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Dynamics and flavor symmetry realizations in chiral QCD

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The phases and symmetries of a wide class of strongly-interacting chiral gauge theories are investigated. The basic constraints arise from the 't Hooft anomaly matching condition, which is often difficult to satisfy, and as a consequence, restricts severely possible infrared behavior of the systems. Two mechanisms, the color-flavor locking and dynamical Abelianization, acting singly or in combination, emerge from our analysis as powerful ideas which help to find a solution.

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1. Introduction

Our world has a nontrivial chiral structure. The macroscopic structures such as biological bodies often have approximately left-right symmetric forms, but not exactly. At the molecular level, $O(10^{-6}\text{cm})$, the structure of DNA has a definite chiral spiral form. At the microscopic length scales of the fundamental interactions, $O(10^{-16}\text{cm})$, the left- and right-handed quarks and leptons have different couplings to the $SU(3) \times SU_L(2) \times U_Y(1)$ gauge bosons.

In spite of these, and in spite of an almost half-century of studies of vectorlike gauge theories like $SU(3)$ quantum chromodynamics (QCD), based on straightforward approximate calculations (lattice simulations), and despite beautiful theoretical developments in models with $\mathscr{N} = 2$ supersymmetries (which are all vectorlike), surprisingly little is known today about strongly-coupled chiral gauge theories.[1] Perhaps it is not senseless to try to understand better this class of gauge theories, which Nature might be making use of, in a way as yet unknown to us.

To be concrete, we shall limit ourselves to $SU(N)$ gauge theories with a set of Weyl fermions in a complex representation of $SU(N)$. Also only asymptotically free models will be considered, as weakly coupled infrared-free theories can be reliably analyzed in perturbation theory, as in the case of the standard electroweak model.

The gauge interactions in these models become strongly coupled in the infrared. There are no gauge-invariant bifermion condensates, no mass terms or potential terms (of renormalizable type) can be added to deform the theories, no θ parameter exists. No center symmetry is present. The vacuum is unique.

For simplicity we shall restrict ourselves to various irreducible [2] $SU(N)$ chiral theories, with N_ψ fermions $\psi^{\{ij\}}$ in the symmetric representation, N_χ fermions $\chi_{[ij]}$ in the anti anti-symmetric representation, and a number of antifundamental (or fundamental) multiplets, η_i^a (or $\tilde{\eta}^{ai}$). The number of the latter is fixed so that the gauge group be anomaly free. We call these (N_ψ, N_χ) model. The question of our interest are:

(i) Does the system confine?

(ii) Does the system experience a dynamical Higgs phenomenon?

(iii) Does the system flow into an IR fixed-point CFT?

(iv) Does the chiral flavor symmetry remain unbroken, or if spontaneously broken, how?

(v) If there are more than one apparently possible dynamical scenarios, all consistent with e.g., 't Hooft anomaly matching conditions [2], the a "theorem" [3], or other criteria such as the ACS condition [4], which one is actually realized in the infrared?

(vi) Does the system dynamically generate hierarchically disparate mass scales, such as in the "tumbling" scenarios [5]?

(vii) Do the systems simplify in the large N limit, or is a planar equivalence to $\mathscr{N} = 1$ supersymmetric models valid (e.g., [9])?

[1] For a partial list of references on the earlier studies of chiral theories, see [5]-[8].

[2] Namely we do not consider addition of fundamental-antifundamental pairs of fermions. The models of this type, in the simplest cases $(N_\psi, N_\chi) = (1,0), (0,1)$, have been studied in [4].

