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Pseudo-scalar Higgs boson form factors at 3 loops in QCD

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In the limit of large top quark mass, using an effective Lagrangian that only admits light degrees of freedom we compute the three-loop QCD corrections to the quark and gluon form factors of a pseudo-scalar Higgs boson. Using the universal infrared factorisation properties, the three-loop operator mixing and finite operator renormalisation are re-derived thereby confirming the results in the operator product expansion. We find that our findings are consistent with the anomaly equation. We derive the hard matching coefficient in soft-collinear effective theory in order to perform the matching of the SCET-based resummation onto the full QCD calculation up to the three loop order.

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1. Introduction

Form factors play an important role in the computation of higher order radiative corrections to scattering processes at the Large Hadron Colliders (LHC). They are nothing but matrix elements of local field operators between on-shell quark and gluon states. In the present work, we compute the three-loop QCD corrections to the quark and gluon form factors for pseudo-scalar operators. They appear in effective field theory descriptions of extensions of the Standard Model [1, 2, 3, 4, 5, 6, 7, 8]. The recent discovery of a Standard-Model-like Higgs boson at the LHC [9, 10] prompted the community to study the properties of the discovered boson in order to identify either with lightest scalar or pseudo-scalar Higgs bosons of extended models. These corrections can be used to obtain the next-to-next-to-next-to leading order and leading log (N^3LO and N^3LL) gluon fusion cross sections [11, 12] for pseudo-scalar Higgs boson production, thereby reducing the theoretical uncertainities resulting from renormalisation and factorisation scales. We use the universal infra-red (IR) pole structure of the form factors to determine the ultraviolet (UV) renormalisation constants and mixing of the effective operators up to three loop level. The finite renormalisation constant, known up to the three loops [13], that preserves one loop nature of the chiral anomaly, is shown to be consistent with anomalous dimensions of the overall renormalisation constants. We also derive the hard matching coefficients for N^3LL resummation in soft collinear effective theory (SCET).

2. Effective theory and Form Factors

The effective Lagrangian [14] describing the interaction between a pseudo-scalar Φ^A and QCD particles reads:

$$\mathscr{L}^A_{\rm eff} = \Phi^A(x)\Big[-\frac{1}{8}C_G O_G(x) - \frac{1}{2}C_J O_J(x)\Big] \tag{2.1}$$

where the operators are defined as

$$O_G(x) = G^{\mu\nu}_a \tilde{G}_{a,\mu\nu} \equiv \varepsilon_{\mu\nu\rho\sigma} G^{\mu\nu}_a G^{\rho\sigma}_a\,, \qquad O_J(x) = \partial_\mu\left(\bar{\psi}\gamma^\mu\gamma_5\psi\right)\,. \tag{2.2}$$

The constants C_G and C_J are Wilson Coefficients that depend on the mass of the top quark m_t. C_G does not receive any QCD corrections beyond one loop due to the Adler-Bardeen theorem [15], but C_J does through the strong coupling constant. Defining $a_s \equiv g_s^2/(16\pi^2) = \alpha_s/(4\pi)$, they are given by

$$\begin{aligned} C_G &= -a_s 2^{\frac{5}{4}} G_F^{\frac{1}{2}} \cot\beta \\ C_J &= -\left[a_s C_F\left(\tfrac{3}{2} - 3\ln\tfrac{\mu_R^2}{m_t^2}\right) + a_s^2 C_J^{(2)} + \cdots\right] C_G\,. \end{aligned} \tag{2.3}$$

$G^{\mu\nu}_a$ and ψ represent gluonic field strength tensor and light quark fields, respectively and G_F is the Fermi constant and $\cot\beta$ is the mixing angle in the Two-Higgs-Doublet model. a_s is renormalised at the scale μ_R.

The computation of higher order terms in dimensional regularisation when γ_5 is present involves a proper definition in $d \neq 4$. We have followed the most practical and self-consistent definition of γ_5 which was introduced by 't Hooft and Veltman through [16]

$$\gamma_5 = i\frac{1}{4!}\varepsilon_{\nu_1\nu_2\nu_3\nu_4}\gamma^{\nu_1}\gamma^{\nu_2}\gamma^{\nu_3}\gamma^{\nu_4}\,. \tag{2.4}$$

Here, $\varepsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor where the indices span over d dimensions.

The calculation of the unrenormalized pseudo-scalar form factors up to three loops are done using the method that was followed to compute three-loop scalar and vector form factors [17, 18]. The transition matrix elements are generated using QGRAF [19]. The resulting large number of scalar integrals are expressible in terms of a much smaller set of scalar integrals, called master integrals (MIs) using integration-by-parts (IBP) [20, 21] and Lorentz invariance (LI) [22] identities. While the LI identities are not linearly independent from the IBP identities [23], they do however help to get the solution efficiently. Using the Laporta algorithm, [24], a reduction to MIs [25, 26, 27, 28, 29] is accomplished with the help of the packages LiteRed [30, 31]. We use Reduze2 [32, 33] for momentum shifts.

