of $Q_{\text{eff}} = 4$ and m = 340 GeV. The results from the full amplitude calculation and from the EFT with/without form factors are provided.

4.3 Final results

We show in Table 2 the final sensitivies in terms of exotic couplings ζ_1, ζ_2 for various scenarios of statistics and pileup. In addition, we consider the case where at least one photon is explicitely required to convert to an electron-positron pair in the detector so that the vertex of the interaction

Luminosity	300 fb^{-1}	300 fb^{-1}	300 fb^{-1}	3000 fb^{-1}
pile-up $<\mu>$	50	50	50	200
coupling (GeV^{-4})	≥ 1 conv. γ 5 σ	≥ 1 conv. γ 95% CL	all γ 95% CL	all γ 95% CL
ζ_1 f.f.	$1.5 \cdot 10^{-13}$	$7.5 \cdot 10^{-14}$	$4 \cdot 10^{-14}$	$3.5 \cdot 10^{-14}$
ζ_1 no f.f.	$3.5 \cdot 10^{-14}$	$2.5 \cdot 10^{-14}$	$1 \cdot 10^{-14}$	$1.5 \cdot 10^{-14}$
ζ_2 f.f.	$2.5 \cdot 10^{-13}$	$1.5 \cdot 10^{-13}$	$8.5 \cdot 10^{-14}$	$7 \cdot 10^{-14}$
ζ_2 no f.f.	$7.5 \cdot 10^{-14}$	$4.5 \cdot 10^{-14}$	$2.5 \cdot 10^{-14}$	$2.5 \cdot 10^{-14}$

Table 2: Final 5 σ discovery and 95% Confidence Level (CL) limit on the EFT exotic couplings ζ_1,ζ_2 for different assumption in terms of statistics and pileup at the 14 TeV LHC.

is located with a very good precision and can be compared with proton timing measurement. This technique may be very useful to reject further non-collision background such as beam halo, which is difficult to predict at the moment [25]. Anomalous 4γ couplings down to 10^{-14} GeV^{-4} can be probed, reaching the predictions of a large panel of extra-dimension models in the multi-TeV range [2, 3]. In Figure 7, we show the final sensitivities for new heavy charged particles (vectors, left, fermions, right) in the case of a pileup $<\mu>= 50$, an integrated luminosity of 300 fb^{-1} and no particuliar requirement on photon conversion status. The predictions are computed from the full amplitude calculation and cross-checked with the EFT prediction with no form factors at high mass. Vectors (fermions) with an effective charge $Q_{\rm eff} = 5$ can be discovered up to $m = 810$ (460) GeV. Even if the current constraints from the LHC on benchmark models such as composite Higgs are typically stronger, our results are completely model-independent and thus provide interesting complementary constraints.

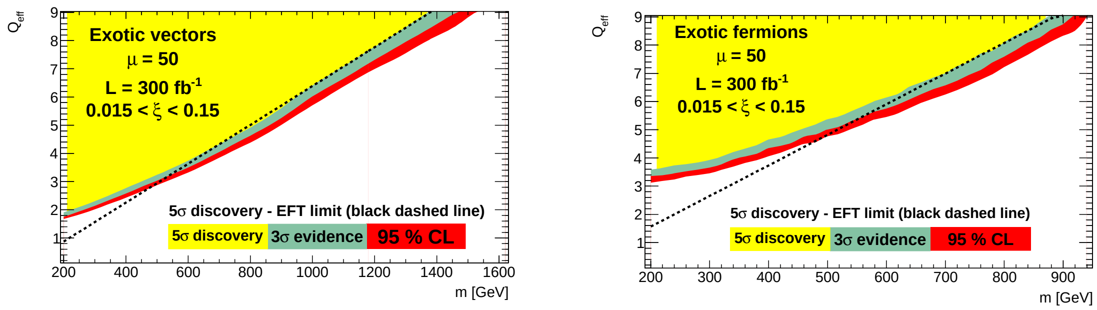

Figure 7: Final 5 σ discovery, 3 σ evidence and 95% Confidence Level (CL) limits on new heavy charged vectors (left) and fermions (right) at the 14 TeV LHC assuming a pileup $<\mu>= 50$ and an integrated luminosity of 300 fb^{-1}. The sensitivities are computed from the full amplitude calculation and cross-checked with the EFT 5 σ prediction with no form factors applied at high mass.

5. Conclusion and plans

We presented an estimation of the discovery potential for light-by-light scattering at the 14 TeV LHC. The irreducible background predicted by the Standard Model was found to be negligible, however, backgrounds due to intact protons from pileup interactions are sizable. Proton tagging seems absolutely compulsory to get rid of them and will soon be possible at the ATLAS and CMS-TOTEM experiments. After selection cuts, no background remains and anomalous 4γ

couplings down to 10^{-14} GeV^{-4} can be probed. In addition, model-independent constraints can be set on any new charged particles only based on their charge, mass and spin. Results based on full amplitude calculation in the case of neutral particle contributions is expected to be published in the coming year. Sensitivities to higher spin resonances are promising but require important theoretical developments. Finally, the three-photon final-state at the LHC may also show a good sensitivity to anomalous 4γ couplings and should be scrutinized with a special care in the coming runs.

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