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Non-Abelian Higgs Theory in a Strong Magnetic Field and Confinement

Hideo Suganuma*,
Department of Physics & Division of Physics and Astronomy, Graduate School of Science, Kyoto University, Kitashirakawaoiwake, Sakyo, Kyoto 606-8502, Japan
E-mail: suganuma@scphys.kyoto-u.ac.jp

The non-abelian Higgs (NAH) theory is studied in a strong magnetic field. For simplicity, we study the SU(2) NAH theory with the Higgs triplet in a constant strong magnetic field $\vec{B}$, where the lowest-Landau-level (LLL) approximation can be used. Without magnetic fields, charged vector fields $A_\mu^{\pm}$ have a large mass M due to Higgs condensation, while the photon field A_μ remains to be massless. In a strong constant magnetic field near and below the critical value $eB_c \equiv M^2$, the charged vector fields $A_\mu^{\pm}$ behave as 1+1-dimensional quasi-massless fields, and give a strong correlation along the magnetic-field direction between off-diagonal charges coupled with $A_\mu^{\pm}$. This may lead a new type of confinement caused by charged vector fields $A_\mu^{\pm}$.

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1. Introduction

In the Weinberg-Salam model, through the Brout-Englert-Higgs mechanism [1], all the weak bosons ($W_\mu^\pm$ and Z_μ) acquire a large mass and can propagate to very short distance, and only photon remains massless and can propagate to long distance.

Also for quantum chromodynamics (QCD), in the context of the dual superconductor picture for quark confinement [2, 3, 4], by taking the maximally Abelian (MA) gauge, off-diagonal gluons acquire a large effective mass of about 1GeV and become infrared inactive [5], and only diagonal gluons contribute to long-distance physics, which is called "Abelian dominance" [3, 4, 5, 6, 7].

In fact, QCD in the MA gauge and the non-Abelian Higgs (NAH) theory such as the Weinberg-Salam model are similar from the viewpoint of large mass generation of off-diagonal gauge bosons $A_\mu^\pm$, and sequential off-diagonal inactiveness and low-energy Abelianization, although the NAH theory does not exhibit charge confinement.

However, this situation can be drastically changed in the presence of a strong magnetic field for the NAH theory, as will be discussed. In this paper, we study the NAH theory in a strong magnetic field and consider a new type of confinement caused by charged vector fields $A_\mu^\pm$. We here note that "magnetic properties of quantum systems" or "quantum systems in external magnetic fields" are also interesting subjects in various fields in physics [8, 9, 10, 11, 12, 13, 14, 15].

2. SU(2) Non-Abelian Higgs Theory with Higgs Triplet

We start from the standard SU(2) non-Abelian Higgs (NAH) theory with the SU(2) gauge field $A_\mu^{\rm SU(2)} = A_\mu^a T^a \in su(2)$ and the SU(2) Higgs-scalar triplet Φ^a ($a = 1,2,3$),

$$\mathscr{L}_{\rm NAH} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2} D_\mu^{\rm SU(2)} \Phi^a D^\mu_{\rm SU(2)} \Phi^a - \frac{\lambda}{4}(\Phi^a\Phi^a - v^2)^2, \tag{2.1}$$

where the SU(2) covariant derivative $D_\mu^{\rm SU(2)} \equiv \partial_\mu + ieA_\mu^{\rm SU(2)}$ satisfies $D_\mu^{\rm SU(2)}\Phi^a = \partial_\mu \Phi^a - e\varepsilon^{abc}A_\mu^b \Phi^c$, and the SU(2) field strength $G_{\mu\nu} \equiv \frac{1}{ie}[A_\mu^{\rm SU(2)}, A_\nu^{\rm SU(2)}]$ is written as $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - e\varepsilon^{abc}A^b_\mu A^c_\nu$.

