

CP-odd Higgs boson production in two-photon processes

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We discuss the CP-odd Higgs boson production through the two-photon processes in $e\gamma$ collisions. First we briefly review the Standard Model Higgs boson production in $e\gamma$ collisions, with special attention to transition form factor and differential production cross section. We then study the production of the CP-odd Higgs boson A^0 which appears in the extended Higgs sector such as the Minimal Supersymmetric Standard Model (MSSM) or in the Two-Higgs Doublet Models (2HDM). The electroweak one-loop contributions to the scattering amplitude for $e\gamma \rightarrow eA^0$ as well as the transition form factor are calculated and expressed in an analytical form. We found that one-loop contribution only comes from top-quark loops. There are no contributions from W-boson loop nor from stop loop. Numerical analysis for the production cross section is presented. It turns out that the $\gamma^* \gamma$ -fusion is far more dominant over the $Z^* \gamma$ -fusion.

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1. Introduction

The Higgs boson with mass about 125 GeV was discovered by ATLAS and CMS at LHC [1] and its spin, parity and couplings were examined [2]. Now it would be intriguing to study its nature in e^+e^- collisions provided by such as linear collider [3]. Along with e^+e^- collider, other options such as e^-e^- , $e^-\gamma$ and $\gamma\gamma$ colliders have also been discussed. See Refs. [4]-[8] and the references therein. We first summarize what we have studied in [9, 10] about standard model (SM) Higgs boson H production through two-photon processes in $e\gamma$ collisions where the scattered electron detected in the final state (single tagging) shown in Fig.1. We calculated the so-

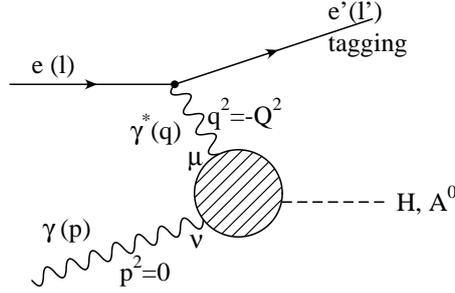


Figure 1: SM Higgs or CP-odd Higgs boson production in $e\gamma$ collision

called “transition form factor” as well as the production cross section, especially their Q^2 (virtual photon mass squared) dependence. It turned out at one-loop order the W -boson loop gives dominant contribution compared to the top-quark loop contribution.

We then extend our argument to CP-odd Higgs boson production in two-photon process of $e\gamma$ collisions [11]. In contrast to SM Higgs boson H , CP-odd Higgs A^0 has much simpler structure in transition form factor. We found that one-loop contribution only comes from top-quark loops. There are no contributions from W -boson loop nor from stop loop. Numerical analysis for the production cross section is presented. It turns out that the $\gamma^*\gamma$ -fusion is far more dominant over the $Z^*\gamma$ -fusion.

2. SM Higgs production in $e\gamma$ collision

The scattering amplitude for $e(l) + \gamma(p) \rightarrow e'(l') + H(q+p)$ shown in Fig.2 is given by

$$\langle e'H|T|e\gamma\rangle = \bar{u}(l')(-ie\gamma^\mu)u(l)\frac{-i}{q^2+i\epsilon}A_{\mu\nu}\epsilon^\nu(p) \quad (2.1)$$

where $\epsilon^\nu(p)$ is the polarization vector of the incident real photon. Here we have introduced the tensor $A_{\mu\nu}$ which can be decomposed due to gauge invariance as

$$A_{\mu\nu}(q,p) = (g_{\mu\nu}(q\cdot p) - p_\mu q_\nu)S_1(m^2, Q^2, m_H^2) + \left(q_\mu p_\nu - \frac{q^2}{q\cdot p}p_\mu p_\nu\right)S_2(m^2, Q^2, m_H^2) \quad (2.2)$$

where $q^2 = -Q^2 < 0$, $p^2 = 0$ and $(q+p)^2 = p_H^2 = m_H^2$. We denote collectively the mass of the intermediate particle in the loop by m . Since $p^\nu\epsilon_\nu(p) = 0$, only S_1 is relevant for the scattering