The UV renormalisation for the form factors requires the renormalisation of the coupling constant and of the operators. The formalism used for the γ_5 matrix fails to preserve the anti-commutativity of γ_5 with γ^μ in d dimensions violating Ward identities. To rectify this, one needs to introduce a finite renormalisation constant Z_5^s [13, 34] in addition to other constants $Z_{ij}, i, j = G, J$, i.e., $[O_i]_R = Z_{ij}[O_j]_B$ with $Z_{JG} = 0, Z_{JJ} \equiv Z_5^s Z_{\overline{MS}}^s$. These constants are already available to the required accuracy [13, 35] to obtain UV finite form factors of the pseudo-scalar operators to three loop level in QCD. In this article, we have recalculated them from our on-shell amplitudes computed up to the three loop level using the structure of infra-red poles. The infrared divergences in QCD amplitudes exhibit an universal behaviour. The very first successful proposal was by Catani [36] (see also [37]) at one and two loop level in QCD. The structure of the single pole in quark and gluon form factors in terms of soft and collinear anomalous dimensions was first observed in [38] up to two loop level and at three loop, it was established in the article [39]. The generalisation to Catani's proposal beyond two loops was achieved by Becher and Neubert [40] and by Gardi and Magnea [41]. We have calculated all the required renormalisation constants Z_{ij} from the consistency conditions on the universal structure of the infrared poles of the UV renormalised form factors. In addition, we have used the anomaly equation to determine Z_5^s. We find that they are in agreement with those derived using a completely different approach. Using these constants, we obtain the UV-finite form factors. They are presented in [42].

The renormalisation group invariance of the anomaly equation [13], i.e.,

$$[O_J]_R = a_s \frac{n_f}{2} [O_G]_R \,. \tag{2.5}$$

gives

$$\gamma_{JJ} = \frac{\beta}{a_s} + \gamma_{GG} + a_s \frac{n_f}{2} \gamma_{GJ} \,. \tag{2.6}$$

where the anomalous dimensions γ_{ij} are defined as

$$\mu_R^2 \frac{d}{d\mu_R^2} Z_{ij} \equiv \gamma_{ik} Z_{kj} \qquad\qquad i, j, k = G, J. \tag{2.7}$$

We find that our results on γ_{GG} and γ_{GJ} are in agreement to $\mathscr{O}(a_s^2)$ with [13] and to $\mathscr{O}(a_s^3)$ with [35]. It was observed through explicit computation in the article [13] that $\gamma_{GG} = -\beta/a_s$ holds true up to two loop level. The results from [35] based on the operator product expansion show that it is also valid at three loops. Here, through explicit calculation, we arrive at the same conclusion. The anomalous dimension thus obtained up to the three loop level are not only in complete agreement with the earlier results but also consistent with the above anomaly equation, i.e., in the limit of $d \to 4$. This serves as one of the most crucial checks on our computation.

The relation between the QCD form factors and those of the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory was found in [43, 44, 45], called the leading transcendentality principle which relates anomalous dimensions of the twist two operators in $\mathcal{N} = 4$ SYM to the leading transcendental (LT) terms of such operators computed in QCD. We find that the LT terms of the diagonal form factors are identical to those of quark and gluon form factors when $C_A = C_F = N$ and $T_f n_f = N/2$ and that LT terms of non-diagonal form factors are identical to each other after replacement of the color factors up to an overall factor 2^l, where l is the number of loops.

The UV renormalised form factors in QCD contain infrared (IR) divergences. Since the IR poles in QCD become UV ones in Soft Collinear Effective Theory (SCET) ([46, 47, 48, 49, 50, 51, 52]), a suitable renormalisation constant can be used to absorb all residual IR poles to obtain finite results. The resulting finite part is the matching coefficient required to perform $\mathrm{N^3LL}$ resummation in SCET for the pseudo-scalar Higgs boson production at the LHC [12].

3. Conclusions

To summarise, we have systematically computed the three loop QCD corrections to pseudo-scalar operators in the effective theory where top quark is integrated out. We have used IBP and LI identities along with the three loop master integrals that are availble to achieve this. Using the universal structure of the IR poles, we could successfully rederive all the UV renormalisation constants which are later used to obtain the UV finite three loop form factors in QCD. We have also studied the leading transcendental behaviour of diagonal and non-diagonal form factors. Using these form factors, we have determined the matching coefficient in SCET up to the three loop level.

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