PROCEEDINGS OF SCIENCE

Yukawa couplings for light stringy states

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This proceeding is based on [1], where we use 4-point amplitudes in order to normalize the vertex operators for massless and massive strings living at D-brane intersections and next to evaluate Yukawa couplings between these states. Using our results we are planning to proceed and evaluate physical processes which might be visible at LHC.

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1. Introduction

Open string compactifications provide a very succesful framework for semi-realistic model building[1]. In this class of models the gauge fields are described by strings with both ends on the same stack of some lower-dimensional hyperplanes, called D-branes, whereas the chiral matter is localised at the intersection of different D-brane stacks. Thus, all the Standard Model fields can be described by open strings with either both of their ends on the same stack (gluons, SU(2) gauge fields and the Hypercharge) or on different stacks of D-branes (quarks, leptons and the Higgs)[2].

A novel property of this class of models is that the string scale can be significantly low, even at a few TeV range [26–28]. Therefore, stringy effects become viable candidates for physics beyond the Standard Model (see or example anomalous Z' [29–42], Kaluza Klein states [43–49], or purely stringy signatures [50–63][3].

A string living at the intersection of two different stacks of D-branes, it vibrates with frequencies proportional to the angle between the branes θ. Thus, a whole tower of massive copies of the lowest/massless mode is living at the same intersection, with masses proportional to $\sqrt{\theta} M_s$. Notice that the first excited mode will be lighter than the standard Regge excitations by a factor θ/π. As a consequence, semi-realistic D-brane configurations predict massive copies for all matter fields of the Standard Model. If the string scale M_s is at a few TeV range, these copies can be very light (*light stringy states*) and they might be visible at LHC [66–69].

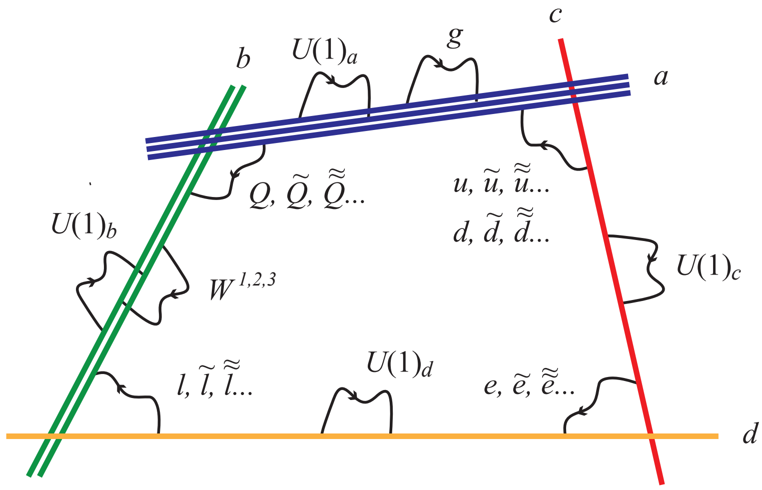

Figure 1: Local D-brane realization of the Standard Model. Each field is accompanied by a whole tower of massive excitations.

In [1] we compute Yukawa couplings between light stringy states and massless ones. In order to determine the correct normalization of the vertex operators (VO) [70,71] of the fields mentioned above, we evaluate four-point amplitudes and by factorizations we extract the desired couplings. In

[1] For original work, see [2–14] and for recent reviews on D-brane model building, see [15–19] and references therein.

[2] For original work on local D-brane configurations, see [20, 21]. For a systematic analysis of local D-brane configurations, see [22–25].

[3] For recent reviews, see [64, 65].

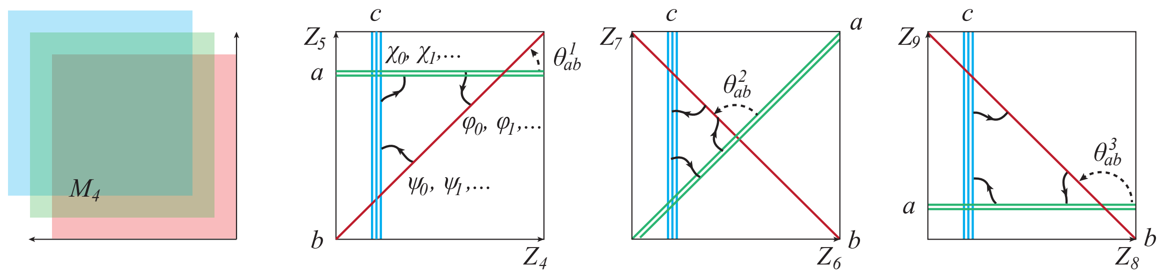

Figure 2: A simple configuration of three stacks of D6-branes in a torus T_I^2. The angles in the figure are large for illustrative purposes.

this proceeding we present few examples which demonstrate our strategy and the rest of the results can be found in the original work [1].