PoS(Confinement2018)048

2. $(N_\psi, N_\chi) = (1,1)$ ("$\psi\chi\eta$") model

First consider the $(1,1)$ model, with left-handed fermion matter fields

$$\psi^{\{ij\}}\ , \qquad \chi_{[ij]}\ , \qquad \eta_i^A\ , \qquad A = 1,2,\ldots 8\ , \tag{2.1}$$

a symmetric tensor, an (anti-)antisymmetric tensor and eight anti-fundamental multiplets of $SU(N)$. The unbroken global symmetry is

$$G_F = SU(8)\times U_1(1)\times U_2(1)\times Z_{N^*} \tag{2.2}$$

$U_{1,2}(1)$ are anomaly free combinations of $U_\psi(1)$, $U_\chi(1)$, $U_\eta(1)$, and Z_{N^*} is some discrete symmetry. They can be taken e.g., as

$$\begin{aligned} U_1(1): &\qquad \psi \to e^{i\frac{\alpha}{N+2}}\psi\ ; \qquad \eta \to e^{-i\frac{\alpha}{8}}\eta\ ; \\ U_2(1): &\qquad \psi \to e^{i\frac{\beta}{N+2}}\psi\ ; \qquad \chi \to e^{-i\frac{\beta}{N-2}}\chi\ . \end{aligned} \tag{2.3}$$

Various possible dynamical possibilities have been discussed in [1].

(i) Confinement phase with no chiral symmetry breaking is excluded by impossibility of finding massless baryons, saturating the G_F anomalies.

(ii) For large N, it was proposed [1] that a possible phase can be described by the nonvanishing bi-fermion condensates

$$\langle\psi^{ij}\eta_j^A\rangle = \Lambda^3 \begin{array}{c} \\ \left(\begin{array}{c} c\mathbf{1}_8 \\ \hline \mathbf{0}_{N-8,8} \end{array}\right) \end{array}^{iA}\ , \qquad \langle\psi^{ik}\chi_{kj}\rangle = \Lambda^3 \begin{array}{c} \\ \left(\begin{array}{c|ccc|c} a\mathbf{1}_8 & & & & \\ \hline & d_1 & & & \\ & & \ddots & & \\ & & & d_{N-12} & \\ \hline & & & & b\mathbf{1}_4 \end{array}\right) \end{array}^{i}_{j}\ , \tag{2.4}$$

with symmetry breaking

$$SU(N)_{\rm c}\times SU(8)_{\rm f}\times U(1)^2 \to SU(8)_{\rm cf}\times U(1)^{N-11}\times SU(4)_{\rm c}\,. \tag{2.5}$$

The massless baryons are

$$\tilde{B}_j^A = \psi^{ik}\chi_{[kj]}\eta_i^A \sim \eta_j^A\ , \qquad (9\le j\le N-4) \tag{2.6}$$

and

$$B^{\{AB\}} = \psi^{ij}\eta_i^A\eta_j^B\ . \tag{2.7}$$

(iii) Another possible phase, for $N\ge 8$, is described by the condensates,

$$\langle\phi^{iA}\rangle = \langle\psi^{ij}\eta_j^A\rangle\ ; \qquad \langle\tilde\phi_j^i\rangle \equiv \langle\psi^{ik}\chi_{kj}\rangle\ . \tag{2.8}$$

where

$$\langle \psi^{ij}\eta_j^A \rangle = \Lambda^3 \left(\begin{array}{c} c\mathbf{1}_8 \\ \hline \\ \mathbf{0}_{N-8,8} \end{array} \right)^{iA} , \qquad \langle \psi^{ik}\chi_{kj} \rangle = \Lambda^3 \left(\begin{array}{c|ccc} \mathbf{0}_8 & & & \\ \hline & d_1 & & \\ & & \ddots & \\ & & & d_{N-8} \end{array} \right)^{i}_{j} . \tag{2.9}$$

The symmetry breaking pattern is:

$$SU(N) \times SU(8) \times U(1)^2 \to SU(8)_{cf} \times U(1)^{N-8} . \tag{2.10}$$

