PROCEEDINGS OF SCIENCE

# Image Denoising Using Shearlet Transform and Nonlinear Diffusion

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In this paper, we present a new image denoising method for removing Gaussian noise from corrupted image by using shearlet transform and nonlinear diffusion. The image is decomposed by the shearlet transform to obtain the shearlet coefficients in each subband; then a diffusion scheme based on statistical property of shearlet coefficients is used to shrink noisy shearlet coefficients. The test shows the proposed method has better performance compared to the relevant methods.

*CENet2015*
*12-13 September 2015*
*Shanghai, China*

[1]Speaker

[2]This work is partially supported by National Nature Science Foundation of China (Grant No. 61401383) and Natural Science Foundation of Xianyang Normal University (Grant No. 14XSYK006) and Qinglan Talent Program of Xianyang Normal University (Grant No. XSYQL201503).





PoS(CENet2015)033

## 1. Introduction

Noise removal for images has received considerable attention in last several decades. A lot of methods are proposed to deal with this problem. Nonlinear anisotropic diffusion methods achieve success for image denoising. It was proposed by Perona and Malik in the early 1990s [1]. The original diffusion model is called Perona-Malik anisotropic diffusion (PMAD). In the process of denoising, the PMAD creates a family of restored signals by starting from the noisy signal and evolving it locally according to a process described by PMAD equation. Afterwards, the PMAD is modified with different purposes [2, 3]. Recently, a new gradient domain method (NGND) proposed by Zhang et al. achieved better performances [4]. On the other hand, since nonlinear diffusion methods cannot preserve the details and textures well, some researchers have introduced the diffusion scheme into the wavelet domain [5].

In this paper, we also present a wavelet domain nonlinear diffusion denoising method. The noisy image is decomposed by using the nonsubsampled shearlet transform (NSST) [6]. The modified NGND scheme is used to remove noise.

The rest of this paper is organized as follows: Section 2 presents the proposed method, Section 3 presents experimental results and the good performance is shown and Section 4 concludes the paper.

## 2. The Proposed Method

Assume an original image $f$ be degraded by additive white Gaussian noise with zero mean and variance $\sigma^2$ and the noise is signal independent. Let $I_0$, $I$ and $n$ represent a noisy NSST coefficient, noise-free coefficient, and noise component, respectively; thus the NSST coefficient of noisy image at the same band can be expressed as

$$I_0(i,j) = I(i,j) + n(i,j) \tag{2.1}$$

### 2.1 The Shearlet Transform

Compared with wavelet transform, the shearlet transform has five properties: well localizing, parabolic scaling, highly directional sensitivity, spatially localizing and optimally sparse [6]. In dimension $n = 2$, the continuous shearlet transform is defined as the mapping

$$SH_{\psi} f(a,s,t) = \langle f, \psi_{a,s,t} \rangle \tag{2.2}$$

where $a > 0$, $s \in R$, $t \in R^2$ and the analyzing elements $\psi_{a,s,t}$ are called shearlets, which are given by

$$\psi_{a,s,t}(x) = \left|\det M_{a,s}\right|^{-\frac{1}{2}} \psi(M_{a,s}^{-1} x - t) \tag{2.3}$$

where $M_{a,s} = \begin{pmatrix} a & \sqrt{a}s \\ 0 & \sqrt{a} \end{pmatrix} = B_s A_a$, where $A_a = \begin{pmatrix} a & 0 \\ 0 & \sqrt{a} \end{pmatrix}$ and $B_s = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$. An anisotropic dilation is produced by $A_a$ and a shearing is produced by $B_s$. The shearlet coefficients of large magnitude come from edges. Each $f \in L^2(R^2)$ can be recovered by

$$f = \int_{R^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \langle f, \psi_{a,s,t} \rangle \psi_{a,s,t} \frac{da}{a^3} ds dt \tag{2.4}$$

By sampling the continuous shearlet transform $SH_{\psi}(a,s,t)$ on appropriate discretizations of the scaling, shear, and translation parameters $a$, $s$, $t$, one obtains a discrete transform. Choose $a = 2^{-2j_1}$ and $s = -l\sqrt{a}$ with $j_1, l \in Z$, we obtain the collection of matrices $M_{2^{-2j_1}, -l}$. Note that

$$M_{2^{-2j_1}, -l}^{-1} = M_{2^{2j_1}, l} = \begin{pmatrix} 2^{2j_1} & l 2^{j_1} \\ 0 & 2^{j_1} \end{pmatrix} = B_0^l A_0^{j_1} \tag{2.5}$$

where $A_0=\begin{pmatrix}4 & 0\\ 0 & 2\end{pmatrix}$ and $B_0=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$ ; therefore, the discrete system of shearlets can be expressed as

$$\psi_{a,s,t}=|\det A_0|^{j_1/2}\psi(B_0^l A_0^{j_1}x-k_1) \tag{2.6}$$

for , $j_1, l \in Z$ , $k_1 \in Z^2$ . By appropriately choosing a $\psi$ , for each $f \in L^2(R^2)$ , we have the reproducing formula

$$f=\sum_{j_1,l\in Z,k_1\in Z^2} <f,\psi_{j_1,l,k_1}>\psi_{j_1,l,k_1} \tag{2.7}$$