The plan of this proceeding is as follows. In Section 2, we describe the set-up of intersecting D6-branes on tori and present the expressions for both massless and massive BRST invariant vertex operators. In Section 3 we compute scattering amplitudes on the disk (tree level). Normalization problems are solved by first considering amplitudes that expose vector boson exchange on one or both channels (s and t).

2. Setup and vertex operators

In order to compute gauge and Yukawa couplings of light massive string states we have to specify our setup. We consider three stacks of D6-branes wrapping factorizable 3-cycles on a six-torus $T^6 = T^2 \times T^2 \times T^2$, which are labeled by a, b and c. In each torus T_I^2 $(I = 1,2,3)$ two generic stacks, let us say a and b, intersecting at an angle $\theta^I_{ab} = \pi a^I_{ab}$ (next we will use the angles a^I_{ab} subtracting π for simplicity). Supersymmetry requires $\pm a^1_{ab} \pm a^2_{ab} \pm a^3_{ab} = 0 \mod 2$, for some choice of signs[4]. For non-vanishing Yukawa couplings, we take

$$a^1_{ab} + a^2_{ab} + a^3_{ab} = 0 \qquad 0 < a^1_{ab} < 1 \qquad 0 < a^1_{bc} < 1 \qquad -1 < a^1_{ca} < 0 \tag{2.1}$$
$$a^1_{bc} + a^2_{bc} + a^3_{bc} = 0 \qquad 0 < a^2_{ab} < 1 \qquad 0 < a^2_{bc} < 1 \qquad -1 < a^2_{ca} < 0 \tag{2.2}$$
$$a^1_{ca} + a^2_{ca} + a^3_{ca} = -2 \qquad -1 < a^3_{ab} < 0 \qquad -1 < a^3_{bc} < 0 \qquad -2 < a^3_{ca} < 0 \tag{2.3}$$

Strings ending on two different stacks of branes give rise to a massless (chiral) spectrum with multiplicity given by the number of intersections and a massive spectrum. The VO's for the lowest string modes (massless and first massive) living at intersections are given by [70]

$$V^{(-1)}_{\phi_0=\phi_0^{ab}} = C_{\phi_0} e^{-\phi_{10}} \phi_0 e^{-\varphi} \sigma_{a^1_{a,b}} \sigma_{a^2_{a,b}} \sigma_{1+a^3_{a,b}} e^{i[a^1_{a,b}\varphi_1 + a^2_{a,b}\varphi_2 + (a^3_{a,b}+1)\varphi_3]} e^{ikX} \tag{2.4}$$
$$V^{(-\frac{1}{2})}_{\psi_0=\chi_0^{bc}} = C_{\psi_0} e^{-\phi_{10}} \psi_0^{\alpha} S_\alpha e^{-\frac{\varphi}{2}} \sigma_{a^1_{b,c}} \sigma_{a^2_{b,c}} \sigma_{1+a^3_{b,c}} e^{i[(a^1_{b,c}-\frac{1}{2})\varphi_1 + (a^2_{b,c}-\frac{1}{2})\varphi_2 + (a^3_{b,c}+\frac{1}{2})\varphi_3]} e^{ikX} \tag{2.5}$$
$$V^{(-\frac{1}{2})}_{\chi_0=\chi_0^{ca}} = C_{\chi_0} e^{-\phi_{10}} \chi_0^{\alpha} S_\alpha e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \sigma_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1 + (a^2_{c,a}+\frac{1}{2})\varphi_2 + (a^3_{c,a}+\frac{1}{2})\varphi_3]} e^{ikX} \tag{2.6}$$

[4]Semi-realistic MSSM constructions on factorizable orbifolds can be found in [22, 72–94].