At the tree level, the Higgs field Φ^a has a vacuum expectation value, and one can set $\langle \Phi^3 \rangle = v\ (\geq 0) \in \mathbf{R}$ and $\Phi^3 = v + \sigma$ in the unitary gauge. Then, one obtains

$$\begin{aligned}
\mathscr{L}_{\rm NAH} = &-\frac{1}{4}[F_{\mu\nu} - ie(A^+_\mu A^-_\nu - A^-_\mu A^+_\nu)]^2 - \frac{1}{2}(D^-_\mu A^+_\nu - D^-_\nu A^+_\mu)(D^\mu_+ A^\nu_- - D^\nu_+ A^\mu_-) \\
&+ (M + e\sigma)^2 A^+_\mu A^\mu_- + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{\lambda}{4}\{(v+\sigma)^2 - v^2\}^2, \\
= &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(D^-_\mu A^+_\nu - D^-_\nu A^+_\mu)(D^\mu_+ A^\nu_- - D^\nu_+ A^\mu_-) + (M + e\sigma)^2 A^+_\mu A^\mu_- \\
&+ ieF^{\mu\nu}A^+_\mu A^-_\nu + \frac{1}{2}e^2[(A^+_\mu A^\mu_+)(A^-_\nu A^\nu_-) - (A^+_\mu A^\mu_-)^2] \\
&+ \frac{1}{2}(\partial_\mu \sigma)^2 - \lambda v^2 \sigma^2 - \lambda v \sigma^3 - \frac{\lambda}{4}\sigma^4,
\end{aligned} \tag{2.2}$$

with the charged vector field $A_\mu^\pm \equiv \frac{1}{\sqrt{2}}(A^1_\mu \pm iA^2_\mu) \in \mathbf{C}$ and its mass $M \equiv ev$. Using the unbroken U(1) gauge (photon) field $A_\mu \equiv A^3_\mu$, we define the U(1) covariant derivative $D^\pm_\mu \equiv \partial_\mu \pm ieA_\mu$ and the U(1) field strength $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, which satisfy $[D^\pm_\mu, D^\pm_\nu] = \pm ieF_{\mu\nu}$.

In the NAH theory, the bilinear part of the charged vector field $A_\mu^\pm$ is expressed by

$$\begin{aligned}\mathcal{L}_{A^\pm 2\mathrm{nd}} &= -\frac{1}{2}(D_\mu^- A_\nu^+ - D_\nu^- A_\mu^+)(D_+^\mu A_-^\nu - D_+^\nu A_-^\mu) + (M+e\sigma)^2 A_\mu^+ A_-^\mu + ieF^{\mu\nu}A_\mu^+ A_\nu^- \\ &= -A_\mu^+ D_\nu^+ (D_+^\mu A_-^\nu - D_+^\nu A_-^\mu) + (M+e\sigma)^2 A_\mu^+ A_-^\mu + ieF^{\mu\nu}A_\mu^+ A_\nu^- \\ &= A_\mu^+ [D_\nu^+ D_+^\nu + (M+e\sigma)^2] A_-^\mu + 2ieF^{\mu\nu}A_\mu^+ A_\nu^- - A_\mu^+ D_+^\mu D_\nu^+ A_-^\nu. \end{aligned} \tag{2.3}$$

Note here that the term "$2ieF^{\mu\nu}A_\mu^+ A_\nu^-$" corresponds to the gyromagnetic ratio $g = 2$ [9, 15] for the charged vector field $A_\mu^\pm$, which is a general property of SU(N) non-Abelian gauge theories, including the NAH theory and QCD.

3. Non-Abelian Higgs Theory in a Strong Magnetic Field

Now, we study the non-Abelian Higgs (NAH) theory in the presence of an external field of A_μ or $F_{\mu\nu}$, e.g., a strong magnetic field B. In this paper, we mainly consider a constant external magnetic field $B(\geq 0)$ in the z-direction, i.e., $F_{12} = B$.

For the simple treatment, while eB can take a large value, the gauge coupling $e(> 0)$ is taken to be small such that one can drop off the $O(e^2)$ self-interaction terms of charged vector fields $A_\mu^\pm$ in the NAH Lagrangian (2.2). Also, λ is taken to be enough large such that v and $M \equiv ev$ are unchanged under the magnetic field.

