

The properties of $D1$ -branes from lattice super Yang–Mills theory using gauge/gravity duality

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The two-dimensional supersymmetric Yang–Mills (SYM) theory with sixteen supercharges at large N and strong 't Hooft coupling is conjectured to be dual to certain supergravity solutions in the decoupling limit. We discretize the gauge theory preserving a subset of supersymmetries on the lattice. Based on the choice of a point in the moduli space for the expansion of the gauge links to target the correct continuum theory, one ends up with different lattice geometries. In our previous work, we explored the free energy and the phase structure on a skewed torus corresponding to A_2^* lattice geometry. Here, we will consider square lattice and calculate the free energy, equation of state and speed of sound in this strongly coupled supersymmetric plasma. Since there is no shear viscosity in two dimensions, we comment on the expectations for the bulk viscosity from the calculations on the dual supergravity side, which unlike the conformal $\mathcal{N} = 4$ SYM case does not vanish and is proportional to the trace of energy-momentum tensor.

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1. Introduction

It is conjectured that the strong coupling, large N limit of supersymmetric theories possessing sixteen supersymmetries admit a holographic dual [1]. In the past couple of years, the program to access and understand the supergravity predictions using numerical simulations of supersymmetric gauge theories have evolved from its nascent stage and good agreement has been observed. Several works [2, 3, 4] spread over the past decade have checked the thermodynamics predicted from the supergravity side with the dual gauge theory observables with remarkable success in (0+1)-dimensions. The general form of the gauge/gravity duality is valid in lower dimensions as well, however, the four-dimensional case is special because $\mathcal{N} = 4$ SYM theory is conformal. Lower dimensions are equally interesting because they can have a rich phase structure and are computationally cheaper to simulate using Monte Carlo methods. Unlike SYM, the theory of strong interactions (QCD) has no known gravity dual, but QCD at high temperatures (about $T \geq 2.0 - 3.0 T_c$) is nearly a conformal field theory and is thought to be in the same universality class as $\mathcal{N} = 4$ SYM. The thermodynamic potentials and transport coefficients have been calculated in QCD [5, 6, 7] and relations to gravity dual via AdS/CFT have been explored. Several important results in strongly coupled QCD have already been obtained using AdS/CFT conjecture, most famously the ratio of shear viscosity (η) to entropic density, η/s . In four dimensions, the only non-trivial viscosity coefficient is η since the bulk viscosity (ζ) vanishes in $\mathcal{N} = 4$ SYM. Recently, we explored the two-dimensional supersymmetric gauge theory on a skewed torus and confirmed the phase transition between two different black hole solutions and computed the dual free energy in both phases [8, 9]. In this proceedings, we *propose* to study thermodynamics of the gauge theory in more detail by not only calculating the internal/free energy but rather the equation of state (EoS) on a square torus (hypercubic trajectories in the moduli space), which in turn will enable us to calculate the speed of the sound, *i.e.* c_s , the simplest transport coefficient.

2. Theoretical background

2.1 Supergravity and its predictions

The IIB supergravity is dual to the ‘decoupling’ limit of N coincident $D1$ -branes [1]. In this limit, finite-energy excitations are considered simultaneously with the limits, $g_{\text{YM}}^2 = \frac{1}{2\pi} \frac{g_s}{\alpha'} = \text{fixed}$ and $\alpha' \rightarrow 0$, where g_s is the string coupling and α' is the ‘Regge slope’. In the case of $D1$ branes, one starts out at weak coupling in the UV with a perturbative description. In the intermediate regime, there is supergravity (SUGRA) description in terms of $D0/D1$ brane solutions and at sufficiently low temperatures, one flows to a free orbifold CFT description. See Figure (1) for a schematic representation of different regimes. The region in which the strongly coupled Yang Mills theory (denoting p to be number of spatial dimensions) is dual to the Type IIA/IIB supergravity is given by,

$$1 \ll \lambda_{\text{eff}} \ll N^{\frac{10-2p}{7-p}} \quad (2.1)$$

where, $\lambda_{\text{eff}} = \lambda_{p+1} \beta^{3-p} = t^{-(3-p)}$, where λ_{p+1} is the coupling in $(p+1)$ -dimensions and t is the dimensionless temperature. This condition reduces to the familiar $1 \ll \lambda_4 \ll N$ in four dimensions. We will refer to λ_2 and λ interchangeably.

