

Nearly perturbative QCD coupling with lattice-motivated zero IR limit

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The product of the gluon dressing function and the square of the ghost dressing function in the Landau gauge can be regarded to represent, apart from the inverse power corrections $1/Q^{2n}$, a nonperturbative generalization $\mathscr{A}(Q^2)$ of the perturbative QCD running coupling $a(Q^2)$ (= $\alpha_{\rm s}(Q^2)/\pi$). Recent large volume lattice calculations for these dressing functions strongly indicate that such a generalized coupling goes to zero as $\mathscr{A}(Q^2) \sim Q^2$ when the squared momenta Q^2 go to zero ($O^2 \ll 1 \text{ GeV}^2$). We construct such a QCD coupling $\mathscr{A}(Q^2)$ which fulfills also various other physically motivated conditions. At high momenta it becomes the underlying perturbative coupling $a(Q^2)$ to a very high precision. And at intermediately low momenta $Q^2 \sim 1 \text{ GeV}^2$ it gives results consistent with the data of the semihadronic τ lepton decays as measured by OPAL and ALEPH. The coupling is constructed in a dispersive way, ensuring as a byproduct the holomorphic behavior of $\mathscr{A}(Q^2)$ in the complex Q^2 -plane which reflects the holomorphic behavior of the spacelike QCD observables. The coupling was applied to the Borel sum rules for τ -decay V + A spectral functions and values for the gluon (dimension-4) condensate and the dimension-6 condensate were extracted, which reproduce the measured OPAL and ALEPH data to a significantly better precision than the perturbative $\overline{\text{MS}}$ coupling (+OPE) approach. The evaluation of the V -channel Adler function, related with the $e^+e \rightarrow$ hadrons ratio, and of the Bjorken polarized sum rule, both at low $Q^2 \sim 1 \text{ GeV}^2$, also give results considerably better than with the usual $\overline{\text{MS}}$ pOCD+OPE approach.

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1. Motivation

In perturbative QCD (pQCD), the running coupling $a(Q^2) \equiv \alpha_s(Q^2)/\pi$ as a function of $Q^2 \equiv$ $-q^2$ has Landau singularities, i.e., singularities outside the negative semiaxis in the complex Q^2 plane, in most renormalization schemes, including \overline{MS} and related schemes. On the other hands, the general principles of quantum field theories, namely locality, microcausality and unitarity, imply that the spacelike physical quantities $\mathscr{D}(Q^2)$, such as current correlators and hadronic structure functions, are holomorphic (analytic) functions of Q^2 in the entire complex plane with the exception of the negative semiaxis, $Q^2 \in \mathbb{C} \setminus (-\infty, -M_{\text{thr}}^2]$, where M_{thr} is a threshold scale of the order of the light meson mass. Such quantities are evaluated as functions of the QCD running coupling $a(\kappa Q^2)$ where $\kappa \sim 1$ is a chosen renormalization scale parameter, $\mathscr{D}(Q^2)_{ev} = \mathscr{F}(a(\kappa Q^2))$. Since $a(\kappa Q^2)$, due to the mentioned Landau singularities, does not share the holomorphic properties that $\mathscr{D}(Q^2)$ must have, the evaluated expressions $\mathscr{D}(Q^2)_{ev.}$ (such as truncated perturbation series for the leading-twist and higher-twist terms) have wrong holomorphic properties. Furthermore, at low $|Q^2|$, due to the vicinity of the Landau singularities the evaluation of $a(\kappa Q^2)$ and thus of $\mathscr{D}(Q^2)_{ev}$. becomes unreliable. For these reasons, it is preferrable to evaluate $\mathscr{D}(Q^2)$ using a holomorphic analog $\mathscr{A}(Q^2)$ of the pQCD coupling $a(Q^2)$, i.e., $\mathscr{A}(Q^2)$ based on $a(Q^2)$, but in contrast to it, has no Landau singularities, i.e., $\mathscr{A}(Q^2)$ is a holomorphic function for $Q^2 \in \mathbb{C} \setminus (-\infty, -M_{thr}^2]$.

