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Measurements of fragmentation functions and implications for semi-inclusive deep-inelastic scattering

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In this talk we will briefly review the present status of fragmentation function measurements with a focus on the more recent measurements of Collins-function related as well as of polarization-averaged single- and di-hadron fragmentation. We will further discuss limitations in the analysis of dihadron-fragmentation data arising through an incomplete integration of the differential cross section imposed by experimental constraints. In particular, we show that the usual application of momentum requirements together with integration over the polar angle of the hadrons in the di-hadron center-of-mass frame leads to a presently uncontrollable mix of various partial-wave components of the di-hadron fragmentation function, prohibiting precision extractions of di-hadron fragmentation functions from e^+e^- annihilation, semi-inclusive deep-inelastic scattering, or proton-proton collision data.

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1. Introduction

Fragmentation functions have been recognized as powerful tools to probe nucleon structure [1] and have received much increased attention in the past decade. Still, the level of knowledge about those lags behind that about parton distributions. In particular, when studying the transverse-momentum structure of the nucleon through semi-inclusive DIS it will be unavoidable to also know the transverse-momentum structure of hadronization as the transverse momentum of the hadrons in semi-inclusive DIS arises through convolutions of the transverse-momentum dependence of both the parton distribution and fragmentation functions. Fragmentation functions are mainly constrained through measurements of semi-inclusive DIS [2, 3, 4, 5] and of hadron production in electron-positron annihilation, especially the very precise data from the *B* factories [6, 7, 8, 9, 10, 11, 12]. [In case of transverse-momentum integrated fragmentation functions, data on hadron production in proton-proton collision is also used in global analyses.] There is, however, a clear lack of data on transverse-momentum dependent hadron production in electron-positron annihilation at this moment, though fortunately this is part of the ongoing program at the *B* factories as well as at BESIII. In addition, charm-quark contributions included in most of the results from electron-positron annihilation limits the precision on light-quark fragmentation functions needed for the high-precision data to come out of the Jefferson Lab 12 GeV program and the envisioned Electron-Ion Collider (EIC).

		quark pol.		
nucleon pol.		U	L	T
	U	f_1		$h_1^\perp$
	L		g_{1L}	$h_{1L}^\perp$
	T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Figure 1: Table of leading-twist TMDs, where the columns and rows identify unpolarized (U), longitudinally (L), and transversely (T) polarized quarks in unpolarized, longitudinally, and transversely polarized nucleons. In black are those distributions that survive integration over transverse momentum, while blue (red) distributions are genuinely transverse-momentum dependent and even (odd) under naive time reversal.

One of the driving forces in fragmentation function studies is their ability to tag the flavor of the hadronizing quark in semi-inclusive DIS measurements but in selected cases also to tag its polarization, a novel way to better constraint the rich world of transverse-momentum distributions (TMDs) depicted in Fig. 1. In particular, chiral-odd distributions in the last column of the *TMD table* require a chiral-odd counterpart in the process, which in semi-inclusive DIS requires fragmentation functions that are sensitive to the transverse quark polarization. In the typical case of observing one hadron in addition to the scattered lepton without specifying the polarization of the hadron produced, two leading-twist fragmentation functions can enter: the chiral-even polarization-averaged $D_1^{q\to h}(z, z^2k_T^2)$, where z is the fractional energy carried away by the hadron and k_T is the transverse momentum aquired, and the chiral-odd Collins fragmentation function $H_1^{\perp,q\to h}(z, z^2k_T^2)$ [13]. Sensitivity to the latter only survives when keeping the dependence on transverse momentum in the process. If one wants to avoid explicit transverse-momentum dependence, but still keep sensitivity to transverse quark polarization at leading twist, at least two hadrons need to be produced [14, 15, 16, 17, 18]. The relative momentum of the two hadrons can be exploited to access the quark polarization of the fragmenting quark. A much discussed example is the chiral-odd di-hadron fragmentation function $H_1^{\sphericalangle,q\to h_1h_2}$ (where the dependence on the kinematic variables has been suppressed here). Unlike $D_1^{q\to h}$ the polarization-averaged counterpart of $H_1^{\sphericalangle,q\to h_1h_2}$, $D_1^{q\to h_1h_2}$, had until recently been unmeasured, but knowledge of it is vital as it enters all spin asymmetries in dihadron production. The progress on measurements sensitive to above-mentioned fragmentation functions in e^+e^- annihilation will be presented next, followed by a discussion of limitations when exploring di-hadron fragmentation in a restricted set of kinematic variables.