Afterwards, the discrete shearlet transform using band-limited functions is proposed [6]. Define $\hat{\psi}^{(0)}(\xi)=\psi_1(\xi_1)\psi_2(\xi_2/\xi_1)$ , $\hat{\psi}^{(1)}(\xi)=\psi_1(\xi_2)\psi_2(\xi_1/\xi_2)$ and $A_1=\begin{pmatrix}2 & 0\\ 0 & 4\end{pmatrix}$ , $B_1=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$ . Given $\psi_{a,s,t}=2^{3j_1/2}\psi^d(B_d^l A_d^{j_1}-k_1)$ , for $d=0,1$, where $j_1\geq 0$ , $-2^{j_1}\leq l\leq 2^{j_1}-1$ , and $k_1\in Z^2$ . Based on $\psi_1$ and $\psi_2$ , filters $v_{j_1}$ and $w_{j_1,l}^{(d)}$ can be found so that $\langle f,\psi_{j_1,l,k_1}\rangle$ = $f*(v_{j_1}*w_{j_1,l}^{(d)})[k_1]$ . This implementation is carried out with two ways. One is the frequency-domain approach and another is the time-domain approach. The author shows the time-domain approach is highly effective for the purpose of denoising. The matlab code of the time-domain approach comes from http://www.math.uh.edu/~dlabate/software.html. In this software, the nonsubsampled shearlet transform using hard thresholding (Shear-HT) is implemented. A Meyer-based shearing filter of size 16 with 16 directions has been applied to the first and second decomposition level with a Meyer-based shearing filter of size 32 with 8 directions applied to the third and fourth decomposition level. In our approach, the same shearlet transform is used.

### 2.2 The Diffusion Scheme in Gradient Domain

Recently, a new gradient-based nonlinear diffusion (NGND) is proposed for image denosing [4]. Unlike previous traditional gradient-based method, the NGND method uses the prefilter in gradient domain which consists of square gradient coefficients. The denoising scheme of NGND is shown as follows

$$f_k(i1,j1)=f_{k-1}(i1,j1)+\Delta t\sum_{u=1}^{4} C_{k-1}^u(i1,j1)(f_{k-1}^u(i1,j1)-f_{k-1}(i1,j1)) \tag{2.8}$$

where $f_k(i1,j1)$ denotes the gray value of pixel at the coordinate $(i1,j1)$ after the *k*th iteration is finished, and $f_0(i1,j1)=f(i1,j1)$. The $f_{k-1}^1(i1,j1)$ , $f_{k-1}^2(i1,j1)$ , $f_{k-1}^3(i1,j1)$ and $f_{k-1}^4(i1,j1)$ represent the gray values of pixels of north, south, east and west of centered pixel at $(i1,j1)$, respectively. The $f_{k-1}^u(i1,j1)-f_{k-1}(i1,j1)$ represents gradient coefficient $\nabla f_{k-1}^u(i1,j1)$ . The $\Delta t$ is the time step. Let $K$ be the gradient threshold, and then the diffusion function can be written as

$$C_{k-1}^u(i1,j1)=\frac{1}{1+\frac{\overline{\nabla f_{k-1}^u(i1,j1)^2}}{K^2}} \tag{2.9}$$

where

$$\overline{\nabla f_{k-1}^u(i1,j1)^2}=(1/m)\sum_{(\overline{i1},\overline{j1})\in\omega}\nabla f_{k-1}^u(\overline{i1},\overline{j1})^2 \tag{2.10}$$

where $\omega$ is a local window centered at processed gradient coefficient $\nabla f_{k-1}^u(i1,j1)$ and $M$ is the size of $\omega$.

Obviously, in NGND, the mean filter is implemented in the gradient domain. The previous prefilter usually operates in the image domain. Although the NGND obtains good performance when compared with the related gradient based method, this method can not still effectively process the images with rich details and textures. Inspired by the previous wavelet domain nonlinear diffusion, we introduce NGND into shearlet transform domain. This is because the shearlet can effectively capture the geometry feature of image and have more directions as said in Section 2.1.