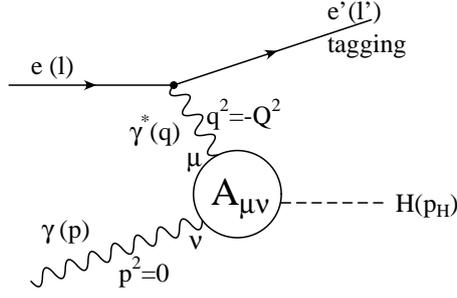


Figure 2: The transition amplitude for Higgs production via virtual and real photon fusion.

amplitude. We define the “transition form factor” $F_i(m^2, Q^2, m_H^2)$ by

$$F_i(m^2, Q^2, m_H^2) = S_1(m^2, Q^2, m_H^2) / \left(\frac{ge^2}{(4\pi)^2} \frac{1}{m_W} \right) \quad (2.3)$$

where $i = 1/2, 1$ for a fermion-loop $F_{1/2}$ and for the W-boson loop F_1 , respectively. e and g are the electromagnetic and weak gauge couplings, respectively, and m_W is the W boson mass.

The total transition form factor is given by

$$F_{\text{total}}(Q^2, m_H^2) = \sum_f N_c e_f^2 F_{1/2}(m_f^2, Q^2, m_H^2) + F_1(m_W^2, Q^2, m_H^2) \quad (2.4)$$

where N_c is the number of the colors (1 for leptons and 3 for quarks) and e_f is the electric charge of the fermion f in the unit of proton charge. Evaluating the production cross section from Eq.(2.4), we found that at the one-loop level the W-loop contribution dominates over top-quark loop [9, 10].

3. MSSM/2HDM and A^0 production

We now consider a minimal extension of the Higgs sector of the Standard Model (SM). Here we investigate the Two-Higgs Doublet Model (2HDM) for the type-II case which includes the MSSM as a special case [12]. We denote the two $SU(2)_L$ doublets H_1, H_2 , with weak hypercharge $Y = -1$ and $Y = 1$, respectively, by the 4 complex scalar fields, $\phi_1^0, \phi_1^-, \phi_2^+, \phi_2^0$ as follows:

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix} = \begin{pmatrix} \phi_1^{0*} \\ -\phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \quad (3.1)$$

where, in the type II model, H_1 (H_2) only couples to the down-type (up-type) quarks and leptons. They acquire the following vacuum expectation values after the spontaneous symmetry breaking:

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \tan \beta = v_2/v_1 \quad (3.2)$$

Then 3 degrees of freedom out of 8 consisting of the 4 complex scalar fields are absorbed by the longitudinal components of W^\pm, Z , and the remaining 5 physical degrees of freedom become the following two charged and three neutral physical Higgs bosons:

$$\text{Charged } H^+, H^-; \quad \text{CP-even } h^0, H^0; \quad \text{CP-odd } A^0 \quad (3.3)$$

Here we are particularly interested in the CP-odd Higgs boson A^0 , and investigate its transition form factor and the production cross section in the $\gamma^*\gamma$ -fusion process.

We now enumerate some characteristics of the coupling of A^0 for the type II case:

- 1) In contrast to the CP-even Higgs bosons h^0 and H^0 , A^0 does not couple to W^+W^- and ZZ pairs at tree level. Hence W -boson and Z -boson one-loop diagrams do not contribute to the A^0 production.
- 2) A^0 does not couple to other two physical Higgs bosons in cubic interactions.
- 3) The couplings of A^0 to the fermions are proportional to the fermion masses. Therefore, we only consider the top quark (for the charged fermion loop diagrams). The A^0 coupling to the top quark with mass m_t is given by $\lambda\gamma_5$ with [12]

$$\lambda = -\frac{gm_t \cot\beta}{2m_W}. \quad (3.4)$$

Here g and m_W are the weak gauge coupling and the weak boson mass, respectively.

- 4) In the case of MSSM, the trilinear A^0 coupling to mass-eigenstate squark pairs $\tilde{q}_i\tilde{q}_i$ ($i = 1, 2$) vanishes [12]. Hence, the scalar top-quark (stop) does not contribute to the A^0 production in $e\gamma$ collisions at one-loop level.