A first version of such a coupling, called Analytic Perturbation Theory (APT), was constructed in [1]. $\mathscr{A}^{(APT)}(Q^2)$ has the same discontinuity along the negative axis $Q^2 = -\sigma < 0$ as the underlying pQCD coupling $a(Q^2)$, but the Landau discontinuities and singularities of $a(Q^2)$ along the positive axis are eliminated in the dispersive integral representation of the coupling. Later, several other couplings were constructed with the dispersive approach, where the discontinuity at low $\sigma > 0$ was changed or parametrized so that the coupling fulfilled certian physically-motivated conditions [2, 3, 4, 5, 6]. These coupling are IR-finite, $\mathscr{A}(0) < \infty$. Analytization of the pQCD beta function $\beta(a)/a$ also leads to a holomorphic coupling $\mathscr{A}(Q^2)$ [7], but it is infinite at $Q^2 = 0$. Lightfront holography approach to QCD [8] gives an IR-finite coupling $\mathscr{A}(Q^2)_{\rm LFH} \propto \exp(-Q^2/Q_0^2)$ where $Q_0 \sim 1$ GeV.

The general algorithm for the construction of the higher power analogs $a(Q^2)^n \mapsto \mathscr{A}_n(Q^2)$ $(\neq \mathscr{A}(Q^2)^n$ in general) in such holomorphic frameworks was presented in [4] (integer *n*) and in [9] (*n* noninteger).

The mentioned dispersive approaches can also be applied directly to (spacelike) physical quantities $\mathscr{D}(Q^2)$ to enforce the correct holomorphic and physical properties, cf. [10, 3, 5]. We will not pursue this line here.

2. Construction of $\mathscr{A}(Q^2)$

Here we will describe briefly the construction of the coupling $\mathscr{A}(Q^2)$ of Refs. [6]. Having the (underlying) pQCD coupling $a(Q^2)$, in a given renormalization scheme, we will impose the following physically-motivated requirements on the coupling $\mathscr{A}(Q^2)$:

1. $\mathscr{A}(Q^2)$ is a holomorphic function for $Q^2 \in \mathbb{C} \setminus (-\infty, -M_{\text{thr}}^2]$.

2. At high $|Q^2| \gg 1$ GeV² we have practically equality $\mathscr{A}(Q^2) = a(Q^2)$ (pQCD at high $|Q^2|$).

3. At intermediate $|Q^2| \sim 1$ GeV², the $\mathscr{A}(Q^2)$ -approach reproduces the well measured semihadronic τ -decay physics.

4. At low $|Q^2| \leq 0.1 \text{ GeV}^2$, we have $\mathscr{A}(Q^2) \sim Q^2$, as suggested by lattice results for the Landau gauge gluon and ghost propagators [12].

It turns out that the property 1 will be a byproduct of the construction of $\mathscr{A}(Q^2)$ by the above properties 2-4.

First, we will explain the property 4. We recall that in pQCD we have for $a(Q^2) \equiv \alpha_s(Q^2)/\pi$

$$a(Q^2) = a(\Lambda^2) Z_{\rm gl}^{(\Lambda)}(Q^2) Z_{\rm gh}^{(\Lambda)}(Q^2)^2 / Z_1^{(\Lambda)}(Q^2)^2, \qquad (2.1)$$

where Z_{gl} , Z_{gh} , Z_1 are the dressing functions of the gluon and ghost propagator, and of the gluonghost-ghost vertex. In the Landau gauge, $Z_1^{(\Lambda)}(Q^2) = 1$ to all orders [13]. Hence

$$\mathscr{A}_{\text{latt.}}(Q^2) \equiv \mathscr{A}_{\text{latt.}}(\Lambda^2) Z_{\text{gl}}^{(\Lambda)}(Q^2) Z_{\text{gh}}^{(\Lambda)}(Q^2)^2 .$$
 (2.2)

$$\mathscr{A}_{\text{latt.}}(Q^2) = \mathscr{A}(Q^2) + \Delta \mathscr{A}_{\text{NP}}(Q^2) .$$
(2.3)

Since $\mathscr{A}_{\text{latt.}}(Q^2) \sim Q^2$ when $Q^2 \to 0$, no finetuning at $Q^2 \to 0$ implies

$$\Delta \mathscr{A}_{\rm NP}(Q^2) \sim Q^2$$
 and $\mathscr{A}(Q^2) \sim Q^2$ $(Q^2 \to 0)$ (2.4)

The coupling $\mathscr{A}(Q^2)$ thus also goes to zero when $Q^2 \to 0$, this is the mentioned property 4.