$$V^{(-\frac{1}{2})}_{\psi_1=\chi_1^{bc}} = C_{\psi_1} e^{-\phi_{10}} \psi_1^{\alpha} S_{\alpha} e^{-\frac{\varphi}{2}} \tau_{a^1_{b,c}} \sigma_{a^2_{b,c}} \sigma_{1+a^3_{b,c}} e^{i[(a^1_{b,c}-\frac{1}{2})\varphi_1+(a^2_{b,c}-\frac{1}{2})\varphi_2+(a^3_{b,c}+\frac{1}{2})\varphi_3]} e^{ikX}$$
$$+ C_{\tilde{\psi}_1} e^{-\phi_{10}} \tilde{\psi}^{\dagger}_{1\dot{\alpha}} C^{\dot{\alpha}} e^{-\frac{\varphi}{2}} \sigma_{a^1_{b,c}} \sigma_{a^2_{b,c}} \sigma_{1+a^3_{b,c}} e^{i[(a^1_{b,c}+\frac{1}{2})\varphi_1+(a^2_{b,c}-\frac{1}{2})\varphi_2+(a^3_{b,c}+\frac{1}{2})\varphi_3]} e^{ikX} \tag{2.7}$$

$$V^{(-\frac{1}{2})}_{\chi_1=\chi_1^{ca}} = C_{\chi_1} e^{-\phi_{10}} \chi_1^{\alpha} S_{\alpha} e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \tau_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1+(a^2_{c,a}+\frac{1}{2})\varphi_2+(a^3_{c,a}+\frac{1}{2})\varphi_3]} e^{ikX}$$
$$+ C_{\tilde{\chi}_1} e^{-\phi_{10}} \tilde{\chi}^{\dagger}_{1\dot{\alpha}} C^{\dot{\alpha}} e^{-\frac{\varphi}{2}} \sigma_{1+a^1_{c,a}} \sigma_{1+a^2_{c,a}} \sigma_{1+a^3_{c,a}} e^{i[(a^1_{c,a}+\frac{1}{2})\varphi_1+(a^2_{c,a}+\frac{1}{2})\varphi_2+(a^3_{c,a}-\frac{1}{2})\varphi_3]} e^{ikX} \tag{2.8}$$

and the masses of the corresponding fields are

$$massless \; : \qquad m^2_{\phi_0} = 0 \quad , \quad m^2_{\psi_0} = 0 \quad , \quad m^2_{\chi_0} = 0 \tag{2.9}$$
$$massive \; : \qquad\qquad , \quad m^2_{\psi_1} = a^1_{bc}/\alpha' \quad , \quad m^2_{\chi_1} = (1-|a^3_{ca}|)/\alpha' \; . \tag{2.10}$$

Having the expression for the VO's of the massless and massive states living at intersections, we evaluate string amplitudes and consequently the Yukawa couplings.

3. Strategy

In principle, we could directly evaluate the Yukawa couplings by 3-point amplitudes. However, these amplitudes are ambiguous due to the unnormalized VO's of the incoming fields. In order to normalize and proceed to physical amplitudes we have to start by evaluating 4-point amplitudes.