In such a system, only the bilinear term of $A_\mu^\pm$ is important, since it includes the coupling with the external photon field A_μ, and the $O(e^2)$ self-interaction terms of $A^\pm$ without A_μ and the Higgs fluctuation σ can be dropped off in the NAH Lagrangian (2.2). Therefore, the NAH theory is mainly expressed by the bilinear term of the charged vector field $A_\mu^\pm$,

$$\mathcal{L}_{A^\pm 2\mathrm{nd}} = A_\mu^+ (D_\nu^+ D_+^\nu + M^2) A_-^\mu + 2ieF^{\mu\nu}A_\mu^+ A_\nu^- - A_\mu^+ D_+^\mu D_\nu^+ A_-^\nu. \tag{3.1}$$

3.1 Field equation for the charge vector field

From the Lagrangian (3.1), the field equations for the charged vector field $A_\mu^\pm$ are given by

$$\begin{aligned}(D_\nu^+ D_+^\nu + M^2) A_-^\mu + 2ieF^{\mu\nu}A_\nu^- - D_+^\mu D_\nu^+ A_-^\nu = 0, \\ (D_\nu^- D_-^\nu + M^2) A_+^\mu - 2ieF^{\mu\nu}A_\nu^+ - D_-^\mu D_\nu^- A_+^\nu = 0.\end{aligned} \tag{3.2}$$

Multiply Eq.(3.2a) by D_μ^+ from the left and using $[D_\mu^\pm, D_\nu^\pm] = \pm ieF_{\mu\nu}$, we obtain

$$ie[D_+^\mu, F_{\mu\nu}]A_-^\nu + M^2 D_\mu^+ A_-^\mu = ie(\partial^\mu F_{\mu\nu})A_-^\nu + M^2 D_\mu^+ A_-^\mu = 0. \tag{3.3}$$

For the constant electromagnetic field $F_{\mu\nu}$, the massive charged-vector field $A_\mu^\pm$ with a non-zero M satisfies the maximally Abelian (MA) gauge condition,

$$D_\mu^+ A_-^\mu = (\partial_\mu + ieA_\mu) A_-^\mu = 0, \tag{3.4}$$

and the field equations (3.2) become

$$(D_\nu^\mp D_\pm^\nu + M^2) A_\mp^\mu \pm 2ieF^{\mu\nu}A_\nu^\mp = [(D_\lambda^\mp D_\pm^\lambda + M^2) g^{\mu\nu} \pm 2ieF^{\mu\nu}] A_\nu^\mp = 0. \tag{3.5}$$

For the constant magnetic field $B = F_{12}(\geq 0)$ in the z-direction, one finds

$$[(D^{\pm}_{\lambda}D^{\lambda}_{\pm}+M^2)g_{\mu\nu} \pm 2ieB\varepsilon_{\mu\nu 3}]A^{\nu}_{\mp} = [(D^{\pm}_{\lambda}D^{\lambda}_{\pm}+M^2)\hat{\eta}+2eB\hat{S}^q_z]_{\mu\nu}A^{\nu}_{\mp} = 0, \tag{3.6}$$

with $(\hat{\eta})_{\mu\nu} = g_{\mu\nu}$ and the spin-charge operator $\hat{S}^q_z \equiv Q\hat{S}_z$. Here, for the charged vector field $A^{\mu}_{\pm}$, Q is the U(1) charge $Q \in \{\pm 1, 0\}$ (in the unit of e), and $(\hat{S}_z)_{\mu\nu} = -i\varepsilon_{\mu\nu 3}$ is a spin operator and its non-zero eigenvalue states are $A^R_{\pm} \equiv \frac{1}{\sqrt{2}}(A^x_{\pm} - iA^y_{\pm})$ $(s_z = 1)$ and $A^L_{\pm} \equiv \frac{-i}{\sqrt{2}}(A^x_{\pm} + iA^y_{\pm})$ $(s_z = -1)$. Thus, we obtain

$$[(D^{\pm}_{\lambda}D^{\lambda}_{\pm}+M^2)\hat{\eta}+2eB\hat{S}^q_z]\mathscr{A}_{\alpha} = 0 \qquad (\alpha = \pm 1, 0) \tag{3.7}$$

in the diagonal basis of $\hat{S}^q_z$ and $\hat{\eta}$. $\mathscr{A}_{\alpha}$ is a linear combination of $A^{\mu}_{\pm}$ so as to satisfy $\hat{S}^q_z\mathscr{A}_{\alpha} = \alpha\mathscr{A}_{\alpha}$:

$$\mathscr{A}_{+1} = A^R_{+} \text{ or } A^L_{-}, \qquad \mathscr{A}_0 = A^z_{\pm},\, A^t_{\pm} \qquad \mathscr{A}_{-1} = A^L_{+} \text{ or } A^R_{-}. \tag{3.8}$$

