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SU(3)-breaking effects and induced second-class form factors in hyperon beta decays from 2+1 flavor lattice QCD

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We discuss the effects of SU(3) symmetry breaking measured in hyperon beta decays from fully-dynamical lattice QCD. Our calculations are carried out with gauge configurations generated by the RBC and UKQCD collaborations with (2+1)-flavors of dynamical domain-wall fermions and the Iwasaki gauge action at two couplings, $\beta = 2.13$ and 2.25. We have estimated the value of the hyperon vector couplings $f_1(0)$ for $\Sigma \to N$ and $\Xi \to \Sigma$ decays with an accuracy of less than one percent. We then find that lattice results of $f_1(0)$ combined with the best estimate of $|V_{us}|$ with imposing CKM unitarity are slightly deviated from the experimental result of $|V_{us}f_1(0)|$ for the $\Sigma \to N$ decay. This discrepancy can be attributed to an assumption made in the experimental analysis on $|V_{us}f_1(0)|$, where the induced second-class form factor g_2 is set to be zero. We report on this matter and show the preliminary results of $g_2(0)$ evaluated in both indirect and direct ways using lattice QCD.

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1. Introduction

The experimental rate of the hyperon beta decays, $B \to bl\bar{\nu}$, is given by

$$\Gamma = \frac{G_F^2}{60\pi^3}(M_B - M_b)^5(1-3\delta)|V_{us}|^2|f_1^{B\to b}(0)|^2(1+\Delta_{\rm RC})\left[1+3\left|\frac{g_1^{B\to b}(0)}{f_1^{B\to b}(0)}\right|^2+\cdots\right], \tag{1.1}$$

where G_F is the Fermi constant measured from the muon lifetime, which already includes some electroweak radiative corrections [1]. The remaining radiative corrections to the decay rate are approximately represented by $\Delta_{\rm RC}$. Here M_B (M_b) denotes the rest mass of the initial (final) octet baryon state. The ellipsis can be expressed in terms of a power series in the small parameter $\delta = (M_B - M_b)/(M_B + M_b)$, which is regarded as a size of flavor SU(3) breaking. The first linear term in δ, which should be given by $-4\delta[g_2(0)g_1(0)/f_1(0)^2]_{B\to b}$ [1] is safely ignored as small as $\mathscr{O}(\delta^2)$ since the nonzero value of the second-class form factor g_2 [5] should be induced at first order of the δ expansion [2]. The absolute value of $[g_1(0)/f_1(0)]_{B\to b}$ can be determined by measured asymmetries such as electron-neutrino correlation. A theoretical attempt to evaluate SU(3)-breaking corrections on the vector coupling $f_1(0)$ [2] is primarily required for the precise determination of $|V_{us}|$.

According to the Ademollo-Gatto theorem (AGT) [6], the value of $f_1(0)$ can start to deviate from the SU(3) Clebsch-Gordan coefficients (hereafter denoted as $f_1^{\rm SU(3)}(0)$) at the second-order in SU(3) breaking. As the mass splittings among octet baryons are typically of the order of 10-15%, an expected size of the second-order corrections is a few percent level. Although either the size or the sign of their corrections was somewhat controversial among various theoretical studies [7], it is found that the second-order corrections of SU(3) breaking on the hyperon vector couplings $f_1(0)$ are negative and its sizes are estimated as about 3% for both $\Sigma \to N$ and $\Xi \to \Sigma$ decays [3] in our previous work using fully-dynamical lattice QCD simulations [8].

2. Numerical Results

We use 2+1 flavor domain-wall fermions (DWF) lattice QCD ensembles generated by the RBC and UKQCD collaborations at two lattice spacings, $a = 0.114$ fm (coarse) [9] and $a = 0.086$ fm (fine) [10]. Their lattice sizes, $L^3 \times T = 24^3 \times 64$ and $32^3 \times 64$, correspond to almost the same physical volumes ($La \approx 2.7$ fm). The dynamical light and strange quarks are described by DWF actions with fifth dimensional extent $L_5 = 16$ and the domain-wall height of $M_5 = 1.8$ for all ensembles. A brief summary of our simulation parameters with 2+1 flavor DWF ensembles appears in Table. 1.