3.1 Scattering Amplitude for $\gamma^*\gamma$ Fusion Process

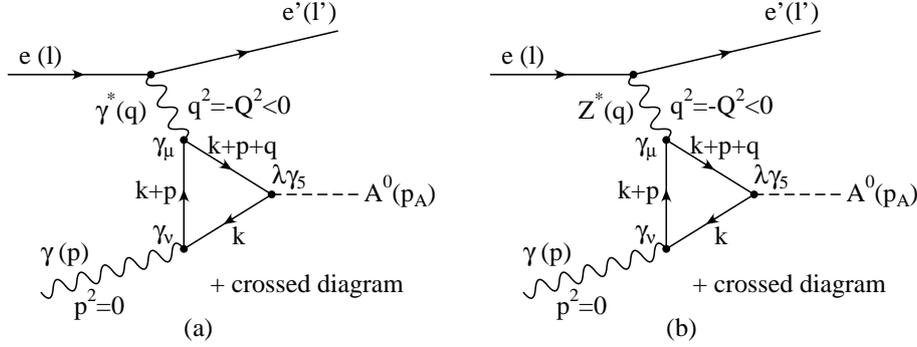


Figure 3: (a) $\gamma^*\gamma$ fusion diagram for $e\gamma \rightarrow e'A^0$ (b) $Z^*\gamma$ fusion diagram for $e\gamma \rightarrow e'A^0$

We first consider the case of $\gamma^*\gamma$ fusion process as shown in Fig.3(a). Since p is the momentum of a real photon, we have $p^2 = 0$ and $p^\nu \varepsilon_\nu(p) = 0$, where $\varepsilon_\nu(p)$ is the photon polarization vector. We set virtual photon momentum $q = l - l'$. Assuming that electrons are massless so that $l^2 = l'^2 = 0$, we introduce the following Mandelstam variables:

$$q + p = p_A, \quad q^2 = -Q^2, \quad p^2 = 0 \quad (3.5)$$

$$s = (l + p)^2 = 2l \cdot p, \quad t = (l - l')^2 = q^2 = -Q^2 = -2l \cdot l', \quad (3.6)$$

$$u = (p - l')^2 = -2l' \cdot p = m_A^2 - s - t. \quad (3.7)$$

where $p_A^2 = m_A^2$ with m_A being the CP-odd Higgs boson mass.

We evaluate the top-loop amplitude for the $\gamma^* \gamma$ fusion diagram as shown in Fig.3(a):

The scattering amplitude for $e(l) + \gamma(p) \rightarrow e'(l') + A^0(p_A)$ is given by

$$\langle e' A | T | e \gamma \rangle_{\gamma^* \gamma} = \bar{u}_{r'}(l') (-ie\gamma_\mu) u_r(l) \frac{-i}{q^2 + i\epsilon} A^{\mu\nu} \varepsilon_\nu(p, \lambda_2) \quad (3.8)$$

where $\varepsilon_\nu(p, \lambda_2)$ is the polarization vector of the incident real photon with momentum p and helicity λ_2 . The $u_r(l)$ ($\bar{u}_{r'}(l')$) is the spinor for the initial (scattered) electron with momentum l (l') and helicity r (r'). The tensor $A_{\mu\nu}$ is given as

$$A_{\mu\nu} = -8ie^2 m_t \lambda \varepsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[k^2 - m_t^2][(k+p)^2 - m_t^2][(k+p+q)^2 - m_t^2]}. \quad (3.9)$$

3.2 One-loop integrals

The above one-loop integral (3.9) is given by the three-point scalar integral by Passarino-Veltman [13]:

$$\frac{1}{(2\pi)^4} \int d^4 k \frac{1}{[k^2 - m_t^2][(k+p_2)^2 - m_t^2][(k+p_1+p_2)^2 - m_t^2]} = \frac{i\pi^2}{(2\pi)^4} C_0(p_1^2, p_2^2, p_3^2, m_t^2, m_t^2, m_t^2) \quad (3.10)$$

