

## Meson correlation functions at high temperature QCD: $SU(2)_{CS}$ symmetry vs. free quarks

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We report on the progress of understanding spatial correlation functions in high temperature QCD. We study isovector meson operators in  $n_f = 2$  QCD using domain-wall fermions on lattices of  $N_s = 32$  and different quark masses. It has previously been found that at  $\sim 2T_c$  these observables are not only chirally symmetric but in addition approximately  $SU(2)_{CS}$  and  $SU(4)$  symmetric. In this study we increase the temperature up to  $5T_c$  and can identify convergence towards an asymptotically free scenario at very high temperatures.

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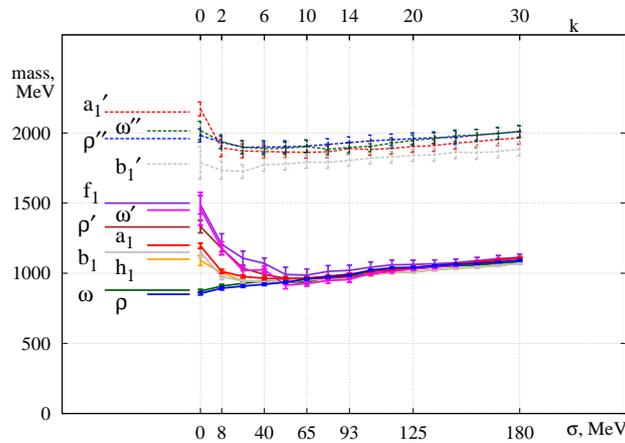
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## 1. Introduction

In a series of numerical experiments, initiated in Ref. [1], some interesting findings have been made [2, 3, 4, 5]: Upon truncating the low-modes of the Dirac operator the spectrum of  $J = 1, 2$  mesons and light baryons revealed a symmetry larger than the chiral symmetry of QCD, see Fig. 1 for the  $J = 1$  results. A priori one expects that such a truncation would lead to a restoration of chiral and possibly of  $U(1)_A$  symmetries in hadrons, if hadrons survive this procedure. Surprisingly larger symmetries,  $SU(2)_{CS}$  and  $SU(2n_f)$ , emerge that contain chiral symmetries of the QCD Lagrangian as subgroups [6, 7]. These symmetries are symmetries of the chromo-electric interaction in QCD, while the chromo-magnetic interaction as well as the quark kinetic term break them. From these results one can conclude that the effects of the chromo-magnetic interaction in QCD are located exclusively in the near-zero modes, while confining chromo-electric interaction is distributed among all modes of the Dirac operator.

While chiral symmetries for meson propagators have been analytically shown to arise upon truncation of the low-lying modes, emergence of  $SU(2)_{CS}$  and  $SU(2n_f)$  is encoded in some matrix elements that include some unknown microscopic dynamics [8]. Given this insight one could expect emergence of the same symmetries in QCD at high temperature without any truncation because the high temperature naturally suppresses the near-zero modes of the Dirac operator.

Recently a full  $n_f = 2$  flavor simulation of QCD using chirally-symmetric Domain-wall fermions at temperatures above the chiral restoration temperature has shown that the spatial correlators of isovector  $J = 1$  mesons approximately feature  $SU(2)_{CS}$  and  $SU(2n_f)$  symmetries [9]. In the present study we extend the temperature range up to  $\sim 5.5T_c$ .



**Figure 1:**  $J = 1$  meson spectrum upon low-mode truncation of the Dirac operator. Figure from Ref. [3].

## 2. Method

Our lattice setup consists of two mass-degenerate flavors of light quarks, which use the Möbius domain wall fermion discretization [10, 11]. The gauge sector is simulated using the tree-level improved Symanzik action. The quark masses are set to values between 2–15 MeV, and the temperature is set by varying strong coupling  $\beta$  and  $N_t$ . More details concerning this set of parameters

$N_s^3 \times N_t$	$\beta$	$m_{ud}a$	$a$ [fm]	# configs	$L_s$	$T$ [MeV]	$T/T_c$
$32^3 \times 8$	4.10	0.001	0.113	800	24	$\sim 220$	$\sim 1.2$
$32^3 \times 8$	4.18	0.001	0.096	230	12	$\sim 260$	$\sim 1.5$
$32^3 \times 8$	4.30	0.001	0.075	260	12	$\sim 320$	$\sim 1.8$
$32^3 \times 8$	4.37	0.001	0.065	77	12	$\sim 380$	$\sim 2.2$
$32^3 \times 6$	4.30	0.001	0.075	270	12	$\sim 440$	$\sim 2.5$
$32^3 \times 8$	4.50	0.001	0.051	197	12	$\sim 480$	$\sim 2.7$
$32^3 \times 4$	4.30	0.001	0.075	200	10	$\sim 660$	$\sim 3.8$
$32^3 \times 4$	4.50	0.001	0.051	209	10	$\sim 960$	$\sim 5.5$