In this study, all three-point correlation functions are calculated with a source-sink separation of 12(15) in lattice units for $24^3(32^3)$ ensembles, which is large enough to suppress the excited state

[1] Conventionally, $(M_B - M_b)/M_B$ is adopted in Eq. (1.1) to be the small parameter [1, 2] However, our definition of the SU(3)-breaking parameter, $\delta = (M_B - M_b)/(M_B + M_b)$ is theoretically preferable for considering the time-reversal symmetry on the matrix elements of hyperon beta-decays in lattice QCD calculations [3, 4]. Accordingly, a factor of $(M_B + M_b)/M_B$ is different in definitions of g_2, g_3, f_2 and f_3 form factors in comparison to those adopted in experiments.

[2] The vector coupling $f_1(0)$ is given by SU(3) Clebsch-Gordan coefficients in the exact SU(3) limit.

[3] In the iso-spin limit ($m_u = m_d$), all hyperon beta-decays can be classified in four types of decays as $\Lambda \to N$, $\Sigma \to N$, $\Xi \to \Lambda$ and $\Xi \to \Sigma$.

β	a^{-1} [GeV]	am_{ud}	$N_{\rm conf}$	MD range	$N_{\rm sep}$	$N_{\rm meas}(f_1)$	$N_{\rm meas}(g_2)$	M_π[GeV]
2.13	1.73(3)	0.005	240	940-5720	20	8	4	0.3292(7)
		0.01	120	5060-7440	20	8	4	0.4214(14)
		0.02	80	1890-3470	20	8	4	0.5569(15)
2.25	2.28(3)	0.004	120	1000-3380	20	8	N/A	0.2902(11)
		0.006	120	1000-3380	20	8	N/A	0.3445(9)
		0.008	120	580-2960	20	8	N/A	0.3926(11)

Table 1: Summary of simulation parameters: the number of gauge configurations, the range, where measurements were made, in molecular-dynamics (MD) time, the number of trajectory separation between each measured configuration, the number of measurements for $f_1(0)$ and $g_2(q^2)$. The table also lists the pion masses [9, 10].

contributions [11, 12]. Our previous results of $f_1(0)$ from the 24^3 ensembles with less number of measurements were published in Ref [8], while preliminary results of $f_1(0)$ from the 32^3 ensembles were first reported in Ref [13]. Details of how to calculate the vector coupling $f_1(0)$ and the induced second-class form factor $g_2(q^2)$ are described in Ref. [4].

2.1 Vector coupling $f_1(0)$

We first show the results of $\tilde{f}_1(0) = f_1(0)/f_1^{\rm SU(3)}(0)$ [4] obtained from both the 24^3 (open circles) and 32^3 (open diamonds) ensembles as a function of the pion mass squared for $\Sigma \to N$ (left panel) and $\Xi \to \Sigma$ (right panel) in Fig 1. In order to estimate $\tilde{f}_1(0)$ at the physical point, we perform a combined global-fit of both coarse and fine lattice data on $\tilde{f}_1(0) = f_1(0)/f_1^{\rm SU(3)}(0)$ as multiple functions of $M_K^2 - M_\pi^2$ and $M_K^2 + M_\pi^2$:

$$\tilde{f}_1(0) = C_0 + (C_1 + C_2 \cdot (M_K^2 + M_\pi^2)) \cdot (M_K^2 - M_\pi^2)^2, \tag{2.1}$$

which form is motivated by AGT [4]. Here, we remark that our simulations are performed with a strange quark mass slightly heavier than the physical mass [9, 10]. To take into account this slight deviation in this global analysis of the chiral extrapolation, we simply evaluate a correction using the Gell-Mann-Oakes-Renner relation for the pion and kaon masses, which corresponds to the quark mass dependence of pseudo-scalar meson masses at the leading order of ChPT [4]. In Fig. 1, the dashed curve shows the fit result at the physical kaon mass, while the difference $\tilde{f}_1(0, M_K^{\rm latt}) - \tilde{f}_1(0, M_K^{\rm phys})$ has been subtracted from raw data points of both the 24^3 and 32^3 lattice simulations. We then get the final results at the physical point as

$$f_1^{\Sigma\to N}(0) = -0.9646(31), \quad f_1^{\Xi\to\Sigma}(0) = +0.9739(26), \tag{2.2}$$

where the quoted errors are only statistical. Those values are consistent with our previous works, which are performed with only the 24^3 ensembles, while the errors are significantly reduced.

