

## Two-Photon exchange in elastic lepton-proton scattering

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The proton elastic form factor ratio can be directly accessed via measurements exploiting polarization degrees of freedom. Compared to the ratio formed with the Rosenbluth-separated form factors from unpolarized measurements Rosenbluth, they show a different trend growing with  $Q^2$ . The proposed explanation is two-photon exchange, which has recently been measured in three precision experiments. From these new data, a hard two-photon exchange effect at the couple-of-percent level can be extracted, in significant disagreement from theoretical calculation. Theory at larger momentum transfer remains untested, as well as the accuracy of the descriptions at lowest  $Q^2$ , relevant for the extraction of the proton's rms. charge and magnetic radii.

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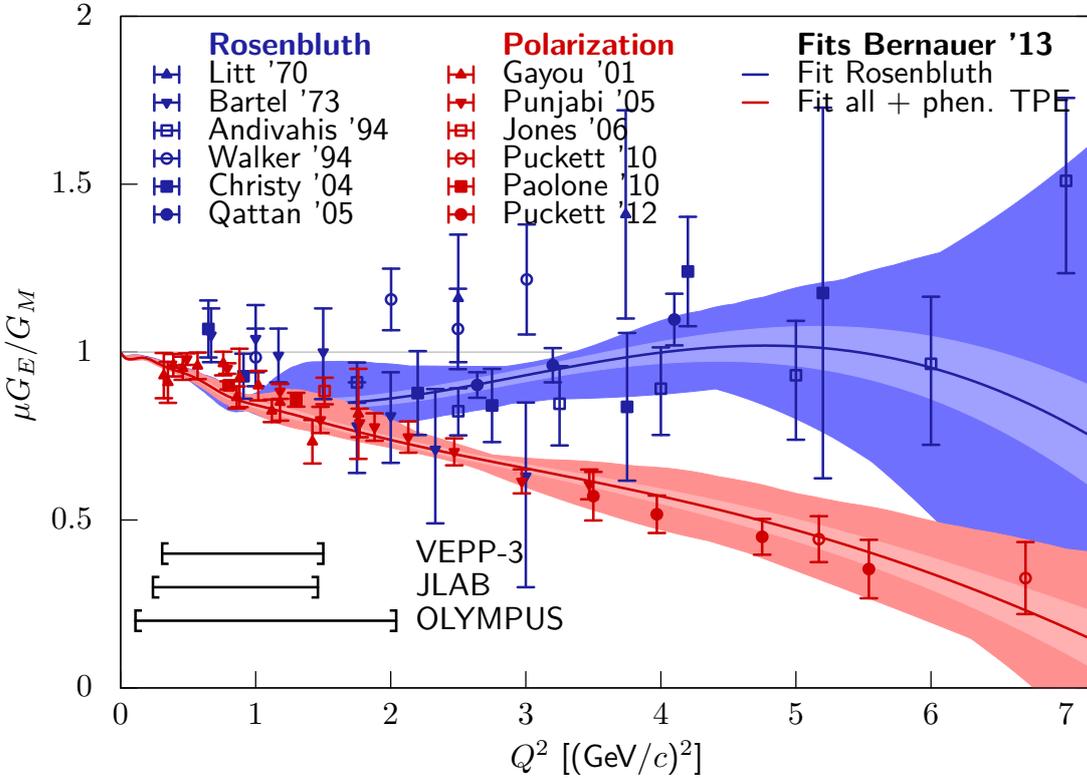
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## 1. Introduction

Proton elastic form factors are classically studied using electron-proton scattering using unpolarized beams and target, producing a wealth of cross section data over an extensive range of (negative) four-momentum transfers,  $Q^2$ . These cross sections can be analyzed in terms of two elastic form factors via the so-called Rosenbluth separation technique.

A more recent technique makes us of beam and/or target polarization to access the form factor ratio.

While the former technique produces form factor ratios in agreement with scaling, i.e., a more or less constant ratio even for large  $Q^2$ , the results from the latter technique exhibit a roughly linear fall-off of the ratio. Figure 1 shows a selection of the available data and recent fits.