### 2.3 The Proposed Denoising Algorithm

Let $\Delta t = 1/5$ , after a series of manipulation, (2.8) can be rewritten as

$$f_k(i1,j1)=\overline{f_{k-1}(i1,j1)}+\frac{1}{5}\sum_{u=1}^{4}\left(\frac{\overline{f^u_{k-1}(i1,j1)^2}}{K^2+\overline{\nabla f^u_{k-1}(i1,j1)^2}}\right)\left(f_{k-1}(i1,j1)-f^u_{k-1}(i1,j1)\right) \quad (2.11)$$

where $\overline{f_{k-1}(i1,j1)}$ is the mean value of $f^1_{k-1}(i1,j1)$, $f^2_{k-1}(i1,j1)$, $f^3_{k-1}(i1,j1)$, $f^4_{k-1}(i1,j1)$ and $f_{k-1}(i1,j1)$. In the shearlet transform domain, the gray value of pixel in image domain in (2.11) is substituted with NSST coefficient. The $\nabla f^u_{k-1}(i1,j1)$ is also substituted with NSST coefficient since the shearlet coefficients of large magnitude usually come from edges. At this time, let $\omega_1$ be a local window which consists of the coordinates of four neighbors of north, south, east and west of processed NSST coefficient and the coordinate of processed NSST coefficient, and then $\overline{I_{k-1}(i,j)}$ denotes the mean value of NSST coefficient in local window $\omega_1$. The $\overline{I_{k-1}(i,j)^2}$ is the mean value of square NSST coefficient in local square window $\omega_2$ centered at the coordinate of processed NSST coefficient; moreover, as NSST has been highly anisotropic, let $\overline{I_{k-1}(i,j)^2}$ substitute $\overline{I^u_{k-1}(i,j)^2}$, which means that nonlinear isotropic diffusion is implemented in shearlet domain. In this sense, in the shearlet domain, (2.11) is expressed as

$$I_k(i,j)=\overline{I_{k-1}(i,j)}+\frac{\overline{I_{k-1}(i,j)^2}}{K^2+\overline{I_{k-1}(i,j)^2}}\left(I_{k-1}(i,j)-\overline{I_{k-1}(i,j)}\right) \quad (2.12)$$

In the shearlet-based Wiener filter (Shear-Wiener), NSST coefficient is processed with Wiener filter [7]. This means that NSST coefficient can be modeled as Gaussian distribution with zero mean. So, the mean value $\overline{I_{k-1}(i,j)}$ of NSST coefficient in local window $\omega_1$ can be approximately considered as zero; thus in the shearlet transform domain, (2.12) is turned into:

$$I_k(i,j)=\frac{\overline{I_{k-1}(i,j)^2}}{K^2+\overline{I_{k-1}(i,j)^2}}I_{k-1}(i,j) \quad (2.13)$$

where $I_k(i,j)$ represents restored NSST coefficient at *k*th iteration. $I_0(i,j)$ denotes original noisy NSST coefficient. In practice, $K^2$ is a constant to be set.

The concrete denoising algorithm is as follows:

Step 1: compute the NSST of noisy image.

Step2: obtain the denoised NSST coefficient by using (2.13). Note that, as usual, in denoising algorithms, the lowpass coefficient is not processed.

Step3: compute the inverse NSST transform by using denoised NSST coefficient.

## 3. The Experimental Results

The test natural images are Barbara and Lena of size 256×256. Two original images are corrupted by additive zero-mean white Gaussian noise (with standard deviation $\sigma = 10, 20$ ). The Shear-HT and NGND method are compared firstly. The experimental results are measured by the peak signal-to-noise ratio (PSNR) in decibels (dB), which is defined as

$$\mathrm{PSNR} = 10\ \log_{10}\frac{255^2}{\mathrm{MSE}} \quad (3.1)$$

where MSE = $(1|rs)\sum_{m=1}^{r}\sum_{n=1}^{s}(f_{m,n}-W_{m,n})^2$ , $f$ is the original noise-free image, $W$ is the estimator of $f$ , $rs$ is the number of pixels. A higher PSNR suggests that the filtered image $W$ is closer to the reference noise-free image $f$ .

As to the proposed method, the sizes of the square window $\omega_2$ are 13×13, 5×5, 5×5 and 5×5 in turn from the coarsest scale to the finest one. The iteration number is 5 and $K^2 = 0.29\ \hat{\sigma}^2$ (the variance $\hat{\sigma}^2$ of noise in NSST domain is estimated by using a Monte-Carlo technique). For NGND, the stopping time is taken when the best PSNRs are obtained. The rest of the parameters are set according to the corresponding literature. For Shear-HT, its parameters are provided in http://www.math.uh.edu/~dlabate/software.html.

| Method | Noise variance | | | |
|---|---|---|---|---|
| | 100 | 400 | 100 | 400 |
| | Lena(256×256) | | Barbara(256×256) | |
| Shear-HT | 33.15 | 29.77 | 32.87 | 28.91 |
| NGND | 33.68(30) | 30.06(28) | 32.74(29) | 28.75(24) |
| Proposed | **33.80**(5) | **30.15**(5) | **33.80**(5) | **30.01**(5). |

**Table 1:** The PSNRs (in dB) presented by Shear-HT, NGND and the proposed method (numbers of iterations used are indicated in parenthesis)

From Table 1, we can find that the proposed method obtains higher PSNRs compared to NGND and Shear-HT [4,6]. Especially for Barbara image, the proposed method obtains about 1dB improvement compared to Shear-HT and NGND methods.