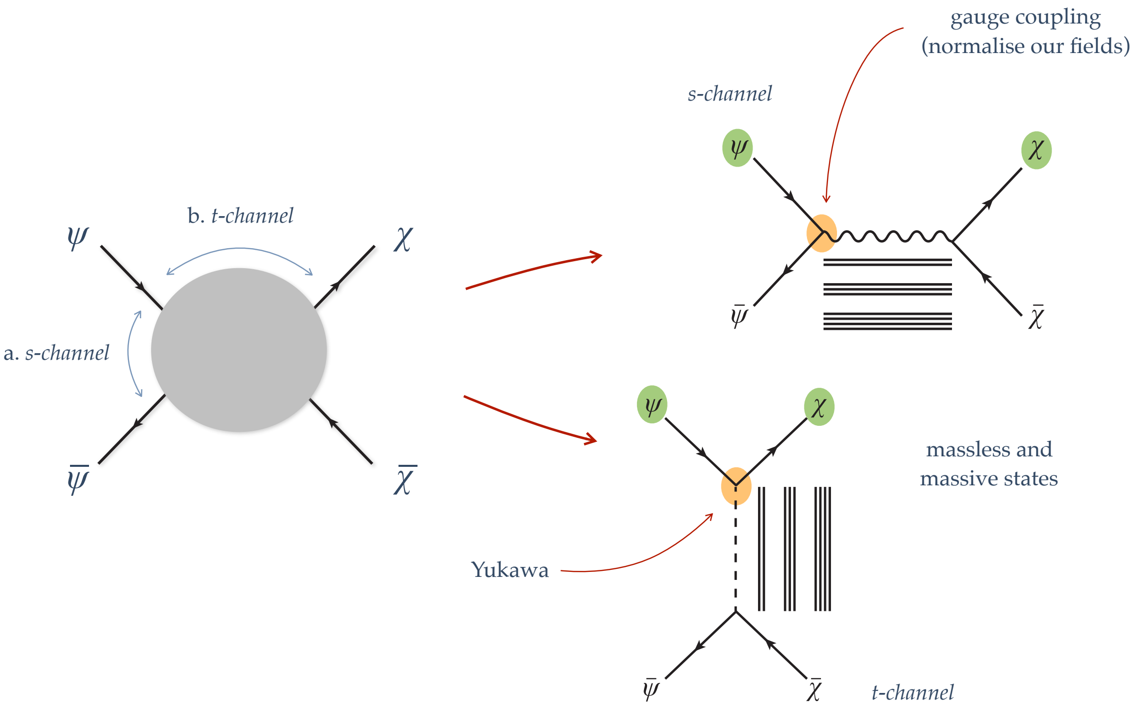

Figure 3: Factorization of the 4-point amplitude: a. At the lowest level of the s-channel we have the exchange of a gauge field which helps to normalise our fields. b. At t-channel we extract the various Yukawa couplings.

The gauge couplings plays a key role to fix the normalization of vertex operators and twist-field correlators that appear in amplitudes with gauge bosons in the external states or propagating in intermediate channels. Once these normalizations are fixed, we extract some of the desired Yukawas via factorization. Other Yukawa's must be extracted from amplitudes without gauge bosons, in this case the previously computed/known Yukawa's replace the gauge couplings in order to fix normalizations. Schematically the strategy is given in the diagram 3.

3.1 The amplitude $\mathscr{A}(\bar{\psi}_0, \psi_0, \chi_0, \bar{\chi}_0)$

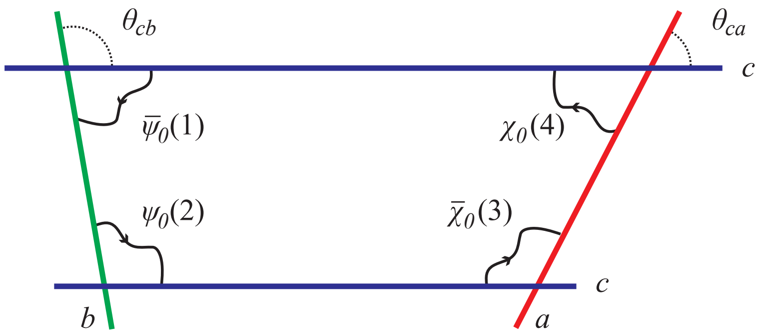

Figure 4: A configuration of branes describing the scattering between the spinors ψ_0 and χ_0.

Following the strategy above we compute specific 4-point amplitudes. At this proceeding we present few examples. More details can be found at our original work [1].

The amplitude $\mathscr{A}(\bar{\psi}_0, \psi_0, \chi_0, \bar{\chi}_0)$ with two-independent angles yields

$$\begin{aligned}\mathscr{A}(\bar{\psi}_0^{cb}, \psi_0^{bc}, \chi_0^{ca}, \bar{\chi}_0^{ac}) &= \langle V_{\bar{\psi}_0}^{(-\frac{1}{2})} V_{\psi_0}^{(-\frac{1}{2})} V_{\chi_0}^{(-\frac{1}{2})} V_{\bar{\chi}_0}^{(-\frac{1}{2})} \rangle \\ &= g_{\rm op}^2 \alpha' \psi_0(2)\cdot\chi_0(3)\bar{\psi}_0(1)\cdot\bar{\chi}_0(4) \int_0^1 dx x^{\alpha' s-1}(1-x)^{\alpha' t-1} \times \\ &\times \prod_{I=1}^{3} \frac{4\pi^2 K_I^{c,ab}\alpha'}{L_{a,I}^{1/4} L_{b,I}^{5/4} L_{c,I}^{1/2}} \frac{\sqrt{\alpha'}}{L_{c,I} G_1^{(I)}(x)} \sum_{n_I, m_I} e^{-S_{\rm Ham}^{(I)}(m_I,n_I)}\end{aligned} \tag{3.1}$$

where L_a, I is the brane a's length in torus I. I_{ab} and similar are the number or intersections between the branes on the torus. $G_1^{(I)}(x)$ are hypergeometric functions $G_1 = {}_2F_1\,(\alpha, 1-\beta; 1; z)$, $G_2 = {}_2F_1\,(1-\alpha, \beta; 1; z)$, $K_I^{c,ab}$ is an overall normalization constant and $S_{\rm Ham}^{(I)}(m_I, n_I)$ is the classical action in "hamiltonian" form:

$$S_{\rm Ham}^{(I)}(m_I,n_I) = \frac{\pi t_I(x)}{\sin\pi|a_{bc}^I|}\left(\frac{4\pi^2\alpha' m_I^2}{L_{c,I}^2} + \frac{\sin^2\pi|a_{bc}^I|L_{b,I}^2}{4\pi^2\alpha'} \frac{I_{ca,I}^2 n_I^2}{\gcd^2(|I_{bc,I}|,|I_{ca,I}|)}\right) - 2\pi i\frac{m_I}{L_{c,I}} f_{\chi\psi,I} \tag{3.2}$$

In the s channel the amplitude shows a gauge boson propagating between parallel branes of type c, we can use the factorization in this channel to fix $K_I^{c,ab} = \frac{L_a^{1/4} L_b^{5/4} L_{c,I}^{1/2}}{(2\pi)^2\alpha'}$. The vertex operators' normalizations obtained from the amplitudes are

$$C_{A_i} = \sqrt{2\alpha'}\prod_I \left[\frac{\alpha'}{L_{i,I}^2}\right]^{1/4} \tag{3.3}$$

$$C_{\chi_0^{ij}} = (\alpha')^{1/4}\sqrt{2\alpha'}\prod_I \left[\frac{\alpha'}{L_{i,I}L_{j,I}}\right]^{1/4} \qquad C_{\phi_0^{ij}} = \sqrt{2\alpha'}\prod_I \left[\frac{\alpha'}{L_{i,I}L_{j,I}}\right]^{1/4} \tag{3.4}$$

$$C_{\chi_1^{ij}} = (\alpha')^{1/4}\sqrt{2\alpha'}\prod_I \left[\frac{\alpha'}{L_{i,I}L_{j,I}}\right]^{1/4} \qquad C_{\tilde{\chi}_1^{ij}} = (\alpha')^{1/4}\sqrt{2\alpha'}\prod_I \left[\frac{\alpha'}{L_{i,I}L_{j,I}}\right]^{1/4} \tag{3.5}$$

The factorization in the t channel exhibits Yukawa couplings. After Poisson resummation we get

$$\mathscr{A}(\bar{\psi}_0,\psi_0,\chi_0,\bar{\chi}_0) = g_{\rm op}^2\alpha'\psi_0(2)\cdot\chi_0(3)\bar{\psi}_0(1)\cdot\bar{\chi}_0(4)\int_0^1 dx\,x^{\alpha' s-1}(1-x)^{\alpha' t-3/2}\prod_{I=1}^{3}\sum_{\tilde{m}_I,n_I}\frac{e^{-S_{\rm Lagr}^{(I)}(\tilde{m}_I,n_I)}}{2\pi\sqrt{I_I(x)}} \tag{3.6}$$

where $I_I(x)$ are combinations of hypergeometric functions and $S_{\rm Lagr}^{(I)}(\tilde{m}_I,n_I)$ is the classical action in “lagrangian” form. The limit $x\to 1$ has two kinds of contributions: one purely quantum, due to the expansion of $I_I(x)$, and one classical, due to the expansion of $t(x)$ in the action. The first three orders correspond to factorizations on the poles for the states ϕ_0, ϕ_1 and ϕ_2. Imposing

$$\mathscr{A}(\bar{\psi}_0,\psi_0,\chi_0,\bar{\chi}_0) \xrightarrow{t\to n a_{ab}^1/\alpha'} |Y_{n00}|^2\psi_0(2)\cdot\chi_0(3)\frac{1}{t-n a_{ab}^1/\alpha'}\bar{\chi}_0(4)\cdot\bar{\psi}_0(1) \tag{3.7}$$

The Yukawa's extracted by factorization are given by

$$|Y_{000}| = g_{\rm op}(2\pi)^{-3/4}\left[\Gamma_{1-a_{ab}^1,1-a_{bc}^1,-a_{ca}^1}\Gamma_{1-a_{ab}^2,1-a_{bc}^2,-a_{ca}^2}\Gamma_{-a_{ab}^3,-a_{bc}^3,-a_{ca}^3}\right]^{1/4}\prod_{I=1}^{3}\exp\left[-\frac{A_{\phi\psi\chi}^{(I)}}{2\pi\alpha'}\right] \tag{3.8}$$