Using the best estimate of $|V_{us}| = 0.2254(8)$ with imposing CKM unitarity [14], we then predict the values $|V_{us} f_1(0)|_{\Sigma\to N} = 0.2174(6)_{V_{us}}(7)_{f_1}$ and $|V_{us} f_1(0)|_{\Xi\to\Sigma} = 0.2195(8)_{V_{us}}(6)_{f_1}$. Although the latter is barely consistent with a single experimental result of $|V_{us} f_1(0)|_{\Xi\to\Sigma} = 0.209(27)$,

[4]Here, $f_1^{\rm SU(3)}(0) = -1$ for $\Sigma \to N$, while $f_1^{\rm SU(3)}(0) = +1$ for $\Xi \to \Sigma$.

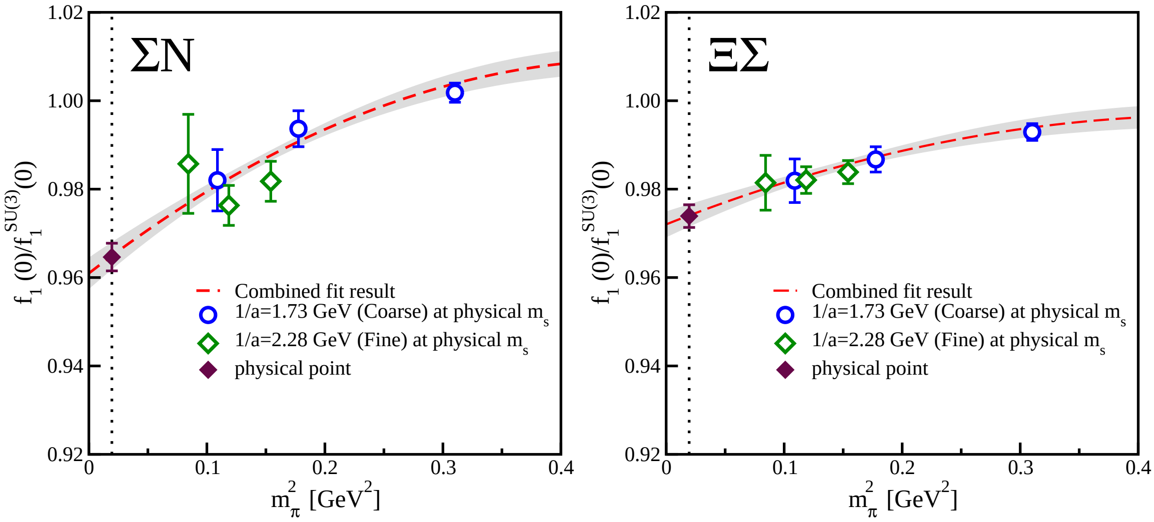

Figure 1: Chiral extrapolation of $\tilde{f}_1(0)$ for $\Sigma \to N$ (left) and $\Xi \to \Sigma$ (right).

the former is slightly deviated from the currently available experimental result of $|V_{us} f_1(0)|_{\Sigma\to N} = 0.2282(49)$ and then reveals a 2σ tension.

This discrepancy might be explained by the following reason. Through a polarized-Σ^- beta-decay experiment, $g_1(0)/f_1(0)$ can be determined as a function of $g_2(0)/f_1(0)$ [1]. This yields the constraint $g_1(0)/f_1(0) - 0.133 g_2(0)/f_1(0) = -0.327(20)$ for $\Sigma \to N$ [15]. Then, the conventional assumption $g_2(0) = 0$ gives the final value of $g_1(0)/f_1(0) = -0.327(20)$, that is used in the experimental analysis on $|V_{us} f_1(0)|_{\Sigma\to N}$ determined from the decay rate (1.1) [1, 15]. The assumption $g_2(0) = 0$ is no longer valid without the exact SU(3) flavor symmetry [5]. Therefore, the 2σ discrepancy may be associated with this assumption.

The value of $g_2(0)$ should be subject to the first order corrections of SU(3) breaking, which are an order of 10-15%. Indeed, non-zero values of $g_2(0)$ are reported as the size of the first order corrections from quenched lattice QCD for both $\Sigma \to N$ [3] and $\Xi \to \Sigma$ [4] channels. On the other hand, a test of the CKM unitary through the first row relation $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$ reaches less than a sub-percent level accuracy using the value of V_{us} given by the average of the K_{l3} and $K_{\mu 2}$ determinations [16]. Therefore, let us now use the CKM unitarity with our theoretical estimate of $f_1(0)$ so as to read off $g_2(0)$ from the $\Sigma \to N$ decay rate and the constraint $|g_1(0)/f_1(0) - 0.133 g_2(0)/f_1(0)|_{\Sigma\to N} = 0.327(20)$ in experiments. We then evaluate the value of $g_2(0) \approx 0.46$, which is roughly consistent with the size of the first order corrections and also the results from quenched lattice QCD [3, 4].