In our present case of e and real γ collision we have

$$C_0(-Q^2, 0, m_A^2, m_t^2, m_t^2, m_t^2) = -\frac{1}{Q^2 + m_A^2} \left\{ \frac{1}{2} g(\rho) + 2f(\tau) \right\} \quad (3.11)$$

where the dimensionless variables τ and ρ are defined as

$$\tau \equiv \frac{4m_t^2}{m_A^2}, \quad \rho \equiv \frac{Q^2}{4m_t^2} \quad (3.12)$$

and the two functions $f(\tau)$ and $g(\rho)$ we have introduced are given by

$$f(\tau) = \left[\sin^{-1} \sqrt{\frac{1}{\tau}} \right]^2 \quad \tau \geq 1 \quad (3.13)$$

$$= -\frac{1}{4} \left[\log \frac{1 + \sqrt{1-\tau}}{1 - \sqrt{1-\tau}} - i\pi \right]^2 \quad \tau < 1 \quad (3.14)$$

$$g(\rho) = \left[\log \frac{\sqrt{\rho+1} + \sqrt{\rho}}{\sqrt{\rho+1} - \sqrt{\rho}} \right]^2 \quad (3.15)$$

Thus we have [11]

$$A_{\mu\nu} = \frac{ge^2 \cot\beta}{(4\pi)^2} \frac{\tau}{2m_W} \frac{\tau}{1+\rho\tau} [g(\rho) + 4f(\tau)] \varepsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \quad (3.16)$$

where we have used the relation $\lambda = -gm_t \cot\beta / 2m_W$. Similar combinations of functions $f(\tau)$ and $g(\rho)$ as in Eq.(3.11) with the time-like virtual mass, appear in the Higgs decay processes $H \rightarrow \gamma^* \gamma$ and $H \rightarrow Z^* \gamma$ in Ref.[14] (see also Ref.[12] for on-shell decays, $H \rightarrow \gamma\gamma$ [15] and $H \rightarrow Z\gamma$).

3.3 Transition Form Factor

We can define the so-called "Transition Form Factor" as in the case of the standard Higgs boson. Now first we note

$$A_{\mu\nu}(q, p) \equiv \tilde{S}(m_t^2, Q^2, m_A^2) \varepsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \quad (3.17)$$

where we now introduce the transition form factor \tilde{F} as

$$\begin{aligned} \tilde{F}(m_t^2, Q^2, m_A^2) &\equiv \tilde{S}_1(m_t^2, Q^2, m_A^2) / \left(\frac{ge^2}{(4\pi)^2} \frac{1}{m_W} \right) \\ &= -\frac{1}{2} \frac{\tau}{1+\rho\tau} [g(\rho) + 4f(\tau)] = 4m_t^2 C_0 \end{aligned} \quad (3.18)$$

Since $\tau = (2m_t/m_A)^2$ we have the following two cases depending on the mass of A^0 .

For $m_A < 2m_t$ *i.e.* $\tau > 1$ we have $f(\tau)$ given by Eq.(3.13) which is a real function. While for $m_A > 2m_t$ *i.e.* $\tau < 1$ we have $f(\tau)$ given by Eq.(3.14) which is a complex function. Therefore, the former is real while the latter becomes complex.

We also introduce the total transition form factor \tilde{F}_{total} which includes the all the flavors of the quark-loop, but dominated by top loop.

$$\tilde{F}_{\text{total}}(Q^2) = \sum_f N_c q_f^2 \tilde{F}(\rho_f, \tau_f) \simeq 3 \cdot \left(\frac{2}{3} \right)^2 \tilde{F}(m_t^2, Q^2, m_A^2) \quad (3.19)$$

We consider the two cases: (a) $m_A = 300$ GeV and (b) $m_A = 400$ GeV shown below (Fig.4(a),(b)).