$$|Y_{100}| = \frac{|Y_{000}|}{\sqrt{a_{ab}^1}}\left[\Gamma_{1-a_{ab}^1,1-a_{bc}^1,-a_{ca}^1}\right]^{1/2}\sqrt{\frac{2A_{\phi\psi\chi}^{(1)}}{\pi\alpha'}} \tag{3.9}$$

$$|Y_{200}| = \frac{|Y_{000}|}{\sqrt{2}a_{ab}^1}\Gamma_{1-a_{ab}^1,1-a_{bc}^1,-a_{ca}^1}\left|\frac{2A_{\phi\psi\chi}^{(1)}}{\pi\alpha'}-1\right| \tag{3.10}$$

where $\Gamma_{a,b,c} = \dfrac{\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(1-a)\Gamma(1-b)\Gamma(1-c)}$, and $A_{\phi\psi\chi}^{(I)}$ is the area of the triangle defined by the three points $f_{\psi,I}$, $f_{\chi,I}$ and $f_{\phi,I}$ in the torus I given by $A_{\phi\psi\chi}^{(I)} = \frac{\sin\pi|a_{bc}^I|\sin\pi|a_{ca}^I|}{2\sin\pi|a_{ab}^I|}f_{\chi\psi,I}^2$.

In order to evaluate the other Yukawas Y_{010}, Y_{001} etc we use supersymmetric Ward identities [1] and we get

$$|Y_{010}| = \frac{|Y_{000}|}{\sqrt{a^1_{bc}}}[\Gamma_{1-a^1_{ab},1-a^1_{bc},-a^1_{ca}}]^{1/2}\sqrt{\frac{2A^{(1)}_{\phi\psi\chi}}{\pi\alpha'}}\ ,\quad |Y_{001}| = \frac{|Y_{000}|}{\sqrt{1+a^3_{ca}}}[\Gamma_{-a^3_{ab},-a^3_{bc},-a^3_{ca}}]^{1/2}\sqrt{\frac{2A^{(3)}_{\phi\psi\chi}}{\pi\alpha'}} \tag{3.11}$$

$$|Y_{020}| = \frac{|Y_{000}|}{\sqrt{2}a^1_{bc}}\Gamma_{1-a^1_{ab},1-a^1_{bc},-a^1_{ca}}\left|\frac{2A^{(1)}_{\phi\psi\chi}}{\pi\alpha'}-1\right|\ ,\quad |Y_{002}| = \frac{|Y_{000}|}{\sqrt{2}(1+a^3_{ca})}\Gamma_{-a^3_{ab},-a^3_{bc},-a^3_{ca}}\left|\frac{2A^{(3)}_{\phi\psi\chi}}{\pi\alpha'}-1\right| \tag{3.12}$$

3.2 Yukawa couplings from amplitudes with massive external legs

The factorization of 4-pt amplitudes with massive external legs yields other Yukawa's that involve more than one massive particle. Following the same steps as before, we fix the normalizations of the vertex operators from amplitudes that include gauge bosons, and we evaluate Yukawa couplings with more than one massive field.

The amplitude $\mathscr{A}(\bar\psi_1,\psi_0,\chi_0,\bar\chi_0)$ reads

$$\begin{aligned}\mathscr{A}(\bar\psi_1,\psi_0,\chi_0,\bar\chi_0) = &\frac{g^2_{\rm op}}{\sqrt{a^1_{bc}}}\alpha'\psi_0(2)\cdot\chi_0(3)\bar\psi_1(1)\cdot\bar\chi_0(4) \\ &\times\int_0^1 dx\, x^{\alpha's-1}(1-x)^{\alpha't-1}\prod_{I=1}^{3}\frac{2\pi\sqrt{\alpha'}}{L_{c,I}G^{(I)}_1(x)}\sum_{m_I}\frac{2\pi\sqrt{\alpha'}m_1}{\sqrt{I_1(x)}L_{c,I}}e^{-S^{(I)}_{\rm Ham}(m_I,n_I)}\end{aligned} \tag{3.13}$$

Similar to the amplitudes with massless external states, we perform a Poisson resummation over the indices m_I and obtain

$$\begin{aligned}\mathscr{A}(\bar\psi_1,\psi_0,\chi_0,\bar\chi_0) = &\frac{g^2_{\rm op}}{\sqrt{a^1_{bc}}}\alpha'\psi_0(2)\cdot\chi_0(3)\bar\psi_1(1)\cdot\bar\chi_0(4)\int_0^1 dx\, x^{\alpha's-1}(1-x)^{\alpha't-3/2-a^1_{ab}} \\ &\times\frac{G^{(1)}_1(x)}{I_1(x)}\prod_{I=1}^{3}\frac{1}{\sqrt{2\pi I_I(x)}}\sum_{\tilde m_I,n_I}\frac{\tilde m_1 L_{c,1}+f_{\chi\psi,1}}{2\pi\sqrt{\alpha'}}e^{-S^{(I)}_{\rm Lagr}(\tilde m_I,n_I)}\end{aligned} \tag{3.14}$$