(iv) Still another option, consistent for any value of N, considered in [1], is that the gauge group dynamically Abelianizes completely, by the adjoint condensates

$$\langle \tilde{\phi}^i_j \rangle \equiv \langle \psi^{ik}\chi_{kj} \rangle = d_j\, \delta^i_j , \qquad \sum_j d_j = 0 ; \qquad i,j = 1,2,\ldots,N , \tag{2.11}$$

with no particular relations among d_j's. We also assume that no color-flavor locking takes place, i.e.,

$$\langle \phi^{iA} \rangle = \langle \psi^{ij}\eta_j^A \rangle = 0 . \tag{2.12}$$

The symmetry breaking occurs as:

$$SU(N)_{\rm c} \times SU(8)_{\rm f} \times U(1)^2 \to \prod_{\ell=1}^{N-1} U_\ell(1) \times SU(8)_{\rm f} \times \tilde{U}(1) , \tag{2.13}$$

where $\tilde{U}(1)$ is an unbroken combination of the two nonanomalous $U(1)$'s, (2.3), with charges:

$$\psi : 2 , \qquad \chi : -2 , \qquad \eta : -1 . \tag{2.14}$$

There are a few more possible dynamical scenarios involving partial color-flavor locking and dynamical Abelianization. G_F anomaly matching is satisfied in all cases, except for the first of the above, scenario (ii): it turns out that the anomaly matching involving a discrete symmetry, $Z_{N^*} \times SU(8)^2$ fails for some N. Except for this, there is for the moment no way of deciding which of the dynamical scenarios is actually realized in the infrared.

3. $(N_\psi, N_\chi) = (1,0)$ model

$$\psi^{\{ij\}} , \quad \eta_i^B , \qquad B = 1,2,\ldots,N+4 , \tag{3.1}$$

or

$$\square\!\square + (N+4)\,\bar{\square} . \tag{3.2}$$

The (continuous) symmetry of this model is

$$SU(N)_{\rm c} \times SU(N+4)_{\rm f} \times U(1) . \tag{3.3}$$

where $U(1)$ is an anomaly-free combination of $U_\psi(1)$ and $U_\eta(1)$, with

$$Q_\psi: \quad N+4;, \qquad Q_\eta: \quad -(N+2)\,. \tag{3.4}$$

There are also discrete symmetries

$$\mathbb{Z}_\psi = \mathbb{Z}_{N+2} \subset U_\psi(1)\,, \qquad \mathbb{Z}_\eta = \mathbb{Z}_{N+4} \subset U_\eta(1)\,. \tag{3.5}$$

(i) It was found [4] that ’t Hooft anomaly matching conditions alone allow, remarkably, for a confinement phase with no bifermion condensates, i.e., with no chiral symmetry breaking. The candidate massless baryons are:

$$B^{[AB]} = \psi^{ij}\eta_i^A\eta_j^B\,, \qquad A,B = 1,2,\ldots,N+4\,, \tag{3.6}$$

fields	$SU(N)_c$	$SU(N+4)$	$\tilde{U}(1)$
ψ	$\square\!\square$	$\frac{N(N+1)}{2}\cdot(\cdot)$	$N+4$
η^A	$(N+4)\cdot\bar{\square}$	$N\cdot\square$	$-(N+2)$
$B^{[AB]}$	$\frac{(N+4)(N+3)}{2}\cdot(\cdot)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$-N$

Table 1: Chirally symmetric phase of the $(1,0)$ model

(ii) Another, natural dynamical scenario is a color-flavor locked Higgs phase, in which bifermion condensate

$$\langle\psi^{\{ij\}}\eta_i^B\rangle = C\,\delta^{jB}\,, \qquad j,B = 1,2,\ldots N\,, \tag{3.7}$$

forms, breaking the color dynamically, and in which the symmetry is reduced to

$$SU(N)_{\rm cf}\times SU(4)_{\rm f}\times U'(1)\,. \tag{3.8}$$

As this forms a subgroup of the full symmetry group, (3.3), it is quite easily seen, by making the decomposition of the fields in the subgroup, that a subset of the same baryons saturate all of the triangles associated with the reduced symmetry group. See Table 2.