Now we will construct $\mathscr{A}(Q^2)$ such that the mentioned properties 2 and 4 can be enforced. The dispersive relation for $a(Q^2)$ is

$$a(Q^2) = \frac{1}{\pi} \int_{\sigma = -Q_{\rm br}^2 - \eta}^{\infty} \frac{d\sigma \rho_a(\sigma)}{(\sigma + Q^2)} \qquad (\eta \to +0), \tag{2.5}$$

where $Q^2 = Q_{br}^2 > 0$ is the branching point for Landau singularities, and $\rho_a(\sigma) \equiv \text{Im } a(Q^2 = -\sigma - i\varepsilon)$ is the discontinuity (spectral) function of *a*. The dispersive relation for the corresponding $\mathscr{A}(Q^2)$ is

$$\mathscr{A}(Q^2) = \frac{1}{\pi} \int_{\sigma = M_{\rm thr}^2 - \eta}^{\infty} \frac{d\sigma \rho_{\mathscr{A}}(\sigma)}{(\sigma + Q^2)} \qquad (\eta \to +0), \tag{2.6}$$

where $\rho_{\mathscr{A}}(\sigma) \equiv \text{Im } \mathscr{A}(Q^2 = -\sigma - i\varepsilon)$. At high positive $\sigma_0 > M_0$ (~ 1 GeV) we expect $\rho_{\mathscr{A}} = \rho_a$, but at low positive $\sigma < M_0^2$ we expect $\rho_{\mathscr{A}} \neq \rho_a$. Here, M_0^2 is a pQCD onset-scale. The a priori unknown behavior of $\rho_{\mathscr{A}}$ in the low- σ regime ($\sigma < M_0^2$) will be parametrized with several delta functions (peaks), specifically three delta functions. This means

$$\rho_{\mathscr{A}}(\sigma) = \pi \sum_{j=1}^{3} \mathscr{F}_{j} \,\delta(\sigma - M_{j}^{2}) + \Theta(\sigma - M_{0}^{2})\rho_{a}(\sigma) \,. \tag{2.7}$$

$$\Rightarrow \mathscr{A}(Q^2) = \sum_{j=1}^3 \frac{\mathscr{F}_j}{(Q^2 + M_j^2)} + \frac{1}{\pi} \int_{M_0^2}^{\infty} d\sigma \frac{\rho_a(\sigma)}{(Q^2 + \sigma)} \,. \tag{2.8}$$

At $|Q^2| > 1$ GeV², $\mathscr{A}(Q^2)$ should practically coincide with pQCD (property 2), so we require

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$$\mathscr{A}(Q^2) - a(Q^2) \sim \left(\frac{\Lambda_L^2}{Q^2}\right)^5 \quad (|Q^2| > \Lambda_L^2 \sim 0.1 - 1 \text{ GeV}^2) .$$
 (2.9)

This (property 2), and the lattice condition $\mathscr{A}(Q^2) \sim Q^2$ at $Q^2 \to 0$ (property 4), give 5 conditions

$$-\frac{1}{\pi} \int_{M_0^2}^{\infty} d\sigma \frac{\rho_a(\sigma)}{\sigma} = \sum_{j=1}^3 \frac{\mathscr{F}_j}{M_j^2}; \qquad (2.10)$$

$$\frac{1}{\pi} \int_{-Q_{\rm br}^2}^{M_0^2} d\sigma \sigma^k \rho_a(\sigma) = \sum_{j=1}^3 \mathscr{F}_j M_j^{2k} \quad (k=0,1,2,3) .$$
(2.11)

But we have 7 parameters, we need 7 conditions, i.e., two more: a) $Q_{\text{max}}^2 \approx 0.135 \text{ GeV}^2$ by lattice calculations, where $\mathscr{A}(Q_{\text{max}}^2) = \mathscr{A}_{\text{max}}$ (extension of property 4). b) \mathscr{A} -coupling framework should reproduce the correct value $r_{\tau}^{(D=0)} \approx 0.20$ (cf. [14]) of the (QCD-part of the) ratio of the semihadronic τ decay width (property 3), where

$$r_{\tau,\text{th}}^{(D=0)} = \frac{1}{2\pi} \int_{-\pi}^{+\pi} d\phi \, (1+e^{i\phi})^3 (1-e^{i\phi}) \, d(Q^2 = m_\tau^2 e^{i\phi}; D=0) \,. \tag{2.12}$$

Here, $d(Q^2; D=0)$ is the massless Adler function, $d(Q^2; D=0) = -1 - 2\pi^2 d\Pi(Q^2; D=0)/d \ln Q^2$ (Π is the vector or axial current correlator), and its perturbation expansion is known up to $\sim a^4$. In our approach, $a(Q^2)^n \mapsto \mathscr{A}_n(Q^2) \ (\neq \mathscr{A}(Q^2)^n)$ [4]

$$d(Q^2; D=0)_{an}^{[4]} = \mathscr{A}(Q^2) + d_1 \mathscr{A}_2(Q^2) + d_2 \mathscr{A}_3(Q^2) + d_3 \mathscr{A}_4(Q^2).$$
(2.13)