**Figure 1:** The proton form factor ratio  $\mu G_E/G_M$ , determined via Rosenbluth-type (blue points, from [1, 2, 3, 4, 5, 6]) and polarization-type (red points, from [7, 8, 9, 10, 11, 12]) experiments. While the former are compatible with a constant ratio, the latter indicate a linear downward trend. Curves represent phenomenological fits [13], to either the Rosenbluth-type world data set alone (blue curves) or to all data (red curves).

This "form factor ratio puzzle" limits the precise determination of form factors at higher  $Q^2$ . Since form factors encode the distribution of charge and magnetization, and their description is an important touchstone for theory, a resolution of this puzzle is highly sought for.

## 2. Two-photon exchange

It was suggested by Blunden et al. [14] and Guichon et al. [15] that hard two-photon exchange (TPE) could be an important effect in Rosenbluth-type experiments. Two-photon exchange is reflected by Feynman diagrams where two photon lines connect the lepton and proton. The interference term of these diagrams with the one photon exchange diagrams appear in the cross section with the order  $\alpha^3$ . While standard radiative corrections (e.g. [16, 17]) include the soft case, i.e. where one of the photons has vanishing momentum, an inclusion of the hard case, where both photons have non-vanishing momenta, might resolve the discrepancy, mostly by correcting down the ratio from unpolarized experiments. The exact division in “soft” and “hard” is somewhat arbitrary and different authors use different definitions.

### 2.1 Theoretical calculations

Most current theoretical calculations are based on two approaches, both based on a model-dependent picture of the nucleon: hadronic calculations, e.g. [18], which are believed valid for  $Q^2$  from 0 up to a couple of  $\text{GeV}^2$ , and GPDs based calculations, e.g. [19], valid for a couple of  $\text{GeV}^2$  and higher.

### 2.2 Phenomenological extraction

Assuming that TPE is indeed the reason for the discrepancy, an expected size for the TPE correction can be extracted from the existing data on the form factor ratio. In [13], the authors built a model based on the following assumptions:

- TPE is the dominant source of the difference.
- TPE affects only the Rosenbluth-type experiments, polarization data is unchanged. This is likely a good approximation, as the effect of TPE on the cross section is magnified in the Rosenbluth separation to a substantially larger effect on  $G_E$  for  $Q^2 \gg 0$ .
- The effect is dominantly linear in  $\varepsilon$ . Precision Rosenbluth experiments have not found any deviation from a straight line in the Rosenbluth separations so far, which sets bounds on the curvature of a TPE introduced correction.
- The correction is zero for forward scattering, i.e., for  $\varepsilon = 1$ .
- The Feshbach Coulomb correction [20] is the correct limit for  $Q^2 \rightarrow 0$ . Modern theoretical calculations all converge to this limit.

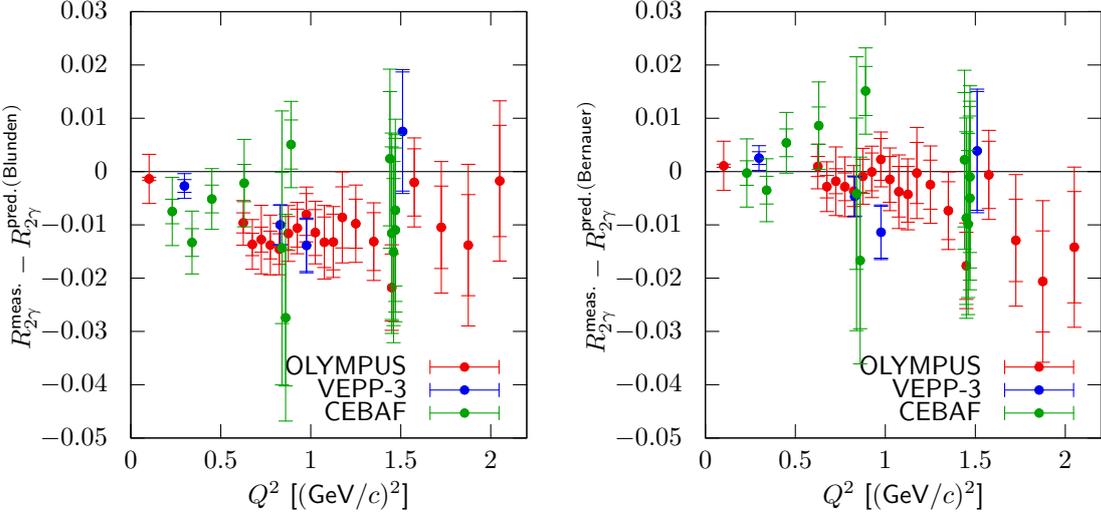
Assuming a correction of the form  $1 + \delta_{TPE}$  to the cross section, with

