PROCEEDINGS OF SCIENCE

Polarization effects in case of very low energy hadron-hadron scattering

Cyril Adamuščín[*]
Institute of Physics, Slovak Academy of Sciences Bratislava, 845 11, Slovakia
E-mail: cyril.adamuscin@savba.sk

Stanislav Dubnička
Institute of Physics, Slovak Academy of Sciences Bratislava, 845 11, Slovakia
E-mail: stanislav.dubnicka@savba.sk

Anna Zuzana Dubničková
Depart. of Theoretical Physics, Comenius University Bratislava, 842 48,Slovakia
E-mail: dubnickova@fmph.uniba.sk

We will analyze the contribution of the polarization of the incoming hadrons to the amplitude of the hadron hadron scattering at the ultra low energy limit. We will show that their polarization does not have significant effect on the amplitude of such scattering. The polarization of the outgoing hadrons is also studied.

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[*]Speaker.

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1. Motivation and Introduction

The electromagnetic (EM) structure of hadrons can be described by their electromagnetic form factors, which are complex functions of one variable – the momentum transferred squared q^2. The EM form factors can be measured in experiments with unpolarized electron beam scattered on the unpolarized hadron target. From the measured differential cross section in such experimental setup two structure function $A(q^2), B(q^2)$ can be extracted using the Rosenbluth technique for $q^2 < 0$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E' \cos^2(\theta/2)}{4E^3 \sin^4(\theta/2)} [A(q^2) + B(q^2) \tan(\theta/2)]. \tag{1.1}$$

The structure functions are directly related to the nucleon EM form factors as

$$A(q^2) = \frac{1}{1+\eta} \left(G_E^2(q^2) + \eta G_M^2(q^2) \right), \;\; B(q^2) = 2\eta G_M^2(q^2), \tag{1.2}$$

where $\eta = -\frac{q^2}{4M^2}$.

The similar relations exist for the annihilation case but $q^2 > 0$. It is also possible to perform scattering experiments with polarized particles, which allow to extract more precise data on EM form factors.

In our paper we study the polarization effects in the measurement of the elastic scattering of two polarized protons at very low energies. A hadron-hadron interaction is generally dominated by the strong interaction. However at energies below 1 MeV is the contribution of the strong interaction negligible and the process can be fully described by the electromagnetic interaction. Therefore we can use the formalism of the nucleon EM form factors to predict the behavior of the nucleons in such scattering.

2. Dynamics

We will analyze an elastic low energy scattering of two longitudinally polarized nucleons in the center of momentum frame. As the interaction of two nucleons with energies ~ 1 MeV is determined dominantly by electromagnetic interaction, we can use one photon exchange approximation and the EM form factors to predict the differential cross section of the investigated process.

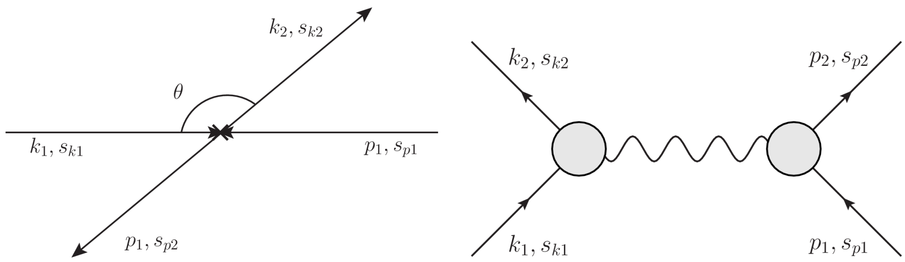

Figure 1: The kinematic and Feynman diagram of the studied process

For the proton (spin=1/2) the on-shell electromagnetic current can be written as

$$J_\mu = \bar{u}(k_2) \left[G_M(q^2)\gamma_\mu - F_2(k_{2\mu} + k_{1\mu})/(2M) \right] u(k_1) \tag{2.1}$$

and the hadron tensor for the first scattered particle (with momentum k)

$$H^k_{\mu\nu} = J_\mu J^*_\nu = \bar{u}(k_2)\left[G_M(q^2)\gamma_\mu - F_2(k_{2\mu}+k_{1\mu})/(2M)\right]u(k_1)\times \\ \times \left[\bar{u}(k_2)\left[G_M(q^2)\gamma_\nu - F_2(k_{2\nu}+k_{1\nu})/(2M)\right]u(k_1)\right]^*. \tag{2.2}$$

