PROCEEDINGS OF SCIENCE

Exotic states and their properties from large-N_c QCD

Wolfgang Lucha
Institute for High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria
E-mail: Wolfgang.Lucha@oeaw.ac.at

Dmitri Melikhov*
Institute for High Energy Physics, Austrian Academy of Sciences, Nikolsdorfergasse 18, A-1050 Vienna, Austria, and
D. V. Skobeltsyn Institute of Nuclear Physics, M. V. Lomonosov Moscow State University, 119991, Moscow, Russia, and
Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria
E-mail: dmitri_melikhov@gmx.de

Hagop Sazdjian
Institut de Physique Nucléaire, CNRS-IN2P3, Université Paris-Sud, Université Paris-Saclay, 91406 Orsay Cedex, France
E-mail: sazdjian@ipno.in2p3.fr

The analysis of two-ordinary-meson scattering amplitudes in the limit of a large number, N_c, of the colour degrees of freedom of quantum chromodynamics, with suitably decreasing strong coupling and all quarks transforming according to the gauge group's fundamental representation, enables us to establish a set of rigorous consistency conditions for the emergence of a tetraquark (*i.e.*, a bound state of two quarks and two antiquarks) as a pole in these amplitudes. For genuinely flavour-exotic tetraquarks, these constraints require the existence of two tetraquark states distinguishable by their preferred couplings to two ordinary mesons, whereas, for cryptoexotic tetraquarks, our constraints may be satisfied by a single tetraquark state, which then, however, may mix with ordinary mesons. For elucidation of the tetraquark features, the consideration of the *subleading* contributions proves to be mandatory: for both variants of tetraquarks, their decay widths fall off like $1/N_c^2$ for large N_c.

The European Physical Society Conference on High Energy Physics
5–12 July 2017
Venice, Italy

*Speaker.

PoS(EPS-HEP2017)390

1. Incentive: Implications of the Large-N_c Limit of QCD on Polyquark Bound States

Tetraquark mesons are exotic bound states of two quarks and two antiquarks hypothesized to be predicted by quantum chromodynamics, the quantum field theory governing the strong interactions. We extract information on general features of tetraquarks by considering four-point Green functions of bilinear quark currents $j_{ij} = \bar{q}_i \mathfrak{A} q_j$ serving as interpolating operators of mesons M_{ij} composed of antiquark $\bar{q}_i$ and quark q_j, $\langle 0|j_{ij}|M_{ij}\rangle = f_{M_{ij}}$, where $i, j, \ldots = 1,2,3,4$ represent the flavour quantum numbers of the quarks and the generalized Dirac matrix $\mathfrak{A}$ fits to the interpolated meson's parity and spin quantum numbers, within that generalization of QCD given by the limit of a large number N_c of colour degrees of freedom, that is, by a quantum field theory based on the gauge group $\mathrm{SU}(N_\mathrm{c})$, with fermions transforming according to the N_c-dimensional fundamental[1] representation of $\mathrm{SU}(N_\mathrm{c})$ [1]; the strong fine-structure coupling $\alpha_\mathrm{s} \equiv g_\mathrm{s}^2/(4\pi)$ is assumed to decrease, for $N_\mathrm{c} \to \infty$, like $\alpha_\mathrm{s} \propto 1/N_\mathrm{c}$.

Systematic application of the limit $N_\mathrm{c} \to \infty$ puts us in a position to discriminate unambiguously two classes of hadrons according to their large-N_c behaviour: those that survive the large-N_c limit as stable bound states and hence may be dubbed as "ordinary", and the others, *i.e.*, those that do not but disappear for $N_\mathrm{c} \to \infty$ [2]. However, although tetraquarks can only appear at an N_c-subleading order [3], in order to enable observability by experiment their decay widths Γ should not grow with N_c [4]. It is straightforward to prove that at large N_c the meson decay constants $f_{M_{ij}}$ rise like $f_{M_{ij}} \propto N_\mathrm{c}^{1/2}$ [1].

2. Analysis of Tetraquark Poles in Meson–Meson Scattering Amplitudes at Large N_c

For well-definiteness, let's base this large-N_c QCD study on a variety of plausible assumptions:

- For the analysis of tetraquarks, the large-N_c limit makes sense and works well; the application of the $1/N_\mathrm{c}$ expansion to tetraquarks is justified and allows us to arrive at reliable conclusions.
- In the large-N_c limit, poles interpretable as tetraquark bound states exist in the complex plane.
- In the series expansions in powers of $1/N_\mathrm{c}$ of those n-point Green functions which potentially accommodate tetraquark poles, the tetraquark states T arise at the lowest possible $1/N_\mathrm{c}$ order.
- The masses, m_T, of the tetraquark states, T, do not grow with N_c but remain finite for $N_\mathrm{c} \to \infty$.