In the limit $x\to 1$ the leading term is the massless pole due to the chiral exchange in the (a,b) sector. We have fixed all the normalizations that appear in the amplitudes and we already know the Yukawa's $Y^*_{010}Y_{000}$, thus factorization in this channel can be used to check that normalizations are consistent. The subleading terms determine the Yukawa Y_{110}

$$|Y_{110}| = |Y_{000}|\left|\frac{2A^{(1)}_{\phi\psi\chi}}{\pi\alpha'}-1\right|\frac{1}{\sqrt{a^1_{ab}a^1_{bc}}}\Gamma_{1-a^1_{ab},1-a^1_{bc},-a^1_{ca}} \tag{3.15}$$

We can obtain the Yukawas Y_{101} and Y_{011} from two amplitudes with massive bosons in the external states. As for the massless case, we can use Ward identities to relate these to amplitudes with only fermions. The explicit expressions of the amplitudes read

$$\mathscr{A}(\bar{\chi}_1^{ac},\chi_0^{ca},\phi_0^{ab},\bar{\phi}_0^{ba}) = \frac{g_{\rm op}^2}{\sqrt{1+a_{ca}^3}}\alpha'\bar{\chi}_1^{ac}(1)\not{k}_4\chi_0^{ca}(2)\int_0^1 dx x^{\alpha' s-1}(1-x)^{\alpha' t-5/2-a_{bc}^3}\times$$
$$\times\frac{G_1^{(3)}(x)}{I_3(x)}\prod_{I=1}^3\frac{1}{\sqrt{2\pi I_I(x)}}\sum_{\tilde{m}_I,n_I}\frac{\tilde{m}_3 L_{a,3}+f_{\psi\phi,3}}{2\pi\sqrt{\alpha'}}e^{-S_{\rm Lagr}^{(I)}(\tilde{m}_I,n_I)} \tag{3.16}$$

$$\mathscr{A}(\bar{\phi}_1,\phi_0,\chi_0,\bar{\chi}_0) = \frac{g_{\rm op}^2}{\sqrt{a_{ab}^1}}\alpha'\bar{\chi}_0^{cb}(4)\not{k}_2\bar{\chi}_0^{bc}(3)\int_0^1 dx x^{\alpha' s-1}(1-x)^{\alpha' t-5/2-a_{ca}^1}\times$$
$$\times\frac{G_1^{(1)}(x)}{I_1(x)}\prod_{I=1}^3\frac{1}{\sqrt{2\pi I_I(x)}}\sum_{\tilde{m}_I,n_I}\frac{\tilde{m}_1 L_{b,1}+f_{\chi\phi,1}}{2\pi\sqrt{\alpha'}}e^{-S_{\rm Lagr}^{(I)}(\tilde{m}_I,n_I)} \tag{3.17}$$

The limit $x\to 1$ yields two new Yukawa's

$$|Y_{011}| = \frac{|Y_{000}|}{\sqrt{a_{bc}^1(1+a_{ca}^3)}}[\Gamma_{1-a_{ab}^1,1-a_{bc}^1,-a_{ca}^1}\Gamma_{-a_{ab}^3,-a_{bc}^3,-a_{ca}^3}]^{1/2}\sqrt{\frac{2A_{\phi\psi\chi}^{(1)}}{\pi\alpha'}\frac{2A_{\phi\psi\chi}^{(3)}}{\pi\alpha'}} \tag{3.18}$$

$$|Y_{101}| = \frac{|Y_{000}|}{\sqrt{a_{ab}^1(1+a_{ca}^3)}}[\Gamma_{1-a_{ab}^1,1-a_{bc}^1,-a_{ca}^1}\Gamma_{-a_{ab}^3,-a_{bc}^3,-a_{ca}^3}]^{1/2}\sqrt{\frac{2A_{\phi\psi\chi}^{(1)}}{\pi\alpha'}\frac{2A_{\phi\psi\chi}^{(3)}}{\pi\alpha'}} \tag{3.19}$$

The amplitude $\mathscr{A}(\bar{\psi}_1,\psi_0,\chi_0,\bar{\chi}_1)$ allows us to determine the Yukawa's Y_{111} and Y_{211}.