2. The experiments at the *Beauty* and *Charm* factories

The new data coming from e^+e^- annihilation are either from BaBar [19] or Belle [20] at the B factories (PEP-II and KEK-B) or from BESIII [21]. The B factories operated mainly at or close to the center-of-mass energy of the $\Upsilon(4S)$, corresponding $\sqrt{s} = 10.58$ GeV. While the bulk of the data set is at the $\Upsilon(4S)$ peak, additional samples of about 10% are taken mostly slightly below that energy. The amount of data is immense: about 1 ab^{-1} for Belle and roughly half of that for BaBar. The detectors have a high degree of hermiticity allowing for a broad range of fragmentation-function measurements, while the main focus has been on various aspects of B and charm physics. Being above the threshold for $b\bar{b}$ production, all but top quarks are produced in the collision. Removing heavy-quark contributions is thus one of the challenges in dealing with the B factory data, which is mainly addressed through the event-shape variable *thrust*. Large values of thrust correspond to 2-jet–like events, while smaller values to spherical events typical for B decays.

BESIII on the other hand is located at BEPC, an asymmetric variable-energy e^+e^- collider that runs in a range of 2–4.6 GeV center-of-mass energy. As a result $b\bar{b}$ production does not play a role and charm production can be avoided by taking data below the charm threshold, which graciously simplifies interpretation of results. On the other hand, the hadron multiplicity per event is low and events are much more spherical prohibiting a clear identification of the quark–anti-quark jets. The results presented here and recently published in Ref. [22] are based on a data sample of 62 pb^{-1} taken at $\sqrt{s} = 3.65$ GeV away from resonances. As such only fragmentation of *up*, *down* and *strange* quarks has to be considered in the interpretation of these data.

3. Single-hadron production

Earlier global fits of polarization-averaged single-hadron fragmentation functions (see, e.g., Refs. [23, 24]) were limited to e^+e^- annihilation data that covered mainly low values of z and larger center-of-mass energies. However, semi-inclusive DIS data at typical fixed-target energies are at a lower scale and cover larger ranges in z. Those e^+e^- annihilation data also limited the analysis of evolution and gluon fragmentation, which enters the cross section only at higher orders. The more recent and very precise data from BaBar and Belle on inclusive production of charged pion and kaon, as well as in case of BaBar on proton/anti-protons [8, 10], had a tremendous impact on the fragmentation function extractions. As discussed, e.g., in Refs. [25, 26, 27], the addition of the B-factory measurements contributes to significant reductions of the uncertainties, especially in the gluon and light-quark fragmentation functions. Comparing those data to the LEP data clearly indicate QCD scaling violations, and the different electroweak coupling at the different scales even

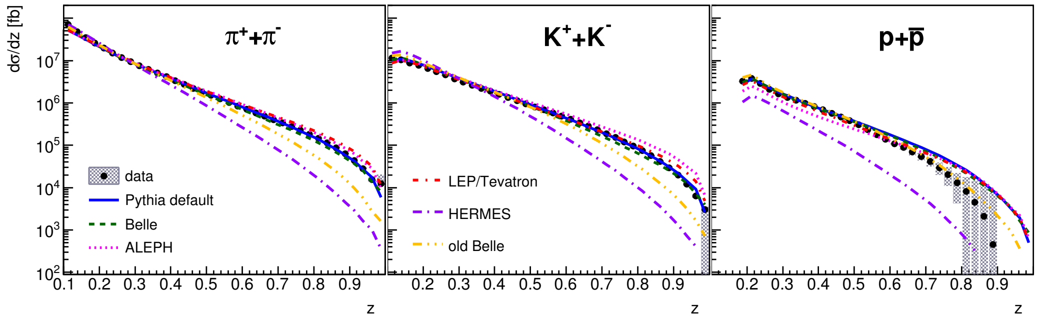

Figure 2: Cross section for inclusive production of single charged pions, kaons as well as (anti)protons [9] as a function of the fractional energy z, compared to different PYTHIA tunes.

allow for some flavor separation without using data from other, mainly semi-inclusive DIS, processes. Recently, in the process of analyzing inclusive production of hadron pairs, the Belle collaboration revisited the single-hadron cross-section measurement and included, in a somewhat coarser kinematic binning, also results for protons and anti-protons [9]. The cross sections are shown in Fig. 2 in comparison with various PYTHIA [28] tunes. While the data for pions and kaons are reasonably well described, protons are more difficult to reproduce and most PYTHIA tunes overshoot the data at medium to large z. Altogether, single inclusive hadron production in e^+e^- annihilation is presently the best understood process among all the e^+e^- annihilation data on fragmentation functions. A grain of salt might be an apparently not well understood radiative correction as that applied in the data analysis (up to 30%) had to be undone in the phenomenology [29]. Furthermore, single-hadron production is neither well-suited for flavor separation of fragmentation functions nor very useful to tag quark polarization.