4. $(2,0)$ model

An interesting generalization of the above is the model with matter fermions

$$\psi^{\{ij,m\}}\,, \quad \eta_i^B\,, \qquad m=1,2\,, \quad B=1,2,\ldots,2(N+4)\,, \tag{4.1}$$

or

$$2\,\square\!\square + 2(N+4)\,\bar{\square}\,. \tag{4.2}$$

The (continuous) symmetry of this model is

$$SU(N)_{\rm c}\times SU(2)_f\times SU(2N+8)_{\rm f}\times U(1)\,, \tag{4.3}$$

fields	$SU(N)_{cf}$	$SU(4)_f$	$U'(1)$
ψ	$\square\!\square$	$\frac{N(N+1)}{2}\cdot(\cdot)$	1
η^{A_1}	$\overline{\square\!\square} \oplus \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$N^2\cdot(\cdot)$	-1
η^{A_2}	$4\cdot\bar{\square}$	$N\cdot\square$	$-\frac{1}{2}$
$B^{[A_1B_1]}$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$\frac{N(N-1)}{2}\cdot(\cdot)$	-1
$B^{[A_1B_2]}$	$4\cdot\bar{\square}$	$N\cdot\square$	$-\frac{1}{2}$

Table 2: Color-flavor locked phase in the $(1,0)$ model. A_1 or B_1 stand for $A,B=1,2,\ldots,N$. A_2 or B_2 the rest of the flavor indices.

where $U(1)$ is an anomaly-free combination of $U_\psi(1)$ and $U_\eta(1)$,

$$U(1): \qquad \psi \to e^{i\alpha/2(N+2)}\psi\,, \qquad \eta \to e^{-i\alpha/2(N+4)}\eta\,. \tag{4.4}$$

It turns out that, in contrast to the $(1,0)$ model, it is not possible to find a confining, chirally symmetric phase with no fermion condensates: there are no candidate massless, color-singlet baryons made out of $\psi^{\{ij,m\}}$ and η_i^B which could saturate the anomalies.

Instead, we find that the system could flow into a double $SU(N)$ color-flavor-flavor locked phase, with condensates,

$$\langle\psi^{\{ij,1\}}\eta_j^B\rangle = C\,\delta^{i,B}\,, \qquad j,B=1,2,\ldots N\,, \tag{4.5}$$

$$\langle\psi^{\{ij,2\}}\eta_j^B\rangle = C\,\delta^{i,B-N}\,, \qquad j=1,2,\ldots N\,, \quad B=N+1,\ldots,2N \tag{4.6}$$

The symmetry is

$$SU(N)_c\times SU(2)_f\times SU(2N+8)_f\times U(1)_f \to SU(N)_{cf}\times\tilde{U}(1)\times SU(2)_{ff}\times SU(8)\,, \tag{4.7}$$

where $SU(2)_{ff}$ is a linear combination of $SU(2)_f$ and

$$SU(2)\subset SU(2N)\subset SU(2N+8) \tag{4.8}$$

which exhange the first and second N flavors. The charges of the unbroken $SU(2)$ are:

$$\begin{pmatrix}\psi^{ij,1}\\ \psi^{ij,2}\end{pmatrix}\sim\underline{2}\,; \qquad \begin{pmatrix}\eta_i^{A\le N}\\ \eta_i^{N\le A\le 2N}\end{pmatrix}\sim\underline{2}^* \tag{4.9}$$

the $\tilde{U}(1)$ charges are as before,

$$\psi: \quad 1\,; \qquad \eta^{B\le 2N}: \quad -1\,; \qquad \eta^{B>2N}: \quad -\frac{1}{2}\,. \tag{4.10}$$