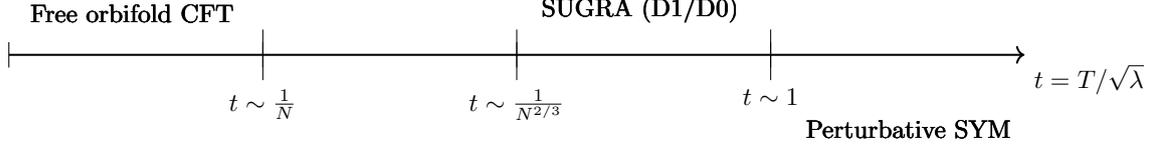


Figure 1: The different limits of the $\mathcal{N} = (8, 8)$ SYM theory. We will focus on the region $1/N^{2/3} < t \ll 1$.

Assuming the event horizon of the black hole geometry is at $U = U_0$ (see [1, 10] for details). Then, we can calculate the temperature associated with the supergravity metric T as,

$$T = \frac{(7-p)U_0^{\frac{5-p}{2}}}{4\pi\sqrt{d_p}\lambda_{p+1}} \quad (2.2)$$

The corresponding energy can be easily calculated and gives,

$$\frac{E}{N^2} \Big|_{\text{Dp-brane}} = \frac{(9-p)U_0^{7-p}L^p}{2^{11-2p}\pi^{\frac{13-3p}{2}}\Gamma\left(\frac{9-p}{2}\right)\lambda_{p+1}^2} \quad (2.3)$$

It can be further shown that,

$$\frac{E}{S} = \left(\frac{9-p}{14-2p}\right)T \quad ; \quad E - P = \frac{4P}{5-p} \quad (2.4)$$

where, S is the entropy, E is the energy, and P is pressure times the volume (pV). The speed of sound c_s is then,

$$c_s = \sqrt{\frac{\partial P}{\partial E}} = \sqrt{\frac{5-p}{9-p}} \quad (2.5)$$

The hydrodynamical coefficients for general Dp-branes, with $p \geq 2$ was calculated in [11]. However, the case $p = 1$ is special. It is the only odd p , with $p < 5$ which is not conformal. Also, there is no shear viscosity in two dimensions. For D1-branes, it was found in [12] that the speed of sound is $c_s = \sqrt{\frac{1}{2}}$. The entropy density and bulk viscosity are given by ¹,

$$s = \frac{2^4\pi^{5/2}N^2T^2}{3^3\sqrt{\lambda}} \quad ; \quad \zeta = \frac{2^2\pi^{3/2}N^2T^2}{3^3\sqrt{\lambda}} \quad (2.6)$$

and hence, the ratio $\zeta/s = 1/4\pi$ similar to four dimensions but with η replaced by ζ . One would ideally expect that $c_s = \sqrt{\frac{1}{2}}$ will be obtained for a conformal fluid in (2+1)-dimensions, so it is interesting that this result was obtained in a two-dimensional SYM theory in a regime where it is *not* conformal. This has been discussed in [13] where it was argued that the hydrodynamical properties of non-conformal branes is fully determined in terms of conformal hydrodynamics. The focus of the lattice calculations will be to calculate, c_s , over the entire region, where the D1-description is valid and provide a numerical outlook on this issue.

In the well-studied $p = 0$ case, there is a single phase since the temporal direction corresponding to the black hole horizon is always *deconfined*. For $p = 1$, there is an intricate phase structure corresponding to topology changing transitions [14] also known as the *black hole/black string* tran-

¹Note that there is a typo in Equation (5.7) of [12]

sition. Using holography, this is conjectured to be dual to the *deconfinement* transition in the gauge theory which for the two-dimensional SYM theory is expected to occur around $r_x^2 = c_{\text{gravity}} r_\tau$ ($r_x = \sqrt{\lambda}L$, $r_\tau = \sqrt{\lambda}\beta$), and $c_{\text{gravity}} \approx 2.45$ for the square torus [15, 16].