4. Hadron-pair production

As pointed out, even though single-hadron production is theoretically well understood and its phenomenology backed by an immense data set, it is limited when turning to flavor separation or polarization studies. There exists a much richer landscape when hadron pairs are considered. Pairs of almost back-to-back hadrons have been intensively studied in the quest of unraveling the transverse-spin structure of nucleons by means of the Collins effect [13]. However, production of hadron pairs had been discussed already much earlier [30] and can be used as a probe of parton-flavor dependence of the hadronization process. Pairs of hadrons produced in the fragmentation of the same quark lead to the class of dihadron fragmentation functions. These are particularly interesting in the study of the chiral-odd transversity because dihadron fragmentation can access at leading twist chiral-odd distributions in collinear factorization. For the distinction of single-hadron and dihadron fragmentation, a hemisphere definition is employed in experiments. The thrust axis, or better the plane perpendicular to the thrust axis, discriminates hadrons within the same from hadrons in opposite hemispheres. Alternatively, the collinearity of the two tracks is used: for instance in the analysis by BESIII presented below, a minimum opening angle between the two pions of 120 degrees is required. However, there is no clear procedure how to compare results meaningfully when obtained using different topological requirements. In general, production of hadrons in opposite hemispheres are much better described via products of single-hadron fragmentation functions, while hadron pairs in the same hemisphere by dihadron fragmentation functions.

One of the main interests in looking at almost back-to-back hadron pairs is due to the Collins effect as it paves a way to tag transverse quark polarization. It is through the angular correlations of the two hadrons, either using the momentum direction of one hadron as a reference axis or the thrust axis, that one can access transverse-polarization effects [13, 31]. It would go beyond the scope of these proceedings to review in detail all the results on the Collins effect available. Instead, a more detailed discussion can be found elsewhere [32]. However, it is worthwhile to point out that in recent years progress has been made in going from charged-pion pairs and Collins asymmetries integrated over transverse momentum to meson combinations involving kaons and to keeping the explicit dependence on transverse momentum. This allows a much more rigorous testing of models and parametrizations for the Collins function including the dependence on quark flavors that are not part of the valence structure of nucleons, and also of deviations from the traditionally used Gaussian Ansatz for the transverse-momentum dependence. As an example of recent results, the Collins asymmetries from BESIII [22] are presented in Fig. 3. There are two important features that set these results apart from those of Belle and BaBar. First, as the center-of-mass energy is much lower, the thrust axis cannot be used as a reliable estimator of the $q\bar{q}$-axis. This requires using one hadron for defining the reference axis.[1] Even though this involves a more intricate convolution of the transverse-momentum dependences [31], it is theoretically cleaner as not introducing a dependence on the topological event quantity thrust. On the other hand, being below the charm threshold means that the hadron pairs originate from *uds* events only, much closer to the situation of typical fixed-target semi-inclusive DIS experiments. Being at a lower scale also permits in principle to engage in evolution studies using e^+e^- annihilation data only.

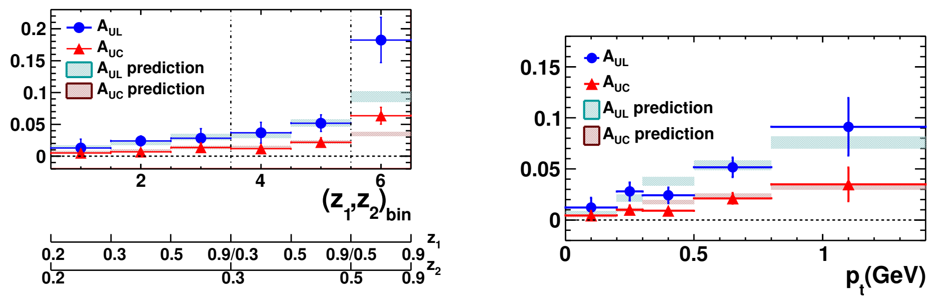

Figure 3: Collins asymmetries for charged-pion pairs [unlike-sign over like-sign pairs (A_{UL}) and unlike-sign over charge-averaged pairs (A_{UC})] as a function of (z_1, z_2) as indicated (left), and as a function of transverse momentum (right) [22]. Also depicted are the model predictions of Ref. [33].