**Table 1:** Lattices used in this work:  $L_s$  is the length in the fifth dimension of the Möbius domain wall fermions.

can be found in Ref. [12, 13] and Table 1. The (pseudo)critical temperature for this ensembles is  $T_c = 175 \pm 5$  MeV. In total this allows us to access temperatures from  $T \simeq 1.2 - 5.5T_c$ .

In this work we focus on the local isovector bilinears

$$\mathcal{O}_\Gamma(x) = \bar{q}(x) \vec{\tau} \Gamma q(x), \quad (2.1)$$

where  $\Gamma$  denotes any element of the Clifford algebra and  $\vec{\tau}$  are the generators in flavor space. For spatial measurements in  $z$ -direction we sum over orthogonal lattice slices in  $x, y, t$ -direction

$$C_\Gamma(n_z) = \sum_{n_x, n_y, n_t} \langle \mathcal{O}_\Gamma(n_x, n_y, n_z, n_t) \mathcal{O}_\Gamma(\mathbf{0}, 0)^\dagger \rangle. \quad (2.2)$$

The gamma structures are grouped into the following objects:

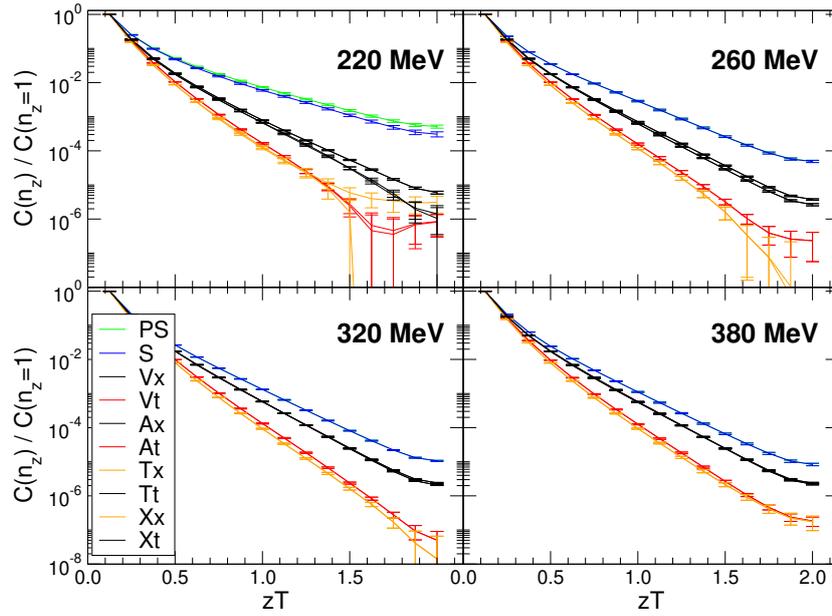
- Scalar ( $S$ ,  $\Gamma = \mathbf{1}$ ) and Pseudoscalar ( $PS$ ,  $\Gamma = \gamma_5$ )
- Vector ( $\mathbf{V}$ ,  $\Gamma = \gamma_k$ ) and Axial-vector ( $\mathbf{A}$ ,  $\Gamma = \gamma_k \gamma_5$ )
- Tensor-vector ( $\mathbf{T}$ ,  $\Gamma = \gamma_k \gamma_3$ ) and Axial-tensor-vector ( $\mathbf{X}$ ,  $\Gamma = \gamma_k \gamma_3 \gamma_5$ )

as well as the non-propagating  $\Gamma = \gamma_3$  and  $\Gamma = \gamma_3 \gamma_5$ . The vector indices take on values  $k = 1, 2, 4$ , which we denote as  $x, y$  and  $t$  respectively. *E.g.* the first component of the Axial-vector would be  $A_x$ .