3.2 Eigenstates and the Landau level

Now, we consider the eigenstate of the charged vector field $A^{\pm}_{\mu}$ in the constant magnetic field, by investigating the operator in the field equation (3.7) for the charged vector field $\vec{A}_{\pm}$,

$$\begin{aligned}\hat{K} &\equiv -(D^{\pm}_{\lambda}D^{\lambda}_{\pm}+M^2)+2eB\hat{S}^q_z = -\partial_t^2+\partial_z^2+\{(\partial_x \pm ieA_x)^2+(\partial_y \pm ieA_y)^2\}-M^2+2eB\hat{S}^q_z \\ &= \hat{p}_t^2-\hat{p}_z^2-2eB\left(\hat{a}^{\dagger}\hat{a}+\frac{1}{2}\right)-M^2+2eB\hat{S}^q_z,\end{aligned} \tag{3.9}$$

where $\hat{p}_{\mu} \equiv i\partial_{\mu}$ and $\hat{a} \equiv \frac{1}{\sqrt{2eB}}[(\hat{p}_x \pm eA_x) - i(\hat{p}_y \pm eA_y)]$. Here, $\hat{K}$ has a one-dimensional harmonic oscillator in the x, y direction [9, 11], and we introduce the diagonal basis such as $\hat{a}^{\dagger}\hat{a}|n\rangle = n|n\rangle$. Then, the eigenvalue of $\hat{K}$ is given by

$$K(p_t, p_z, n, s_z) \equiv p_t^2 - p_z^2 - 2eB\left(n+\frac{1}{2}-s_z\right) - M^2 \quad (n = 0, 1, 2, \dots;\ s_z = \pm 1, 0) \tag{3.10}$$

for the n-th Landau level with the eigenvalue s_z of $\hat{S}^q_z$. The eigenstate $|p_t, p_z, n, s_z\rangle$ which satisfies

$$\hat{K}|p_t, p_z, n, s_z\rangle = K(p_t, p_z, n, s_z)|p_t, p_z, n, s_z\rangle \tag{3.11}$$

is expressed in the coordinated space as

$$\psi_{p_t p_z n s_z}(x) \equiv \langle x^{\mu}|p_t, p_z, n, s_z\rangle = \langle t|p_t\rangle\langle z|p_z\rangle\langle x, y|n\rangle\chi_{s_z} = e^{-ip_t t}e^{ip_z z}\psi_n(x,y)\chi_{s_z}, \tag{3.12}$$

with the harmonic-oscillator eigenstate $\psi_n(x,y)$ and the spin eigenstate χ_{s_z}, satisfying $\hat{S}^q_z\chi_{s_z} = s_z\chi_{s_z}$. For the external U(1) gauge (photon) field $A_{\mu} \equiv A^3_{\mu}$, we take the symmetric gauge, $(A_x, A_y) = \frac{B}{2}(y, -x)$ with $A_t = A_z = 0$. (Physical results never depend on the remaining U(1) gauge choice.) Then, the harmonic-oscillator eigenstate $\psi_n(x,y)$ is written in 2-dimensional polar coordinates by

$$\psi_n(x,y) = \frac{1}{l}\sqrt{\frac{n!}{(n-m)!}}e^{-\frac{r^2}{4l^2}}\left(\frac{r}{\sqrt{2}l}\right)^{|m|}L_n^{|m|}\left(\frac{r^2}{2l^2}\right)\frac{e^{im\varphi}}{\sqrt{2\pi}} \quad (|m| \leq n) \tag{3.13}$$

with $r \equiv \sqrt{x^2+y^2}$ and the associated Laguerre polynomial L_n^m. Here, $l \equiv \frac{1}{\sqrt{eB}}$ is a typical length of the minimal cyclotron orbit in the magnetic field B.

For the on-mass-shell state of the charged vector field $A^{\pm}_{\mu}$ satisfying the field equation (3.7), its energy p_t is given by [9]

$$p_t^2 = p_z^2 + 2eB\left(n + \frac{1}{2} - s_z\right) + M^2 \quad (n = 0, 1, 2, ...;\ s_z = \pm 1, 0) \tag{3.14}$$

for the Landau level with n and s_z. In the strong magnetic field, the lowest Landau level (LLL) with $n = 0$ and $s_z = 1$ gives a dominant contribution, and therefore one can use the LLL approximation [12], keeping only the LLL contribution. Here, the LLL wave-function is written by

$$\psi_{LLL}(x,y,z) \equiv \psi_0(x,y) e^{ip_z z} \chi_{s_z=1} = \sqrt{\frac{eB}{2\pi}} e^{-\frac{x^2+y^2}{4l^2}} e^{ip_z z} \chi_{s_z=1}, \tag{3.15}$$

which is localized within the order of the length $l \equiv \frac{1}{\sqrt{eB}}$ in the x, y direction. The LLL energy p_t is

$$p_t^2 = p_z^2 - eB + M^2 = p_z^2 - \mu^2, \quad \mu \equiv \sqrt{M^2 - eB}. \tag{3.16}$$