$$\mathscr{A}(\bar{\psi}_1,\psi_0,\chi_0,\bar{\chi}_1) = \frac{g_{\rm op}^2}{\sqrt{a_{bc}^1(1+a_{ca}^3)}}\alpha'\psi_0(2)\cdot\chi_0(3)\bar{\psi}_1(1)\cdot\bar{\chi}_1(4)\int_0^1 dx x^{\alpha' s-1}(1-x)^{\alpha' t-1}$$
$$\times\prod_{I=1}^3\frac{4\pi^2\sqrt{\alpha'}}{L_{c,I}G_1^{(I)}(x)}\sum_{m_I,n_I}\frac{\alpha' m_1 m_3}{\sqrt{I_1(x)I_3(x)}L_{c,1}L_{c,3}}e^{-S_{\rm Ham}^{(I)}(m_I,n_I)} \tag{3.20}$$

Once again the amplitude does not expose gauge boson exchange, since the sum over the lattice forbids it. To study the t channel we perform the usual Poisson resummation

$$\mathscr{A}(\bar{\psi}_1,\psi_0,\chi_0,\bar{\chi}_1) = \frac{\alpha' g_{\rm op}^2}{\sqrt{a_{bc}^1(1+a_{ca}^3)}}\psi_0(2)\cdot\chi_0(3)\bar{\psi}_1(1)\cdot\bar{\chi}_0(4)\int_0^1 dx x^{\alpha' s-1}(1-x)^{\alpha' t-5/2+a_{ab}^2}$$
$$\times\frac{G_1^{(1)}(x)G_1^{(3)}(x)}{I_1(x)I_3(x)}\prod_{I=1}^3\frac{1}{\sqrt{2\pi I_I(x)}}\sum_{\tilde{m}_I,n_I}\frac{(\tilde{m}_1 L_{c,1}+f_{\psi\chi,1})(\tilde{m}_3 L_{c,3}+f_{\psi\chi,3})}{4\pi^2\alpha'}e^{-S_{\rm Lagr}^{(I)}(\tilde{m}_I,n_I)} \tag{3.21}$$

Factorizing this amplitude we obtain the Yukawa's

$$|Y_{111}| = \frac{|Y_{000}|}{\sqrt{a_{ab}^1 a_{bc}^1(1+a_{ca}^3)}}\Gamma_{1-a_{ab}^1,1-a_{bc}^1,-a_{ca}^1}\Gamma_{-a_{ab}^3,-a_{bc}^3,-a_{ca}^3}^{1/2}\left|\frac{2A_{\phi\psi\chi}^{(1)}}{\pi\alpha'}-1\right|\sqrt{\frac{2A_{\phi\psi\chi}^{(3)}}{\pi\alpha'}} \tag{3.22}$$

$$|Y_{211}| = \frac{|Y_{000}|}{\sqrt{2}a_{ab}^1\sqrt{a_{bc}^1(1+a_{ca}^3)}}\Gamma_{1-a_{ab}^1,1-a_{bc}^1,-a_{ca}^1}^{3/2}\Gamma_{-a_{ab}^3,-a_{bc}^3,-a_{ca}^3}^{1/2}\left|\frac{2A_{\phi\psi\chi}^{(1)}}{\pi\alpha'}-3\right|\sqrt{\frac{2A_{\phi\psi\chi}^{(1)}}{\pi\alpha'}\frac{2A_{\phi\psi\chi}^{(3)}}{\pi\alpha'}} \tag{3.23}$$

4. Conclusions

In [1] we have evaluated the Yukawa couplings between light stringy states and SM fields. Even though the tower of massive stringy states looks like KK towers, there are differences. First of all the mass gaps at KK models are typically universal. In intersecting D-brane models each tower of states lives on different intersections with unique angle and therefore unique mass gap. Second the mass gaps in KK modes are typically proportional to the moding ($\sim m/R$) whereas in intersecting D-brane models they are proportional to the square root of the moding ($\sim \sqrt{m}M_s$). Finally, interactions which are prohibited due to conservation of internal momentum (for example the decay of an excited KK mode to two unexcited modes) are allowed in intersecting D-brane models (in analogy with the previous, the Yukawa $Y_{|100|}$ is different from zero (3.9)).

Our results can be used in order to built an effective field theory including SM fields and light stringy states and study specific decays. If the string scale is at the few TeV range, decays of such particles might be the first stringy effect which might be visible at LHC or future experiments.

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