2.2 Induced second-class form factor $g_2(q^2)$

The general form of the baryon matrix element for hyperon beta decay, $B \to b$, is composed of the vector and axial-vector transitions, $\langle b(p')|V_\mu(x) + A_\mu(x)|B(p)\rangle$, which are described by six form factors: f_1, f_2 and f_3 for the vector part and g_1, g_2 and g_3 for the axial-vector part. All six form factors in the hyperon beta decay can be measured in lattice QCD simulations [3, 4]. Hereafter, we focus on the axial-vector part of the baryon matrix element. We adopt the local axial current $A_\mu(x) = \bar{u}(x)\gamma_\mu\gamma_5 s(x)$ for the current operator and then define the finite-momentum three-point function with the baryon interpolating operators $\mathcal{O}_B$ and $\mathcal{O}_b$ for the initial (B) and final (b) states that carry fixed momentum $\mathbf{p}$ and $\mathbf{p}'$, respectively. The current operator, hence, has a three-dimensional momentum transfer $\mathbf{q} = \mathbf{p} - \mathbf{p}'$. In this study, the z direction is chosen as the polarized direction. We thus have three-types of the projected correlation functions with a projection operator $\mathcal{P}_z^5 = (1+\gamma_4)\gamma_5\gamma_z$ for decay process $B(\mathbf{p}) \to b(\mathbf{p}')$:

$$\Lambda_L^{A,B\to b}(q^2,q_z) \propto \frac{1}{4}\mathrm{Tr}\left\{\mathcal{P}_z^5\langle\mathcal{O}_b(t_{\rm sink},\mathbf{p}')A_z(t,\mathbf{q})\overline{\mathcal{O}}_B(t_{\rm src},-\mathbf{p})\rangle\right\}, \tag{2.3}$$

$$\Lambda_T^{A,B\to b}(q^2,q_z) \propto \frac{1}{4}\mathrm{Tr}\left\{\mathcal{P}_z^5\langle\mathcal{O}_b(t_{\rm sink},\mathbf{p}')A_{x,y}(t,\mathbf{q})\overline{\mathcal{O}}_B(t_{\rm src},-\mathbf{p})\rangle\right\}, \tag{2.4}$$

$$\Lambda_0^{A,B\to b}(q^2,q_z) \propto \frac{1}{4}\mathrm{Tr}\left\{\mathcal{P}_z^5\langle\mathcal{O}_b(t_{\rm sink},\mathbf{p}')A_t(t,\mathbf{q})\overline{\mathcal{O}}_B(t_{\rm src},-\mathbf{p})\rangle\right\}, \tag{2.5}$$

where $q = p - p'$ and q_z denotes the z-component of $\mathbf{q}$ corresponding to longitudinal momentum. The explicit q_z-dependence, appeared in three-point correlation functions, stems from our choice of the polarized direction. Details of definitions of $\Lambda_i^{A,B\to b}(q^2,q_z)$ $(i = L,T,0)$ are described in Ref. [4].

All three form factors g_1, g_2 and g_3 in the axial-vector matrix element are obtained from appropriate linear combinations of quantities $\Lambda_i^{A,B\to b}(q^2,q_z)$. In this study, we calculate the three nonzero three-momentum transfer $\mathbf{q} = (2\pi/L)\mathbf{n}$ ($\mathbf{n}^2 = 1,2,3$) for the hyperon decay process at the rest flame of the final states ($\mathbf{p}' = \mathbf{0}$). When $|\mathbf{q}| \neq 0$ for $|\mathbf{p}'| = 0$, the induced second-class form factor g_2 can be given by the following combination,

$$g_2^{B\to b}(q^2) = \frac{M_B+M_b}{2M_b}\left[\Lambda_L^{A,B\to b}(q^2,q_z=0) - \Lambda_0^{A,B\to b}(q^2,q_z) - \frac{E_B-M_b}{M_b}\Lambda_T^{A,B\to b}(q^2,q_z)\right], \tag{2.6}$$

where $E_B = \sqrt{M_B^2+\mathbf{p}^2}$ [4]. In addition, we calculate the time-reversal process ($b \to B$) as well as $B \to b$ decay process. Note that the induced second form factor $g_2(q^2)$ are supposed to have a relation $g_2^{B\to b}(q^2) = -g_2^{b\to B}(q^2)$ in our convention.