The density matrix ρ for the polarized particle equals

$$\rho = u(p)\bar{u}(p) = (\hat{p}+M)\frac{1}{2}(1-\gamma_5\hat{s}), \tag{2.3}$$

therefore

$$H^k_{\mu\nu} = Tr\Big[(\hat{k}_2+M_k)\frac{1}{2}(1-\gamma_5\hat{s}_{k2})\left[G_{Mk}(q^2)\gamma_\mu - G_{2k}(k_{2\mu}+k_{1\mu})/(2M)\right]\times \\ \times (\hat{k}_1+M_k)\frac{1}{2}(1-\gamma_5\hat{s}_{k1})\left[G^*_{Mk}(q^2)\gamma_\nu - G^*_{2k}(k_{2\nu}+k_{1\nu})/(2M)\right]\Big]. \tag{2.4}$$

The similar hadron tensor can be derived also for the second scattered particle (p)

$$H^p_{\mu\nu} = Tr\Big[(\hat{p}_2+M_p)\frac{1}{2}(1-\gamma_5\hat{s}_{p2})\left[G_{Mp}(q^2)\gamma_\mu - G_{2p}(p_{2\mu}+p_{1\mu})/(2M)\right]\times \\ \times (\hat{p}_1+M_p)\frac{1}{2}(1-\gamma_5\hat{s}_{p1})\left[G^*_{Mp}(q^2)\gamma_\nu - G^*_{2p}(p_{2\nu}+p_{1\nu})/(2M)\right]\Big]. \tag{2.5}$$

We can calculate the amplitude of the scattering

$$|\mathscr{M}|^2 = \frac{e^4}{q^4}H^k_{\mu\nu}H^p_{\mu\nu} = \frac{e^4M^4}{q^4}D \tag{2.6}$$

and the differential cross section in the center of momentum frame with neglecting the terms $\left(\frac{E_{kin}}{M}\right)^n$ for $n \geq 2$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{|\mathscr{M}|^2}{64\pi^2W^2} = \alpha^2\frac{M^4D}{4W^2q^4}, \tag{2.7}$$

where $M = M_p = M_k$ as we analyze the scattering of the identical particles, $W = E_{p1}+E_{k1}$ is initial energy of the system in the central mass frame and D is naked structure of the amplitude.

3. Kinematics and Calculation

The calculation was performed for two identical particles in the center of momentum frame, where the momenta of the incoming protons can be written as

$$k_1 = (E,0,0,K_1),\ \ p_1 = (E,0,0,-K_1), \tag{3.1}$$

where E, K_1 are energy and momentum of the incoming proton. The polarization four vector is defined as

$$s_p = \left(\frac{\vec{p}\cdot\vec{\xi}}{M}, \vec{\xi} + \frac{\vec{p}(\vec{p}\cdot\vec{\xi})}{M(M+E)}\right). \tag{3.2}$$

For the longitudinal polarization of the incoming particles we get

$$s_{k1} = \frac{1}{M}(K_1,0,0,E),\ \ s_{p1} = \frac{1}{M}(K_1,0,0,-E). \tag{3.3}$$

We have analyzed three possible cases of the outgoing protons polarization:

1. unpolarized outgoing protons
2. longitudinal polarization of the outgoing protons (Fig. 2a)
3. polarization along the z-axis (Fig. 2b)

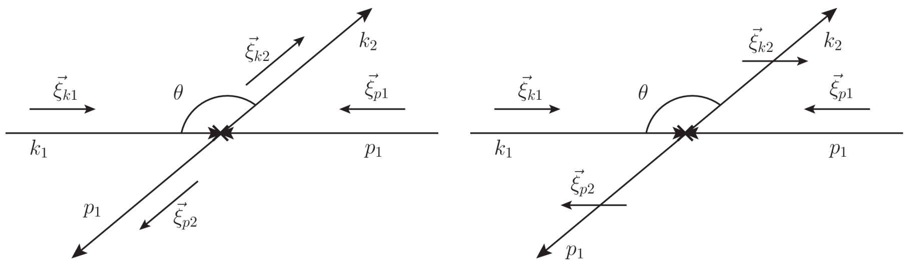

Figure 2: a) $\vec{\xi}_{k2}||\vec{k}_2, \vec{\xi}_{p2}||\vec{p}_2$ b) $\vec{\xi}_{k2} = (0,0,\pm 1), \vec{\xi}_{p2} = (0,0,\mp 1)$

The scalar products of momentum and polarization four vectors were expressed in terms of the incident energy, the mass of scattered particles and the scattering angle E, M, θ. The momentum m transferred squared q^2 equals

$$q^2 = 2(M^2 - E^2)(1 - \cos^2\theta). \tag{3.4}$$

The calculation itself was preformed using FORM [1].

At first we analyze the case without the polarization of outgoing particles with different directions of the polarization of the incoming particles. In the very low energy case ($E < 1$MeV) the kinetic energy of the incoming particle E_{kin} is negligible to its mass and $E/M \to 1$. Therefore we have calculated the result in two approximations

1. $E = M$.
2. $E = M + E_{kin}$ with neglecting the terms $\left(\frac{E_{kin}}{M}\right)^n$ for $n \geq 2$.