The baryons are

$$B^{A,C}=\psi^{ij,1}\eta_i^{A\le N}\eta_j^C+\psi^{ij,2}\eta_i^{N<A\le 2N}\eta_j^C\,; \qquad C>2N\,; \tag{4.11}$$

which is a $SU(2)$ singlet; the others are

$$B^{[A_1B_1],1} = \psi^{ij,1}\eta_i^{A_1}\eta_j^{B_1}\,, \qquad A_1, B_1 = 1, 2, \ldots, N \tag{4.12}$$

and

$$B^{[A_2B_2],2} = \psi^{ij,2}\eta_i^{A_2}\eta_j^{B_2}\,, \qquad A_2, B_2 = N+1, N+2, \ldots, 2N \tag{4.13}$$

which form a doublet. Their $\tilde{U}(1)$ charges are:

$$B^{A,C}: \quad -\frac{1}{2}\,; \qquad B^{[AB],m}: \quad -1\,. \tag{4.14}$$

fields	$SU(N)_{cf}$	$SU(8)$	$SU(2)$	$\tilde{U}(1)$
ψ	$2\cdot\square\square$	$N(N+1)\cdot(\cdot)$	$\frac{N(N+1)}{2}\cdot\square$	1
η^{A_i}	$2\cdot(\bar{\square\square}\oplus\bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}})$	$2N^2\cdot(\cdot)$	$N^2\cdot\square$	-1
η^C	$8\cdot\bar{\square}$	$N\cdot\square$	$8N\cdot(\cdot)$	$-\frac{1}{2}$
$B^{A,C}$	$8\cdot\bar{\square}$	$N\cdot\square$	$8N\cdot(\cdot)$	$-\frac{1}{2}$
$B^{[A_iB_i],m}$	$2\cdot\bar{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$N(N-1)\cdot(\cdot)$	$\frac{N(N-1)}{2}\square$	-1

Table 3: An $SU(2)$ flavor-flavor locked symmetric phase in the $(2,0)$ model. A_i or B_i $(i=1,2)$ indicate the flavor indices up to $2N$; C the rest, $2N+1,\ldots,2N+8$.

5. $(0,1)$ model

This model was also studied by by Appelquist-Duan-Sannino [4], by Poppitz [6] and by ourselves [1] earlier. The matter fermions are

$$\chi_{[ij]}\,, \qquad \tilde{\eta}^{Bj}\,, \qquad B = 1, 2, \ldots, (N-4)\,. \tag{5.1}$$

The symmetry is

$$SU(N)_{\rm c} \times SU(N-4)_{\rm f} \times U(1)\,, \tag{5.2}$$

where the anomaly free $U(1)$ charge is

$$\chi: \quad N-4\,; \qquad \tilde{\eta}^{Bj}\;: \quad -(N-2)\,. \tag{5.3}$$

$$b_0 = 11N - (N-2) - (N-4) = 9N + 6\,. \tag{5.4}$$

There are also discrete symmetries

$$\mathbb{Z}_\chi = \mathbb{Z}_{N-2} \subset U_\psi(1)\,, \qquad \mathbb{Z}_\eta = \mathbb{Z}_{N-4} \subset U_\eta(1)\,. \tag{5.5}$$

fields	$SU(N)_c$	$SU(N-4)$	$U(1)$
χ $\tilde{\eta}^A$	$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$ $(N-4)\cdot\square$	$\frac{N(N-1)}{2}\cdot(\cdot)$ $N\cdot\bar{\square}$	$N-4$ $-(N-2)$
$B^{\{AB\}}$	$\frac{(N-4)(N-3)}{2}\cdot(\cdot)$	$\square\square$	$-N$

Table 4: Confinement and unbroken symmetry in the $(0,1)$ model

(i) As in the $(1,0)$ model discussed earlier, the anomaly matching argument alone allows a chirally symmetric (no fermion condensates) confining vacuum, with massless baryons

$$B^{\{CD\}} = \chi_{[ij]}\,\tilde{\eta}^{iC}\tilde{\eta}^{jD}\,, \qquad C,D = 1,2,\ldots(N-4)\,, \tag{5.6}$$

assumed to be symmetric in CD. See Table 4.