From Fig. 1, we can see that Fig. 1c with Shear-HT and Fig. 1d with NGND oversmooth the features of image; furthermore, the line texture is blurred in the upper right corner with Fig. 1d. For this, NGND produced bad results. The proposed method achieves the best visual effects since the textures and edges are preserved well when the noise is removed.

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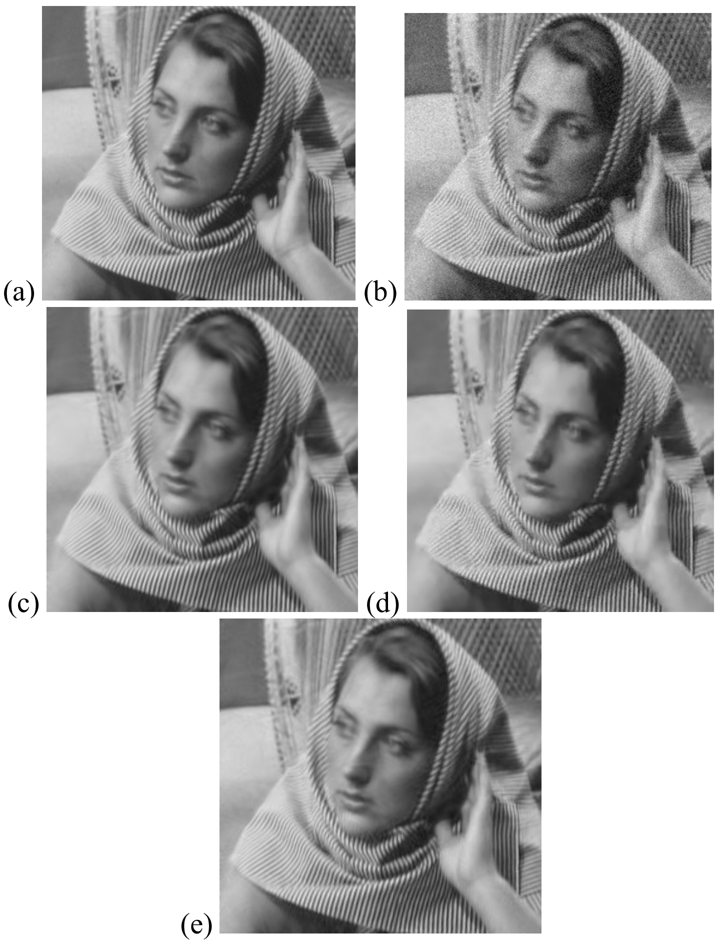


**Figure 1:** Comparison of the restoration results from the different methods. Zoom into file for a better view. (a) original Barbara (256×256) image. (b) noisy image ($\sigma = 10$). (c-e) shows restored Barbara images using Shear-HT, NGND and Proposed, respectively.

In order to further verify the performance of proposed method, the context based diffusion in stationary wavelet domain (SWCD) and the Shear-Wiener are also used to compare [5, 7]. For the lack of source codes, the comparisons are only in term of the reported PSNRs. From Table 2 obviously, the proposed method outperforms SWCD and Shear-Wiener in PSNR.

| Method | Noise variance | | | |
|---|---|---|---|---|
| | 100 | 400 | 100 | 400 |
| | Lena(512×512) | | Barbara(512×512) | |
| SWCD | 32.4 | 29.79 | 26.24 | 25.28 |
| Shear-Wiener | 35.17 | 31.84 | 32.56 | 28.75 |
| Proposed | **35.50**(5) | **32.46**(5) | **34.13**(5) | **30.54**(5) |

**Table 2:** The PSNRs (in dB) presented by SWCD, Shear-Wiener and the proposed method (numbers of iterations used are indicated in parenthesis)

## 4. Conclusion

This paper presents an effective image denoising algorithm. In this method, the shearlet transform is used to decompose noisy image so that the details and textures of image can be captured more accurately. The diffusion scheme has also improved the edge enhancement and iterative noise reduction features when compared to wavelet shrinkage denoising method; in this sense, the proposed method has advantage over other related denoising algorithm. The tests show the power of the proposed method.

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