### 3. Results

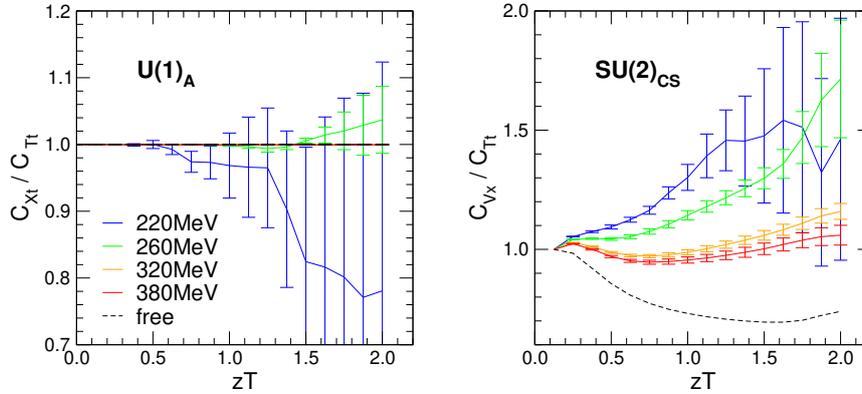
Figures 2 and 3 summarize findings for  $T \leq 2T_c$  from [9]. Fig. 2 shows that correlation functions of operators connected by both  $U(1)_A$  transformations ( $PS$  and  $S$ ,  $\mathbf{T}$  and  $\mathbf{X}$ ) as well as  $SU(2)_L \times SU(2)_R$  symmetry ( $\mathbf{V}$  and  $\mathbf{A}$ ) get degenerate at temperatures  $T > 220$  MeV. This is a clear signal for restoration of the flavor chiral symmetry, as well as for at least approximate restoration of  $U(1)_A$  symmetry.

However a larger degeneracy is seen. *E.g.* correlators for  $V_x$  and  $T_t$ , as well as  $V_t$  and  $T_x$  get approximately degenerate at  $T \sim 380$  MeV. Such degeneracies indicate emergence of the  $SU(2)_{CS}$  and  $SU(4)$ . Fig. 3 shows the ratio of corresponding correlators connected by  $U(1)_A$  and  $SU(2)_{CS}$



**Figure 2:** Normalized spatial correlation functions for operators and ensembles up to  $T \sim 380$  MeV.

transformations. It can be seen that while the  $U(1)_A$  symmetry is "exactly" restored, a degeneracy of correlators connected by the  $SU(2)_{CS}$  transformation is only approximate: There are still symmetry breaking effects at the level of 5%.



**Figure 3:** Detailed ratio for correlation functions of operators connected by  $U(1)_A$  as well as  $SU(2)_{CS}$  symmetries.

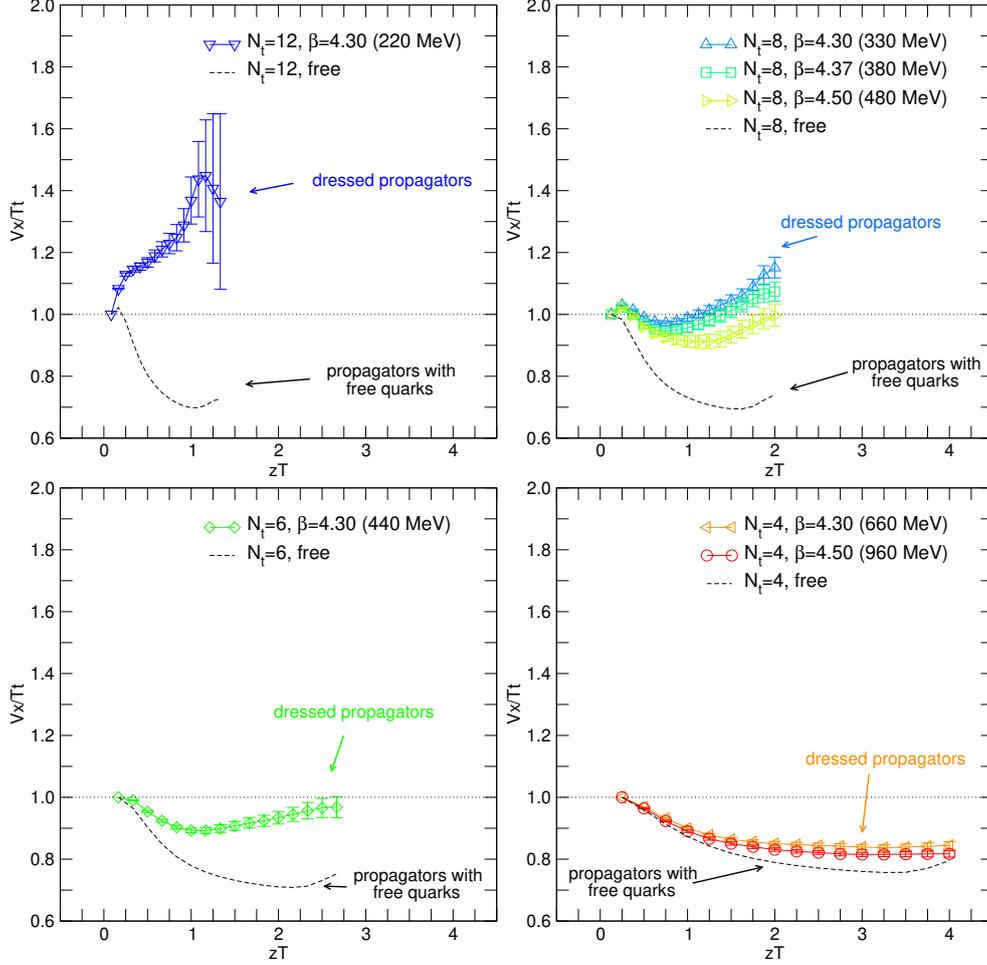
The  $SU(2)_{CS}$  ratio of correlators for all available lattice ensembles is shown in Fig. 4. The data therein supports approximate  $SU(2)_{CS}$  symmetry in the region around  $2T_c$ . The interacting correlators approach the free quark limit at very high temperatures  $T > 700$  MeV, where  $SU(2)_{CS}$  is broken. The same ratio for free quarks, *i.e.* non-interacting quarks, is given for each lattice geometry. The ratio for free quarks is always below 1, which is in agreement with the analytic observation that the free (massless) Dirac Lagrangian  $\mathcal{L} = \bar{\Psi}i\not{\partial}\Psi$  breaks  $SU(2)_{CS}$  symmetry.