(ii) Another natural hypothesis is that this system develops a condensate of the form

$$\langle \chi_{[ij]}\,\tilde{\eta}^{Bj}\rangle = \mathrm{const.}\,\Lambda^3\delta_i^B\,; \qquad i,B = 1,2,\ldots,N-4\,, \tag{5.7}$$

namely,

$$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \otimes \square \to \bar{\square} \oplus \ldots\,. \tag{5.8}$$

The symmetry is broken as

$$SU(N)_{\mathrm{c}} \times SU(N-4)_{\mathrm{f}} \times U(1) \to SU(N-4)_{\mathrm{cf}} \times U(1)' \times SU(4)_{\mathrm{c}}\,. \tag{5.9}$$

A subset of the massless baryons (5.6) saturate all the anomalies associated with $SU(N-4)_{\mathrm{cf}} \times U(1)'$. See Table 5. As noted by Appelquist, Duan, Sannino [4], there remains the $\chi_{i_2 j_2}$ fermions which remain massless and strongly coupled to the $SU(4)_c$. We may assume that $SU(4)_c$ confines, and the condensate

$$\langle \chi\chi \rangle \neq 0\,, \tag{5.10}$$

in

$$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \otimes \overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}} \to \overline{\begin{smallmatrix}\square\\\square\\\square\\\square\end{smallmatrix}} \oplus \ldots\,, \tag{5.11}$$

forms and $\chi_{i_2 j_2}$ acquire dynamically mass.

6. Conclusion

Many other systems have been analyzed ($(1,1)$, $(1,0)$, $(2,0)$, $(3,0)$ $(0,1)$, $(0,2)$ $(0,3)$, $(2,1)$, $(1,-1)$, etc.) and in more details. Let us summarize some of the lessons learned:

fields	$SU(N-4)_{cf}$	$U'(1)$	$SU(4)_c$
$\chi_{i_1 j_1}$	$\overline{\tiny\yng(1,1)}$	N	$\frac{(N-4)(N-5)}{2}\cdot(\cdot)$
$\chi_{i_1 j_2}$	$4\cdot\overline{\tiny\yng(1)}$	$\frac{N}{2}$	$(N-4)\cdot\overline{\tiny\yng(1)}$
$\chi_{i_2 j_2}$	$\frac{4\cdot 3}{2}\cdot(\cdot)$	0	$\overline{\tiny\yng(1,1)}$
$\tilde{\eta}^{A,i_1}$	${\tiny\yng(2)}\oplus{\tiny\yng(1,1)}$	$-N$	$(N-4)^2\cdot(\cdot)$
$\tilde{\eta}^{A,i_2}$	$4\cdot{\tiny\yng(1)}$	$-\frac{N}{2}$	$(N-4)\cdot{\tiny\yng(1)}$
$B^{\{AB\}}$	${\tiny\yng(2)}$	$-N$	$\frac{(N-4)(N-3)}{2}\cdot(\cdot)$

Table 5: Color-flavor locking in the $(0,1)$ model. The color index i_1 or j_1 runs up to $N-4$. The rest is indicated by i_2 or j_2.

1. Confining vacuum without chiral symmetry breaking turns out to be rather exceptional (only $(1,0)$ and $(0,1)$ models allow the matching with such a hypothesis); the dynamical Higgs phase seems to appear equally in many models, and perhaps, more natural.

2. In many models color-flavor (or even color-flavor-flavor) locking hypothesis allows one to achieve the anomaly matching.

3. Dynamical Abelianization $SU(N) \to U(1)^{N-1}$ (full or partial) due to some bifermion condensate in adjoint representation, is another important mechanism for many systems to be described consistently in the infrared.

4. Complementarity [10] may or may not work.

5. The large N planar equivalence to supersymmetric $SU(N)$ Yang-Mills does not hold in the $\psi\chi\eta$ model (cfr. [9]).

6. a-theorem is always satisfied in the proposed dynamical scenarios, whereas the ACS criterion [4] sometimes fails.

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