Apart from the free/internal energy, EoS and c_s , there are other interesting observables to measure using lattice calculations to compare to their corresponding gravity predictions. One of these include the Wilson loops proposed in [17, 18] for supersymmetric gauge theories which also include the contribution from the $(9 - p)$ adjoint scalars (Φ). It is defined as follows,

$$W = \frac{1}{N} \text{Tr} \hat{P} \exp \left[\oint_C d\tau \left(A_\mu(x) \dot{x}^\mu + \hat{\theta}^i |\dot{x}| \Phi_i(x) \right) \right], \quad (2.7)$$

where $\hat{\theta}$ is the unit vector and C is the contour which is parametrized by $x^\mu(\tau)$. It is normalized such that large N limit is well-defined. We mention the prediction for this observable obtained using supergravity calculations for $p < 3$, where only the $p = 0$ case has yet been discussed using numerical simulations [19]

- $p = 0 : \log \langle W \rangle = 1.89 t^{-3/5}$
- $p = 1 : \log \langle W \rangle = 1.54 t^{-1/2}$
- $p = 2 : \log \langle W \rangle = 1.15 t^{-1/3}$

Generally, $\log \langle W \rangle \sim t^{-(3-p)/5-p} \sim \lambda_{\text{eff}}^{1/(5-p)}$. Note that for $\mathcal{N} = 4$ SYM, this gives the $\sqrt{\lambda}$ dependence.

2.2 Finite temperature supersymmetric gauge theory

We consider the maximally supersymmetric Yang–Mills theory on two-torus ($S_\beta^1 \times S_L^1$) with anti-periodic boundary conditions for the fermions along the time cycle ($\beta = 1/T$) and denote the trace of the energy-momentum tensor (also known as ‘trace anomaly’ or ‘interaction measure’) by $\Delta = E - P$, where E and P are defined as,

$$E = T^2 \left. \frac{\partial \ln Z}{\partial T} \right|_V ; \quad P = VT \left. \frac{\partial \ln Z}{\partial V} \right|_T \quad (2.8)$$

Using the approximation for Eq. (2.8) for homogeneous systems as,

$$P \approx T \ln Z, \quad (2.9)$$

we can deduce an expression that relates the pressure to Δ given by,

$$\begin{aligned} \frac{\Delta}{T^2} &= \frac{E}{T^2} - \frac{P}{T^2} \\ &= T \frac{\partial}{\partial T} \left(\frac{P}{T^2} \right) \end{aligned} \quad (2.10)$$

Integrating Eq.(2.10) gives,

$$\frac{P(T)}{T^2} - \frac{P(T_0)}{T_0^2} = \int_{T_0}^T dT' \frac{1}{T'^3} \Delta(T') \quad (2.11)$$

In principle, this relation will help us determine the EoS and the speed of sound. The range of temperatures ($t = T/\sqrt{\lambda}$ and we set $\lambda = 1$) considered for the numerical integration would have to satisfy, $1/N < t < \alpha^2/c_{\text{gravity}}$ with $t \ll 1$.

3. Lattice action

The lattice $\mathcal{N} = 4$ super Yang–Mills action based on topological twisting formulation can be written as a sum of \mathcal{Q} -exact and \mathcal{Q} -closed terms. The details (and, devils) can be found elsewhere [20, 21]. For a recent review, see [22]. The action is given by,

$$S_{\text{exact}} = \frac{N}{4\lambda_{\text{eff}}} \sum_{\mathbf{n}} \text{Tr} \left[-\bar{\mathcal{F}}_{ab}(\mathbf{n}) \mathcal{F}_{ab}(\mathbf{n}) - \chi_{ab}(\mathbf{n}) \mathcal{D}_{[a}^{(+)} \psi_{b]}(\mathbf{n}) - \eta(\mathbf{n}) \bar{\mathcal{D}}_a^{(-)} \psi_a(\mathbf{n}) + \frac{1}{2} \left(\bar{\mathcal{D}}_a^{(-)} \mathcal{U}_a(\mathbf{n}) \right)^2 \right], \quad (3.1)$$