$$\delta_{TPE} = \delta_{\text{Feshbach}} + a(1 - \varepsilon) \ln(1 + b * Q^2), \quad (2.1)$$

the authors could fit the combined world data set with excellent  $\chi^2$ .

### 3. Current status

Three contemporary experiments have tried to measure the size of TPE, based at VEPP-3 [21], Jefferson Lab (CLAS, [22]) and DESY (OLYMPUS, [23]). The next-order correction to the elastic lepton-proton cross section contains terms corresponding to the product of the diagrams of one-photon and two-photon exchange. These terms change sign when switching between  $e^-$  and  $e^+$ . Therefore, the size of TPE can be determined by measuring the ratio of positron to electron scattering:  $R_{2\gamma} = \frac{\sigma_{e^+}}{\sigma_{e^-}} \approx 1 + 2\delta_{TPE}$ .



**Figure 2:** Difference of the data of the three recent TPE experiments [21, 22, 23] to the calculation in [18] (left) and the phenomenological prediction from [24] (right).

In the probed  $Q^2$  region, the hadronic approach should be valid. Since the correction is dependent on two variables, a direct comparison of the data is difficult. Fig. 2 therefore shows the difference of the data of the three experiments to the calculation by Blunden et al. [18] and to the phenomenological prediction by Bernauer et al. [24]. The three data sets are in good agreement with each other. The calculation over-predicts the effect by about 1% for most of the measured kinematics. The phenomenological prediction appears closer for most of the  $Q^2$  range, but is above the data for the largest available  $Q^2$ . While the data at the largest  $Q^2$  is too imprecise for a strong statement, this is worrisome, as this coincides with the opening of the divergence in the fits in Fig. 1. It might be a hint that TPE alone cannot explain the whole discrepancy.

No hard TPE is ruled out by the data. The experiments agree with the phenomenological prediction with a reduced  $\chi^2$  of 0.68. Compared to that, the theoretical calculation (red.  $\chi^2$  of 1.09) is significantly worse, and the large normalization shifts to achieve this  $\chi^2$  is ruled out at the 99.6% confidence level.

The calculations based on GPDs are only valid at higher  $Q^2$  and are so far not tested by any experiment.

For a more in-depth review, see [25]. Without a resolution of the puzzle and a test of TPE at larger  $Q^2$ , the extraction of reliable form factor information is impossible, especially from the high

precision, large  $Q^2$  measurements which are part of the Jefferson Lab 12 GeV program. Clearly, new data are needed.

#### 4. Effects on the radius extraction

While the effect is generally thought of as effecting predominantly large  $Q^2$ , TPE does play a role in the extraction of the radius at low  $Q^2$ . For the electric radius, this effect is small: Not including any correction changes extractions by typically on the order of 0.01 fm, and, because all theoretical prescriptions have the same low- $Q^2$  limit, the extractions in [24] vary by only 0.004 fm between the different prescriptions tested. However, the failure of theory to describe the available TPE data might be seen as casting doubt on the validity of this limit. The MUSE experiment [26] will measure TPE at kinematics relevant for the radius.

For the magnetic radius, the situation is more dire. The current fits are sensitive to the 0.04 fm level. Future experiments aimed at the extraction of the magnetic form factor at small  $Q^2$  necessarily measure at large scattering angles to achieve small  $\varepsilon$  and boost the fraction of the cross section stemming from  $G_M$  and, with that, the precision of the extracted  $G_M$ . Currently, no experiment aimed to measure TPE at these kinematics is planned.

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