Before embarking on elucidating the dynamics of the formation of a tetraquark bound state, the main issue is to single out, in the expansion of a four-point Green function in powers of $1/N_\mathrm{c}$ and α_s, those (large-N_c) Feynman diagrams that might develop tetraquark poles. To this end, we impose, for tetraquarks supposedly consisting of (anti-)quarks of masses m_1, m_2, m_3 and m_4 and, with respect to the s channel, incoming external momenta p_1 and p_2 the following set of basic selection criteria [5]:

1. A tetraquark-phile Feynman diagram depends *nonpolynomially* on its variable $s \equiv (p_1 + p_2)^2$.
2. A tetraquark-phile Feynman diagram supports appropriate four-quark intermediate states and exhibits the corresponding branch cuts starting at the branch points $s = (m_1 + m_2 + m_3 + m_4)^2$.

Only Feynman diagrams complying with both criteria can contribute to the physical tetraquark pole.

[1] For the sake of simplicity, in particular, in order to deal with a unique $N_\mathrm{c} \to \infty$ limit, let us disregard the other logical possibility of fermions transforming according to the $\frac{1}{2} N_\mathrm{c} (N_\mathrm{c} - 1)$-dimensional antisymmetric representation of $\mathrm{SU}(N_\mathrm{c})$.

PoS(EPS-HEP2017)390

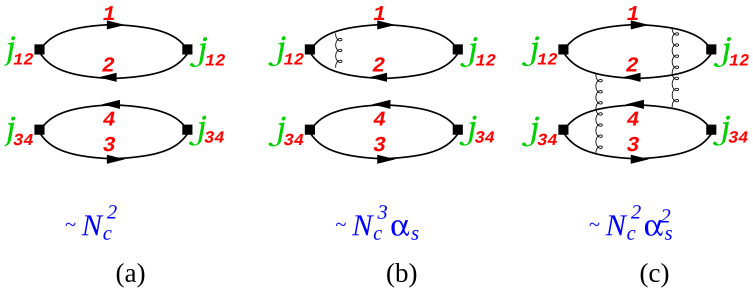

Figure 1: Four-current Green function $\langle j_{12}^\dagger\, j_{34}^\dagger\, j_{12}\, j_{34}\rangle$: N_c-leading (a,b) and N_c-subleading (c) contributions.

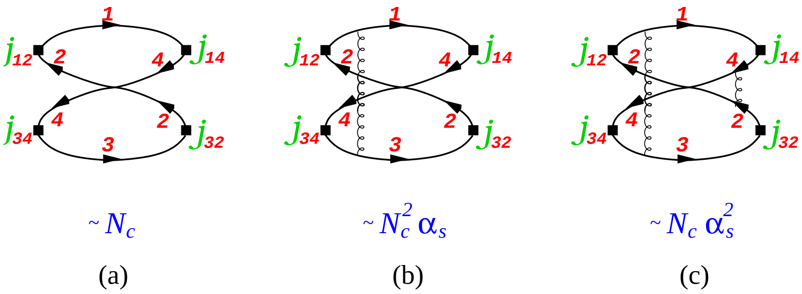

Figure 2: Four-current Green function $\langle j_{14}^\dagger\, j_{32}^\dagger\, j_{12}\, j_{34}\rangle$: N_c-leading (a,b) and N_c-subleading (c) contributions.

Under these premises, we derive, for various classes of tetraquarks T, the large-N_c behaviour of • the tetraquark-phile four-point Green functions, identified by a subscript T, • the amplitudes A for transitions between tetraquark T and two ordinary mesons, and • the tetraquark decay rate $\Gamma(T)$ [5]:

★ For genuinely *exotic* tetraquarks $T = (\bar{q}_1\, q_2\, \bar{q}_3\, q_4)$, involving four different quark flavours, the correlators without (Fig. 1) and with (Fig. 2) a flavour reshuffle behave differently at large N_c:

$$\langle j_{12}^\dagger\, j_{34}^\dagger\, j_{12}\, j_{34}\rangle_T = O(N_c^0)\ , \qquad \langle j_{14}^\dagger\, j_{32}^\dagger\, j_{14}\, j_{32}\rangle_T = O(N_c^0)\ , \qquad \langle j_{14}^\dagger\, j_{32}^\dagger\, j_{12}\, j_{34}\rangle_T = O(N_c^{-1})\ .$$

This observation forces us to conclude that there exist, at least, two different tetraquark states, called T_A and T_B, each with a preferred two-meson decay channel, but with *parametrically* the same decay rate of order N_c^{-2}. Phrased in other words, “always two there are, ... no less” [6]:

$$\begin{aligned} A(T_A \leftrightarrow M_{12}\, M_{34}) &= O(N_c^{-1})\ , \quad A(T_A \leftrightarrow M_{14}\, M_{32}) = O(N_c^{-2}) \quad \Longrightarrow \quad \Gamma(T_A) = O(N_c^{-2})\ ,\\ A(T_B \leftrightarrow M_{12}\, M_{34}) &= O(N_c^{-2})\ , \quad A(T_B \leftrightarrow M_{14}\, M_{32}) = O(N_c^{-1}) \quad \Longrightarrow \quad \Gamma(T_B) = O(N_c^{-2})\ . \end{aligned}$$

The tetraquarks T_A and T_B may mix, with mixing parameter decreasing at least as fast as $1/N_c$.