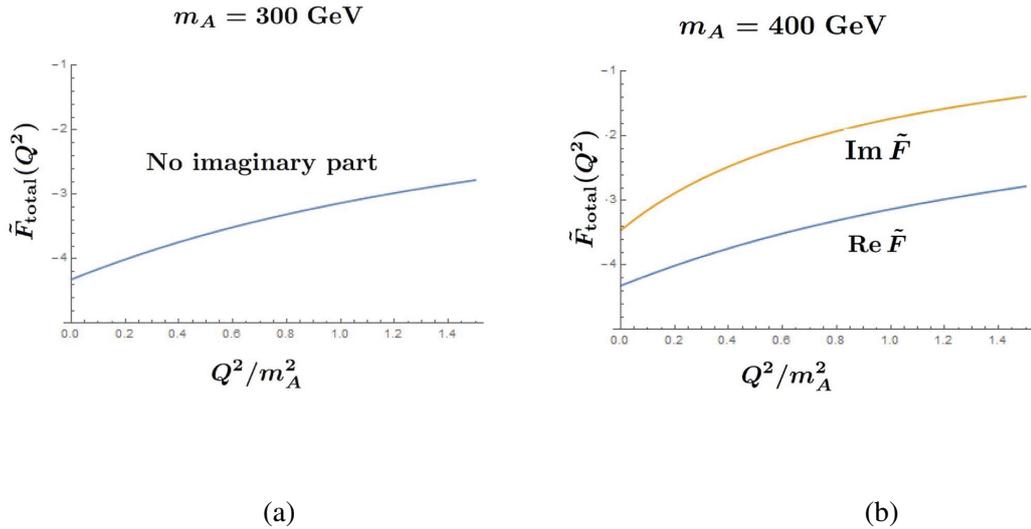


Figure 4: Transition Form Factor for the mass (a) $m_A = 300$ GeV (b) $m_A = 400$ GeV

3.4 Differential cross section

The differential cross section for the CP-odd Higgs production via $\gamma^*\gamma$ fusion in $e\gamma \rightarrow eA^0$ is given by

$$\frac{d\sigma(\gamma^*\gamma)}{dQ^2} = \frac{\alpha_{\text{em}}^3 g^2}{64\pi 4\pi} \frac{1}{Q^2} \left[1 + \frac{u^2}{s^2} \right] \frac{1}{m_W^2} |\tilde{F}_{\text{total}}(Q^2)|^2 \quad (3.20)$$

where $\alpha_{\text{em}} = e^2/4\pi$. Since $s = 2l \cdot p$ and $u = -2p \cdot l'$ we note $u = m_A^2 + Q^2 - s$. Hence we obtain

$$\frac{d\sigma(\gamma^*\gamma)}{dQ^2} / \frac{\alpha_{\text{em}}^3 g^2}{64\pi 4\pi} = \frac{1}{Q^2} \left[1 + \frac{(m_A^2 + Q^2 - s)^2}{s^2} \right] \frac{1}{m_W^2} \left| \frac{4}{3} \tilde{F}(m_t^2, Q^2, m_A^2) \right|^2 \quad (3.21)$$

3.5 Z boson and real γ fusion

The scattering amplitude for $e(l) + \gamma(p) \rightarrow e'(l') + A^0(p_A)$ via $Z^*\gamma$ fusion shown in Fig.3(b) is

$$\langle e'A | T | e\gamma \rangle_{Z^*\gamma} = \frac{g}{4 \cos \theta_W} \bar{u}_{r'}(l') (i\gamma_\mu) (f_{Ze} + \gamma_5) u_r(l) \frac{-i}{q^2 - m_Z^2} \tilde{A}^{\mu\nu} \varepsilon_\nu(p, \lambda_2) \quad (3.22)$$

where

$$\tilde{A}_{\mu\nu} = 8e \frac{g}{4 \cos \theta_W} m_t \lambda f_{Zt} \frac{1}{16\pi^2} C_0(-Q^2, 0, m_A^2, m_t^2, m_t^2, m_t^2) \varepsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta \quad (3.23)$$

and e - e - Z coupling f_{Ze} and t - t - Z coupling f_{Zt} are given by

$$f_{Ze} = (-1 + 4 \sin^2 \theta_W), \quad f_{Zt} = (1 - \frac{8}{3} \sin^2 \theta_W). \quad (3.24)$$

Note that the scattering amplitudes both from $\gamma^*\gamma$ -fusion and $Z^*\gamma$ -fusion should be added up. When taking absolute square of the amplitude we get an interference term, which turns out to be positive, in addition to the $\gamma^*\gamma$ - as well as $Z^*\gamma$ - fusion terms as follows:

$$\frac{d\sigma}{dQ^2}(\text{total}) = \frac{d\sigma}{dQ^2}(\gamma^*\gamma\text{-fusion}) + \frac{d\sigma}{dQ^2}(Z^*\gamma\text{-fusion}) + \frac{d\sigma}{dQ^2}(\text{Interference}) \quad (3.25)$$