These 7 conditions (with $r_{\tau,\text{th}}^{(D=0)} = 0.201$) then give the values of the parameters of the coupling (cf. Ref. [6] 2nd entry)

$$M_0^2 = 8.719 \text{ GeV}^2;$$
 $M_1^2 = 0.053 \text{ GeV}^2,$ $M_2^2 = 0.247 \text{ GeV}^2,$ $M_3^2 = 6.341 \text{ GeV}^2;$
 $\mathscr{F}_1 = -0.0383 \text{ GeV}^2,$ $\mathscr{F}_2 = 0.1578 \text{ GeV}^2,$ $\mathscr{F}_3 = 0.0703 \text{ GeV}^2.$

The underlying $(N_f = 3)$ pQCD coupling *a* was constructed in the (4-loop) lattice MiniMOM scheme [11], because the lattice results [12] for $\mathscr{A}_{latt.}(Q^2)$ (at low positive Q^2) were obtained in this scheme. We do, however, rescale Q^2 from the MiniMOM (Λ_{MM}) to the usual $\Lambda_{\overline{MS}}$ -scale convention. The results are presented in Fig. 1. We note that all the locations of the delta functions, $\sigma_j = M_j^2$ (j = 1, 2, 3), and M_0^2 , turned out to be positive, i.e., the coupling $\mathscr{A}(Q^2)$ has no cut along the positive axis and is thus, as a consequence, holomorphic. The threshold mass is $M_{\text{thr}} = M_1 \approx 0.23$ GeV, which is, as expected, in the regime of the light meson masses.

3. Conclusions

A QCD coupling $\mathscr{A}(Q^2)$ was constructed, in the lattice MiniMOM scheme, rescaled to the usual $\Lambda_{\overline{\text{MS}}}$ -scale convention. It has the following properties:

A) $\mathscr{A}(Q^2)$ reproduces the pQCD results at high momenta $|Q^2| > 1 \text{ GeV}^2$.

B) $\mathscr{A}(Q^2) \sim Q^2$ at low momenta $|Q^2| \lesssim 0.1 \text{ GeV}^2$, as suggested by high-volume lattice results.

C) $\mathscr{A}(Q^2)$ at intermediate momenta $|Q^2| \sim 1 \text{ GeV}^2$ reproduces the well the measured physics of the inclusive semihadronic τ -lepton decay.

D) $\mathscr{A}(Q^2)$, as a byproduct of construction, possesses the attractive holomorphic behavior shared by QCD spacelike physical quantities.



Figure 1: Coupling \mathscr{A} at positive Q^2 (solid curve) and the underlying pQCD coupling *a* (light dashed curve), both in 4-loop MiniMOM scheme. Included is \mathscr{A}_2 (dashed curve) which is the \mathscr{A} -analog of power a^2 , and the naive (i.e., unusable) power \mathscr{A}^2 (dot-dashed curve). The usual $\overline{\text{MS}}$ scheme pQCD coupling \overline{a} (dotted curve) is also included. The lattice coupling $\mathscr{A}_{\text{latt.}}$ (Ref. [12], first entry), with Q^2 rescaled as explained in the text, is presented as points with bars.

The usual $\overline{\text{MS}}$ pQCD coupling $a(Q^2; \overline{\text{MS}}) \equiv \alpha_s(Q^2; \overline{\text{MS}})/\pi$ shares with the coupling \mathscr{A} only the property A (high-momentum), but on the other three properties it is either worse (point C) or it fails (points B and D).

We applied the Borel sum rules to τ -decay V + A spectral functions, and we obtained values for the gluon (dimension-4) condensate and the dimension-6 condensate by fitting to the measured OPAL and ALEPH data. The fitting turned out to be significantly better than with the perturbative $\overline{\text{MS}}$ coupling (+OPE) approach. Further, when we compared the obtained theoretical and the experimental V-channel Adler function $\mathcal{D}_V(Q^2)$, related with the $e^+e \rightarrow$ hadrons ratio, at low $Q^2 \sim 1 \text{ GeV}^2$, the results were considerably better than with the usual $\overline{\text{MS}}$ pQCD+OPE approach. We refer for details to Refs. [6] (in the 3-loop and 4-loop MiniMOM, respectively). Recently we also applied the approach with this coupling to the study of the Bjorken polarized sum rule at low Q^2 [15], and the results are significantly better than with the $\overline{\text{MS}}$ pQCD coupling.

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- Gorazd Cvetič
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