Much less information had been available on the unpolarized sector of hadron pair production. At LEP some attempts had been made to tag parton flavor by accompanying hadrons, e.g., at OPAL [34] using the method of leading-particle tagging proposed in Ref. [35]. Hadrons in opposite hemispheres are predominantly produced by different partons and can thus be used to further constrain the flavor dependence of fragmentation functions, but also to study the transverse-momentum

[1] An approach that is also used at Belle and BaBar, as an alternative to using the thrust axis.

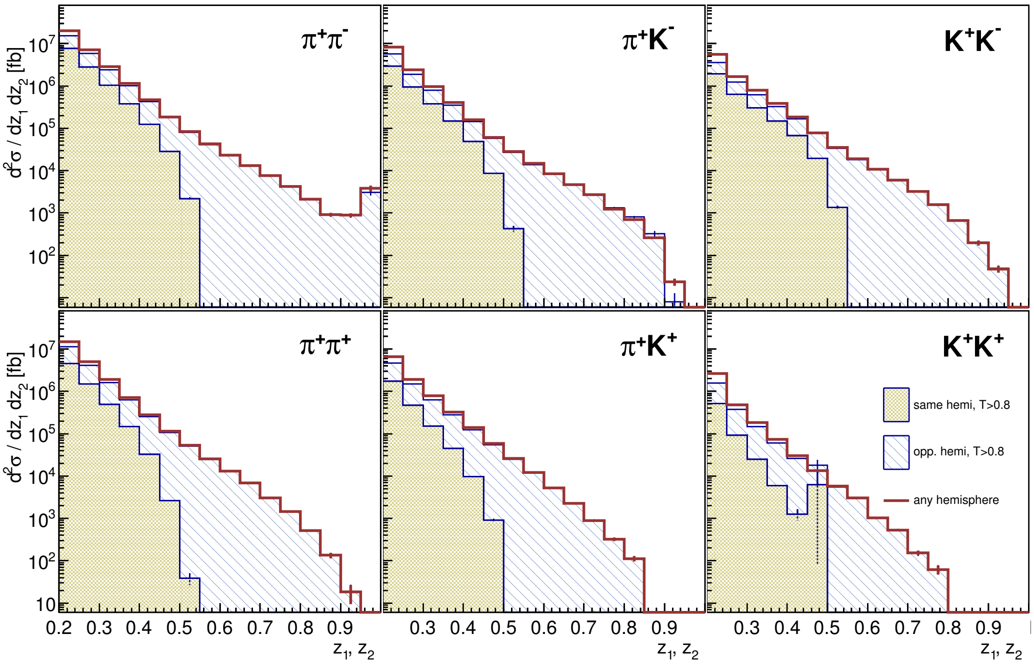

Figure 4: Inclusive dihadron cross sections for different meson combinations and topologies as indicated, plotted as a function of $z_1 = z_2$ [9]. Note that for the unlike-hadron pairs a factor two is missing in the cross sections. The drop of the cross sections of same-hemisphere pairs around 0.5 is due to the kinematic limit of having not more than two hadrons with more than half the energy available in one hemisphere.

dependence of D_1 [36]. First attempts for the latter using among others the rather limited data from TASSO [37, 38] were recently presented in Ref. [39]. The recent results from Belle on inclusive cross sections of hadron pairs [9] do not focus on the transverse momentum dependence (those are subject to ongoing analyses), but on the flavor dependence of hadronization and the hemisphere decomposition of hadron pair production. A clear hierarchy depending on topology and hadron species can be made out in Fig. 4. The cross section is larger for pairs in opposite hemispheres because the total collision energy is available. Also pairs of oppositely charged hadrons are favored as is expected from fragmentation of oppositely charged $q\bar{q}$ pairs. Moreover, pairs involving kaons are less abundant, in particular in a region where the strange quark needs to be produced in the fragmentation process, e.g., for same-sign kaon pairs or for the cases where $u\bar{u}$ and $d\bar{d}$ are tagged by leading pions in the opposite hemisphere.

Turning to dihadron fragmentation functions one has to realize that until now, PYTHIA simulations were used to extract a model for the unpolarized dihadron fragmentation functions when using dihadron fragmentation to probe chiral-odd distributions by combining the azimuthal asymmetries in e^+e^- annihilation [40] with the spin asymmetries in semi-inclusive DIS [41, 42] (and lately also from pp collision). This situation has changed recently[2] with the publication of the invariant-mass and fractional-energy dependence of inclusive production of di-hadrons by Belle [43].