4. Numerical Analysis

4.1 production cross section

First let us note that the contribution from the $\gamma^*\gamma$ fusion is far more dominant over that from $Z^*\gamma$ -fusion as well as the interference term. We have shown the differential cross sections for the three process in Fig.5 in the case of $\sqrt{s} = 500\text{GeV}$, $m_t = 173\text{GeV}$, $m_A = 400\text{GeV}$, $\cot\beta = 1$. (In fact, the cross sections are proportional to $\cot^2\beta$.) We observe that at $Q^2 = 1000$ (5000) GeV^2 the ratio of $d\sigma/dQ^2(Z^*\gamma)$ to $d\sigma/dQ^2(\gamma^*\gamma)$ is 4.3×10^{-6} (5.2×10^{-5}) and $d\sigma/dQ^2(\text{Interference})$ to $d\sigma/dQ^2(\gamma^*\gamma)$ is 4.1×10^{-3} (1.4×10^{-2}). Thus the Z^* -boson exchange reaction does not actually affect the $\gamma^*\gamma$ exchange process (Fig.3). This means that the transition form factor makes sense for the A^0 production in $e\gamma$ collision.

Now we shall focus on the $\gamma^*\gamma$ fusion process based on the formula for the production cross section Eq.(3.20). In Fig.6 we have plotted the differential production cross section of A^0 with

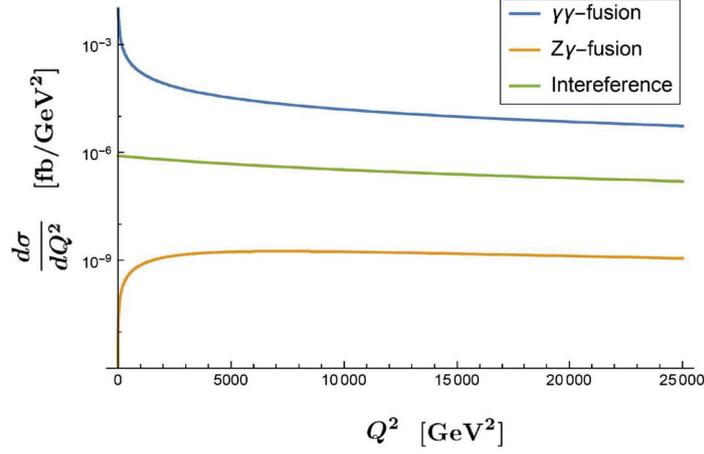


Figure 5: Comparison of the contributions from 3 processes: $\gamma^*\gamma$ -fusion, $Z^*\gamma$ -fusion and Interference for $\sqrt{s} = 500\text{GeV}$, $m_t = 173\text{GeV}$, $\cot\beta = 1$, $m_A = 400\text{GeV}$.

mass $m_A = 200, 300, 400\text{ GeV}$, for $\sqrt{s} = 500\text{ GeV}$ and $m_t = 173\text{ GeV}$. We find that for this kinematical region the production cross section for A^0 increases as m_A gets larger which looks somewhat unexpected result.

We can examine this behaviour in more detail by computing the differential cross section for fixed Q^2 , which we take to $(100)^2\text{ GeV}^2$.

4.2 The A^0 mass dependence of the production cross section

We have plotted the A^0 mass dependence of the differential cross section $d\sigma/dQ^2$ for $Q^2 = (100)^2\text{ GeV}^2$ in Fig.7(a) as well as that for the total cross section σ_{total} in Fig.7(b). As m_A varies across the $t\bar{t}$ threshold $2m_t \approx 346\text{GeV}$, the differential cross section $d\sigma/dQ^2$ increases in the region $m_A < 2m_t$, and it turns to decrease when m_A goes beyond $2m_t$.

In both cases, we see the strong kink structure corresponding to the threshold effect at $m_A = 2m_t \approx 346\text{ GeV}$.

5. Conclusion

In this talk we have investigated the possible production of the CP-odd Higgs boson A^0 which would appear in the 2HDM/MSSM through $e\gamma$ collisions. In contrast to the SM Higgs boson H as well as to the CP-even Higgs boson h^0 and H^0 , the A^0 does not couple to W^+W^- and ZZ pairs because of the CP-odd nature. Hence at one-loop order W^\pm bosons do not contribute to triangle diagrams for the A^0 production, and only top-quark one-loop triangle diagram is relevant. There is no scalar top-quark (stop) contribution. The transition form factor shows much simpler structure.