★ For *cryptoexotic* tetraquarks $T = (\bar{q}_1\, q_2\, \bar{q}_2\, q_3)$, with quark flavour of the ordinary mesons M_{13}, the correlators without (Fig. 3) and with (Fig. 4) a flavour reshuffle have similar N_c behaviour:

$$\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle_T = O(N_c^0)\ , \qquad \langle j_{13}^\dagger\, j_{22}^\dagger\, j_{13}\, j_{22}\rangle_T = O(N_c^0)\ , \qquad \langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle_T = O(N_c^0)\ .$$

The implied N_c constraints may be solved by a single tetraquark state T decaying according to

$$A(T \leftrightarrow M_{12}\, M_{23}) = O(N_c^{-1})\ , \quad A(T \leftrightarrow M_{13}\, M_{22}) = O(N_c^{-1}) \quad \Longrightarrow \quad \Gamma(T) = O(N_c^{-2})\ ,$$

and mixing with ordinary mesons M_{13} with mixing strength dropping not slower than $1/\sqrt{N_c}$.

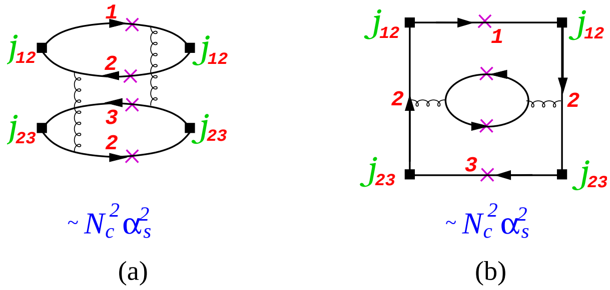

Figure 3: Four-current Green function $\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle$: some N_c-leading contributions potentially capable of developing a cryptoexotic tetraquark pole, with constituents identified by purple crosses on their propagators.

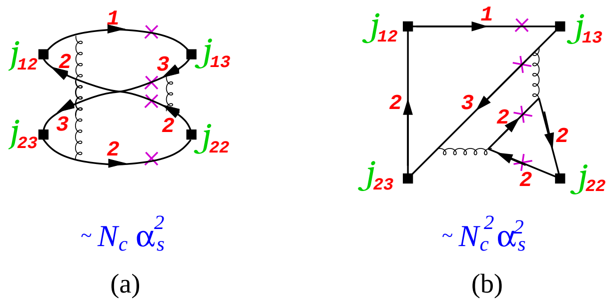

Figure 4: Four-point Green function $\langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle$: N_c-leading (b) and N_c-subleading (a) contributions that potentially support a cryptoexotic tetraquark pole with quark content fixed by the purple-crossed propagators.

3. Summary: Insights on Minimum Numbers and Decay Rates of Tetraquark Types

(Crypto-) exotic tetraquarks T *are* narrow: their decay widths $\Gamma(T)$ vanish in the limit $N_c \to \infty$. Unlike earlier claims [7], they have widths of order $1/N_c^2$. If exotic, they come in two versions, with N_c-dependent branching ratios. Our results [5] generalize ones got for special cases or channels [8].
Acknowledgement. D.M. was supported by the Austrian Science Fund (FWF), project P29028-N27.

References

[1] G. ’t Hooft, Nucl. Phys. B **72** (1974) 461; E. Witten, Nucl. Phys. B **160** (1979) 57.

[2] R. L. Jaffe, Nucl. Phys. A **804** (2008) 25, arXiv:hep-ph/0701038.

[3] S. Coleman, *Aspects of Symmetry — Selected Erice Lectures* (Cambridge University Press, Cambridge, England, 1985), Chap. 8: $1/N$.

[4] S. Weinberg, Phys. Rev. Lett. **110** (2013) 261601, arXiv:1303.0342 [hep-ph].

[5] W. Lucha, D. Melikhov, and H. Sazdjian, Phys. Rev. D **96** (2017) 014022, arXiv:1706.06003 [hep-ph].

[6] Yoda, JM, in *Star Wars — Episode I: The Phantom Menace* (1999, in a galaxy far, far away), scene 197.

[7] L. Maiani, A. D. Polosa, and V. Riquer, J. High Energy Phys. 06 (2016) 160, arXiv:1605.04839 [hep-ph].

[8] M. Knecht and S. Peris, Phys. Rev. D **88** (2013) 036016, arXiv:1307.1273 [hep-ph]; T. D. Cohen and R. F. Lebed, Phys. Rev. D **90** (2014) 016001, arXiv:1403.8090 [hep-ph].