[2]Albeit after this conference, as such it is only mention here but not discussed further.

5. Limitations of dihadron fragmentation in reduced dimensions

Dihadron fragmentation has been worked out in detail in Refs. [16, 17, 44]. It is conventionally expressed in terms of fragmentation functions that depend on the energy fraction z of the dihadron, the dihadron's invariant mass M_h, the momentum difference ζ, which can be related to $\cos\theta$, where θ represents the polar angle of one hadron in the center-of-mass of the dihadron system, and—in principle—on the transverse momentum of the dihadron system (which will be integrated over in the following).

It is convenient to expand the dependence on ζ in terms of partial waves. Including only s and p waves, as is commonly believed to be a valid approximation for low invariant mass of the dihadron pair, the polarization-averaged dihadron fragmentation functions D_1 can be written as [44]

$$D_1(z,\cos\theta,M_h) \simeq D_{1,oo}(z,M_h) + D_{1,ol}(z,M_h)\cos\theta + D_{1,ll}(z,M_h)\frac{1}{4}\left(3\cos^2\theta - 1\right), \tag{5.1}$$

$$H_1^{\sphericalangle}(z,\cos\theta,M_h) \simeq H_{1,ot}^{\sphericalangle}(z,M_h) + H_{1,lt}^{\sphericalangle}(z,M_h)\cos\theta\,, \tag{5.2}$$

where we have suppressed the dependence on the flavor of the fragmenting quark. Furthermore, $D_{1,oo}(z,M_h)$ receives contributions from both s and p waves:

$$D_{1,oo}(z,M_h) = \frac{1}{4}\left(D^s_{1,oo}(z,M_h) + 3D^p_{1,oo}(z,M_h)\right). \tag{5.3}$$

It should be noted that while it is experimentally possible to disentangle the various partial-wave contributions in Eqs. (5.1,5.2) by angular analysis of the cross section, it is not possible to disentangle the two contributions in Eq. (5.3) without assuming specific (and differing) invariant-mass dependences of those. Those contributions are, however, useful to impose limits on the partial wave contributions. In particular, the relevant limit in the context of this discussion reads [44]

$$-\frac{3}{2}D^p_{1,oo} \le D_{1,ll} \le 3D^p_{1,oo}\,. \tag{5.4}$$

It has been suggested to look at dihadron production integrated over the polar angle (e.g., in Ref. [15]). The obvious advantage of such approach is that most terms in Eqs. (5.1,5.2) will drop out. In particular, from Eq. (5.1) only $D_{1,oo}(z,M_h)$ and from Eq. (5.2) only the sp interference $H_{1,ot}^{\sphericalangle}(z,M_h)$ would survive, facilitating greatly the interpretation of the resulting cross sections and cross-section asymmetries. Unfortunately, experimental constraints severely limit the θ integration, which will be discussed in the following specific example of dihadron production at B-factories and to which phenomenological precision such data could be used to extract $D_{1,oo}(z,M_h)$.

The Belle data [43] on dihadron production in e^+e^- annihilation is differential in z and M_h but has the experimental requirement of a minimum momentum (and thus energy) for each individual hadron. It should be emphasized here that such momentum requirement is nothing unique to data from e^+e^- annihilation but rather imposed by basically all experimental apparatuses. Furthermore, we will concentrate on the case of pairs of oppositely charged pions, which will further simplify the calculations and which is the one with most experimental data and phenomenological analyses. In that case, θ is commonly defined with respect to the π^+ momentum. As discussed in more detail in Ref. [45], the requirement of $z_{\min} \ge 0.1$ for each individual pion reduces the range in θ that can