When the mass of the A^0 boson, m_A is smaller than $2m_t$ the transition form factor is a real function of Q^2 , while if m_A is larger than $2m_t$, the transition form factor becomes complex. The

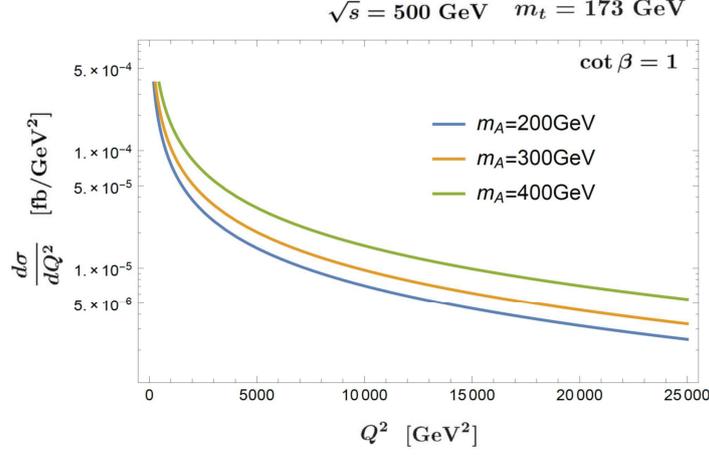


Figure 6: Differential cross section for the production of CP-odd Higgs boson A^0 with mass $m_A = 200, 300, 400$ GeV.

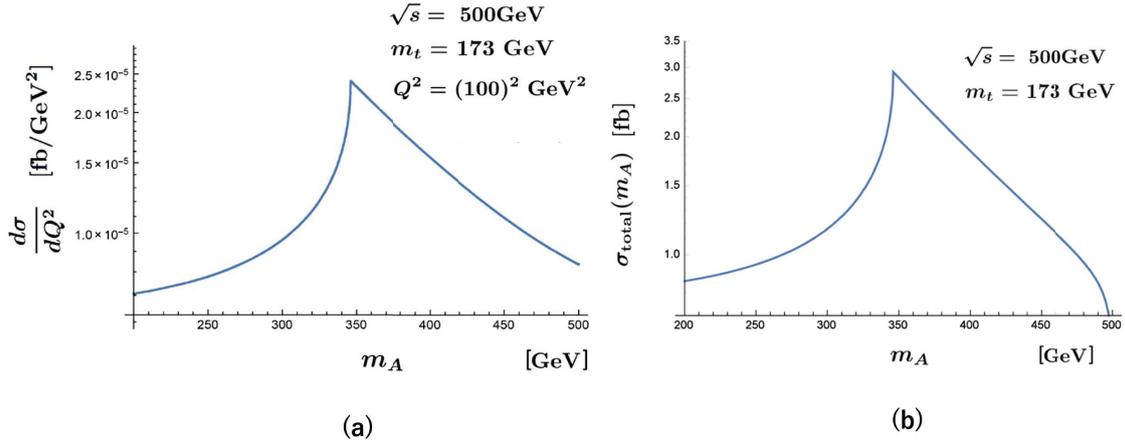


Figure 7: (a) The A^0 mass dependence of the differential cross section for the CP-odd Higgs production with $Q^2 = (100)^2 \text{GeV}^2$ (b) The A^0 mass dependence of the total cross section for the CP-odd Higgs production.

production cross section of CP-odd Higgs boson is given by the absolute square of the transition form factor together with some kinematical factors.

For a fixed value of m_A , the differential production cross section shows a decreasing function of Q^2 . On the other hand, if we fix Q^2 and vary the mass of A^0 , it increases as m_A for $m_A < 2m_t$ and decreases for $m_A > 2m_t$. This feature is common with the total cross section.

In the $e\gamma$ scattering, the contribution from the $\gamma^*\gamma$ fusion is far more dominant over that from $Z^*\gamma$ fusion. Thus actually we only have to consider the photon-exchange diagram, and it makes sense to talk about the transition form factor of A^0 .

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