In Fig. 2, we show our preliminary results of unrenormalized $g_2(q^2)$ for the $\Sigma \to N$ decay as a function of four momentum squared q^2 for the mass of two light degenerate quarks at $am_{ud} = 0.005$ (left panel), 0.01 (central panel) and 0.02 (right panel) with a fixed strange quark mass, $am_s = 0.04$, on the 24^3 ensembles. The size of SU(3) breaking becomes increase from the right panel to the left panel. Accordingly, the non-zero g_2 form factor becomes visible in the left panel and it shows an upward trend toward from finite q^2 to $q^2 = 0$ though it is difficult to determine the precise q^2-dependence of $g_2(q^2)$ within the current statistics. It is observed that the bare value of $g_2(q^2)$ is roughly equal to 0.3 at the smallest q^2-value for $am_{ud} = 0.005$. A factor of $Z_A \approx Z_V \sim 0.72$ must be multiplied to the bare values [5] so as to get the renormalized values of $g_2(q^2)$. Taking into account the fact that the parameter $\delta_{\Sigma N}$ at $am_{ud} = 0.005$ is approximately 0.08, which is about 30% smaller than the physical value of $\delta_{\Sigma N} = 0.12$, the observed non-zero value of $g_2(q^2)$ at $q^2 \sim 0.13$ GeV^2 in the direct calculation are consistent with an indirect estimation of $g_2(0)$ at the physical point with the CKM unitarity constraint.

[5] In this study, we use the vector and axial-vector local currents, which receive finite renormalization relative to their continuum counterparts. However, the well-preserved chiral and flavor symmetries of DWFs yield a common renormalization: $Z_V = Z_A$, up to higher-order discretization errors, $\mathcal{O}(a^2)$ in the chiral limit [11].

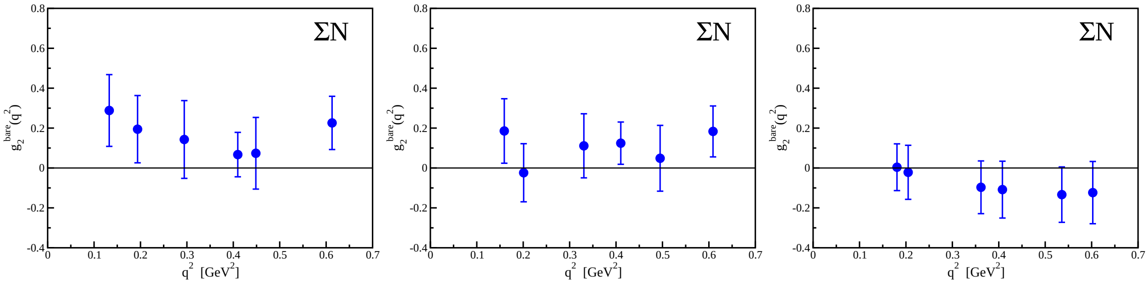

Figure 2: Induced second-class form factor $g_2^{\rm bare}(q^2)$ at $am_{ud} = 0.005$ (left), 0.01 (center) and 0.02 (right) for the 24^3 ensembles.

3. Summary

We have studied the SU(3) breaking effects on the hyperon beta decays using 2+1 flavor dynamical lattice QCD. The theoretical estimate of the hyperon vector coupling $f_1(0)$ reaches a sub percent level accuracy. Then, we found that the current $\Sigma \rightarrow N$ data with lattice input of $f_1(0)$ moves slightly off the CKM unitarity condition. Conversely, we think that this observation would expose a size of the induced second-class form factor g_2, which was less-known and ignored in experiments [1]. We then estimate it as roughly $g_2(0) \approx 0.46$ under the CKM unitarity condition. Its size is indeed consistent with the size of the first-order SU(3) symmetry-breaking corrections. It is also found that in lattice direct measurement, non-zero g_2 form factor is likely evident and its size is roughly consistent with the indirect estimation. Thus, it is most likely that the CKM unitarity could be satisfied in the $\Sigma \rightarrow N$ decay within the current experimental accuracy.

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