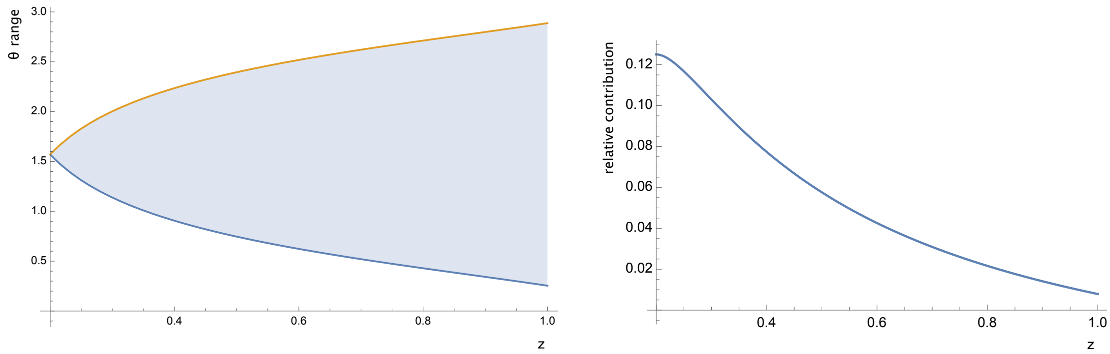

Figure 5: Left: the effect of the requirement of the individual pion's energy on the allowed range of the polar angle θ in the center-of-mass of the two-pion system. The two curves, plotted as a function of the combined z of the the di-pion, delimit the allowed region indicated as shaded area. Right: the strength of the $D^q_{1,ll}$ partial-wave contribution to the (partially integrated) cross section relative to the $D^q_{1,oo}$ contribution, again as a function of z for an invariant mass of 0.5 GeV, where the model of Ref. [46] yields about 0.5 for the ratio $D^q_{1,ll}/D^q_{1,oo}$.

be accessed:

$$|\cos\theta| \leq \frac{z - 2z_{\min}}{\sqrt{[(zE_0)^2 - M_h^2)(M_h^2 - 4m_\pi^2)]}} E_0 M_h \ , \tag{5.5}$$

where E_0 is the initial energy of the fragmenting quark, e.g., $\sqrt{s}/2$, and m_π the charged-pion mass. Equation (5.5) translates into a symmetric range about $\theta = \pi/2$. In particular, the accepted range increases with z of the dihadron, which is depicted on the left of Fig. 5. Being symmetric around $\pi/2$ has the advantage that still some terms in Eqs. (5.1,5.2) will cancel. Nevertheless, there will be contributions that cannot be disentangled from the one of interest, e.g.,

$$\int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta D_1^q(z,\cos\theta,M_h) \simeq \int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta D^q_{1,oo}(z,M_h) + \int_{\cos(\pi-\theta_0)}^{\cos\theta_0} \mathrm{d}\cos\theta D^q_{1,ll}(z,M_h)\frac{1}{4}\left(3\cos^2\theta - 1\right), \tag{5.6}$$

It is clear that the surviving contributions are affected differently by the partial integration. In particular, the contribution of $D^q_{1,oo}$ increases steadily with opening up the θ range until reaching its full size for $\theta_0 = 0$.

At present it is not possible to quantify the effect of the partial integration. Thus there is room for an educated guess based on the model of Ref. [46]. The relative strength of the $D^q_{1,ll}$ contribution can be estimated by relating the two integrals on the r.h.s. of (5.6). Taking, e.g., $M_h = 0.5$ GeV and as a rough estimate of 0.5 for the size of $D^q_{1,ll}$ compared to $D^q_{1,oo}$ at that dihadron mass [46] one finds [45] a relative contribution of up order 10% at low values of z, which is the region that dominates the cross section. At larger values of z, where the θ range opens up and thus suppresses the $D^q_{1,ll}$ term, the relative contribution of the latter quickly drops as depicted on the right of Fig. 5.

This example demonstrates that a precision extraction of, e.g., transversity, by means of dihadron fragmentation functions might be severely hampered as it requires precision knowledge of not only the chiral-odd $H_1^{\sphericalangle,q}$ but in spin asymmetries also of $D^q_{1,oo}$. However, as the elimination of higher partial waves relies on the integration over θ, the minimum fractional-energy requirement in many experimental analyses leads to a surviving contribution from $D_{1,ll}$ in the unpolarized cross section, leaving an ambiguity in the interpretation of the data in absence of precise knowledge about this dihadron fragmentation function. In particular, transversity extracted from spin asymmetries, as performed, e.g., in Ref. [47], using the such-obtained $D_{1,oo}^{q}$ can be easily off by 10%. It should also be mentioned that the partial integration over θ due to the minimum fractional energy requirement leads to a severe underestimation of the true strength of the $D_{1,oo}$ dihadron fragmentation function, especially at low values of z, if the resulting integration range is not properly taken into account.

Last but not least, it is worthwhile to point out that this problem can be avoided in principle by performing a partial-wave analysis. In practice, there is often a limit on how differential the analysis can be performed. Here, dihadron fragmentation poses a challenge due to its dependence on many variables.

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