In contrast to free quarks, the situation for interacting quarks is a little more intricate. The corresponding Lagrangian  $\mathcal{L} = \bar{\Psi}i\not{\partial}\Psi$  in Minkowski space can be written in terms of color-electric

and color-magnetic contributions

$$\mathcal{L} = \bar{\Psi}i\gamma^0 D_0\Psi + \bar{\Psi}i\gamma^i D_i\Psi. \quad (3.1)$$

The first term that includes the color-electric interaction in this equation, is invariant under  $SU(2)_{CS}$  transformations, whereas the second term, with the color-magnetic contributions, is not [7]. Accordingly, a theory with pure color-electric interaction would show  $SU(2)_{CS}$  symmetry in its spectrum and a  $SU(2)_{CS}$  ratio of 1.

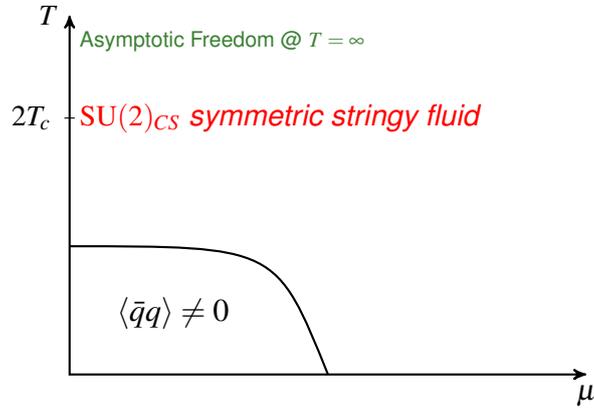


**Figure 4:** Detailed ratio for correlation functions of operators connected by  $SU(2)_{CS}$ . The subplots group lattices of same geometry.

Finally, we provide a possible interpretation of our results in Fig. 4. For temperatures slightly above the chiral restoration, at 220 MeV, the  $SU(2)_{CS}$  ratio is well above 1 (upper left panel), which means that the color-magnetic interaction is large. Increasing the temperature, its role is diminishing and interaction is dominantly color-electric. This can be seen by a  $SU(2)_{CS}$  ratio of approximately 1 at  $\sim 2T_c$  (upper right panel of Fig. 4). Further increasing temperature, as shown in the lower two panels of Fig. 4, the ratio for interacting quarks approaches the limit for free quarks. Here the dynamics is governed by kinetic contributions, which are the same as for free quarks. The

remaining color-electric interaction slowly dies out at very high temperature, and one approaches the asymptotic freedom regime.

This might indicate that elementary objects in QCD at  $T \sim 2T_c$  are not free deconfined quarks but rather quarks with a definite chirality connected by the chromo-electric field [6]. This conclusion remains also true in matter with finite chemical potential [14], see Fig. 5.



**Figure 5:** Sketch of the QCD phase diagram: at  $T \sim 2T_c$  the spectrum shows  $SU(2)_{CS}$  symmetry. Non-vanishing chemical potential has no effect on this property.

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