$$S_{\text{closed}} = -\frac{N}{16\lambda_{\text{eff}}} \sum_{\mathbf{n}} \text{Tr} \left[\epsilon_{abcde} \chi_{de}(\mathbf{n} + \hat{\mu}_a + \hat{\mu}_b + \hat{\mu}_c) \bar{\mathcal{D}}_c^{(-)} \chi_{ab}(\mathbf{n}) \right], \quad (3.2)$$

where λ_{eff} is the dimensionless 't Hooft coupling and we sum over repeated indices. Supersymmetric theories have flat directions which are a problem for numerical simulations and we control this by adding a \mathcal{Q} breaking term to the lattice action (which we extrapolate to zero) as,

$$S_{\text{flat}} = \frac{N}{4\lambda_{\text{eff}}} \mu^2 \sum_{\mathbf{n}, a \neq 3} \text{Tr} \left(\bar{\mathcal{U}}_a(\mathbf{n}) \mathcal{U}_a(\mathbf{n}) - \mathbb{I}_N \right)^2 \quad (3.3)$$

Since we are interested in the dimensionally reduced $\mathcal{N} = 4$ SYM to two dimensions with the same number of supercharges (known as $\mathcal{N} = (8, 8)$ SYM), we dimensionally reduce along the two spatial directions. To have a meaningful dimensional reduction, we have observed that an extra term has to be added to the action given by,

$$S_{\text{center}} = \frac{N}{4\lambda_{\text{eff}}} \mu^2 \sum_{\mathbf{n}, i=x,y} \text{ReTr} \left[\left(\varphi_i(\mathbf{n}) - \mathbb{I}_N \right)^\dagger \left(\varphi_i(\mathbf{n}) - \mathbb{I}_N \right) \right]. \quad (3.4)$$

The lattice supersymmetric theories based on \mathcal{Q} -exact formulation are naturally adapted to non-orthogonal lattices. For example, $\mathcal{N} = 4$ in four dimensions is formulated on A_4^* lattice which has a bigger point group symmetry than the hypercubic lattice. However, we want to study the two-dimensional theory on a square lattice. In [23, 24], it was argued that one can get different lattice geometries by the choice of the expansion point for the fields in the moduli space (the trajectory one follows to the infinity). We add an additional term, $S_{A_2^* \rightarrow \text{hyp}}$, given by,

$$S_{A_2^* \rightarrow \text{hyp}} = \frac{N}{4\lambda_{\text{eff}}} \sigma^2 \sum_{\mathbf{n}} \text{Tr} \left(\bar{\mathcal{U}}_3(\mathbf{n}) \mathcal{U}_3(\mathbf{n}) \right)^2 \quad (3.5)$$

to the action which consists of the gauge links in the extra direction of the skewed geometry. The resulting lattice is square [see Figure (2)] and we keep $\sigma = O(1)$ fixed for all couplings/temperatures.

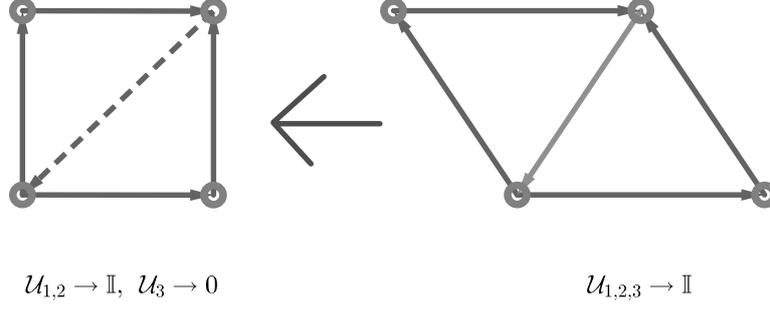


Figure 2: On the right we have A_2^* lattice where the three links are treated equally and expanded symmetrically to target the continuum theory. On the left, by modifying the third link and requiring that it is expanded around zero, we get square lattice. See [23, 24] for details.

The complete lattice action reads,

$$S = S_{\text{exact}} + S_{\text{closed}} + S_{\text{flat}} + S_{\text{center}} + S_{A_2^* \rightarrow \text{hyp}}. \quad (3.6)$$

We are carrying out the numerical simulations using 3.6 on the parallel software `SUSY LATTICE` developed in [25] and will report the results in the future.

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