	1.approx.	2.approx.
D_{UP}	$64F_1^4$	$64(F_1^4 + 4\frac{E_{kin}}{M}(F_1^3(F_1 - F_2) + G_M F_1^3 \cos\theta)$
$D_{\rightarrow\leftarrow}$	$64F_1^4$	$D_{UP} + 64\frac{E_{kin}}{M} G_M^2 F_1^2 (1 - \cos^2\theta)$
$D_{\rightarrow\rightarrow}$	$64F_1^4$	$D_{UP} - 64\frac{E_{kin}}{M} G_M^2 F_1^2 (1 - \cos^2\theta)$
$D_{\leftarrow\leftarrow}$	$64F_1^4$	$D_{UP} - 64\frac{E_{kin}}{M} G_M^2 F_1^2 (1 - \cos^2\theta)$
$D_{\leftarrow\rightarrow}$	$64F_1^4$	$D_{UP} + 64\frac{E_{kin}}{M} G_M^2 F_1^2 (1 - \cos^2\theta)$

where D_{UP} is the result for unpolarized incoming hadrons and the arrows in subscripts denotes the direction of the longitudinal spin orientation of the incoming hadrons. $F_1(q^2), F_2(q^2)$ are Dirac and Pauli EM form factors of the nucleon and the $G_M(q^2)$ is magnetic Sachs form factor of the nucleon.

The longitudinal polarization of the outgoing hadrons means that $\vec{\xi}_{k2}||\vec{k}_2, \vec{\xi}_{p2}||\vec{p}_2$. Therefore the polarization four vectors s_{k2}, s_{p2} take the forms

$$s_{k2} = \pm\frac{1}{M}\left(|\vec{k}_2|, \vec{\xi}E\right), \ s_{p2} = \pm\frac{1}{M}\left(|\vec{p}_2|, -\vec{\xi}E\right). \tag{3.5}$$

We have analyzed four different possible combinations of the outgoing hadron spin orientations

	1.approx.
$D_{(\rightarrow\leftarrow)\rightarrow UP}$	$64F_1^4$
$D_{(\rightarrow\leftarrow)\rightarrow(\rightarrow\leftarrow)}$	$16F_1^4(1+\cos\theta)^2$
$D_{(\rightarrow\leftarrow)\rightarrow(\rightarrow\rightarrow)}$	$16F_1^4(1-\cos^2\theta)$
$D_{(\rightarrow\leftarrow)\rightarrow(\leftarrow\leftarrow)}$	$16F_1^4(1-\cos^2\theta)$
$D_{(\rightarrow\leftarrow)\rightarrow(\leftarrow\rightarrow)}$	$16F_1^4(1-\cos\theta)^2$

For the polarization of the outgoing hadron along z-axis $\vec{\xi}_{k2} = (0,0,\pm 1), \vec{\xi}_{p2} = (0,0,\mp 1)$.

$$s_{k2} = \pm\frac{1}{M}(|\vec{k}_2|\cos\theta, 0, 0, (M+(E-M)\cos\theta)) \tag{3.6}$$

$$s_{p2} = \pm\frac{1}{M}(|\vec{p}_2|\cos\theta, 0, 0, -(M+(E-M)\cos\theta)) \tag{3.7}$$

and again we have analyzed the four different possible combinations of the outgoing hadron spin orientations

	1.approx.	2.approx.
$D_{(\rightarrow\leftarrow)\rightarrow UP}$	$64F_1^4$	$D_{UP}+64\frac{E_{kin}}{M}G_M^2F_1^2(1-\cos^2\theta)$
$D_{(\rightarrow\leftarrow)\rightarrow(\rightarrow\leftarrow)}$	$64F_1^4$	$D_{UP}+64\frac{E_{kin}}{M}G_M^2F_1^2(1-\cos^2\theta)$
$D_{(\rightarrow\leftarrow)\rightarrow(\rightarrow\rightarrow)}$	0	0
$D_{(\rightarrow\leftarrow)\rightarrow(\leftarrow\leftarrow)}$	0	0
$D_{(\rightarrow\leftarrow)\rightarrow(\leftarrow\rightarrow)}$	0	0

4. Conclusions

We have analyzed the very low energy scattering of two protons. We have assumed only the electromagnetic interaction. We have shown that the polarization of the incoming protons does not have significant effect on the amplitude of the investigated process. We have shown that the direction of the polarization of the scattered particle is conserved up to $\mathscr{O}(E_{kin}/M)$. We plan to analyze the same polarization effects for the spin 1 particles.

In this analysis we have omitted the existence of the second diagram with interchanged outgoing nucleons, which should be taken into account due to indistinguishableness of the outgoing nucleons. Its contribution should not change the result with unpolarized outgoing nucleons, however in the second and the third case it will add spin exchange amplitude.

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