3.3 Propagator of the charge vector field

From the Lagrangian (3.1), the propagator $\hat{D}_{\mu\nu}$ of the charged vector field $A^{\pm}_{\mu}$ is expressed as

$$\hat{D}^{-1}_{\mu\nu} = -(D^+_\lambda D^\lambda_+ + M^2) g_{\mu\nu} - 2ieF_{\mu\nu} + D^+_\mu D^+_\nu, \tag{3.17}$$

which actually satisfies $\mathscr{L}_{A^\pm 2\mathrm{nd}} = -A^\mu_+ \hat{D}^{-1}_{\mu\nu} A^\mu_-$. In the RHS of Eq.(3.17), the first two terms are responsible to the physical pole of the charged vector field $A^\mu_\pm$, and the last term $D^+_\mu D^+_\nu$ suffers from gauge transformation.

Here, we consider addition of a gauge-fixing term whose bilinear part of $A^\mu_\pm$ is

$$\mathscr{L}^{\mathrm{g.f.}}_{A^\pm} = -\frac{1}{\alpha}(D^-_\mu A^\mu_+)(D^+_\nu A^\nu_-) = +\frac{1}{\alpha} A^+_\mu D^\mu_+ D^+_\nu A^\nu_-. \tag{3.18}$$

By taking $\alpha = 1$, the total bilinear part of the charged vector field $A^\mu_\pm$ is expressed as

$$\mathscr{L}^{\mathrm{tot}}_{A^\pm 2\mathrm{nd}} \equiv \mathscr{L}_{A^\pm 2\mathrm{nd}} + \mathscr{L}^{\mathrm{g.f.}}_{A^\pm} = A^+_\mu (D^+_\nu D^\nu_+ + M^2) A^\mu_- + 2ieF^{\mu\nu} A^+_\mu A^-_\nu, \tag{3.19}$$

and the charged-vector propagator $\hat{D}_{\mu\nu}$ becomes

$$\hat{D}^{-1}_{\mu\nu} = -(D^+_\lambda D^\lambda_+ + M^2) g_{\mu\nu} - 2ieF_{\mu\nu}. \tag{3.20}$$

For the constant magnetic field $B = F_{12}$, $\hat{D}_{\mu\nu}$ is written with $\hat{\eta}_{\mu\nu} = g_{\mu\nu}$ and $(\hat{S}^q_z)_{\mu\nu} = i\varepsilon_{\mu\nu 3}$ as

$$\hat{D}^{-1}_{\mu\nu} = -(D^+_\lambda D^\lambda_+ + M^2) g_{\mu\nu} - 2eBi\varepsilon_{\mu\nu 3} = -[(D^+_\lambda D^\lambda_+ + M^2)\hat{\eta} + 2eB\hat{S}^q_z]_{\mu\nu}. \tag{3.21}$$

The charged-vector propagator $\hat{D}$ has the space-time structure of

$$\begin{aligned}
\hat{D}^{-1} &\equiv -[(D^+_\lambda D^\lambda_+ + M^2)\hat{\eta} + 2eB\hat{S}^q_z] = [\hat{p}_t^2 - \hat{p}_z^2 - 2eB(\hat{a}^\dagger \hat{a} + 1/2) - M^2]\hat{\eta} - 2eB\hat{S}^q_z \\
&= \begin{pmatrix} -\hat{K}_0 & & & \\ & \hat{K}_0 & -2eBi & \\ & 2eBi & \hat{K}_0 & \\ & & & \hat{K}_0 \end{pmatrix} = \Omega \begin{pmatrix} -\hat{K}_0 & & & \\ & \hat{K}_0 + 2eB & & \\ & & \hat{K}_0 - 2eB & \\ & & & \hat{K}_0 \end{pmatrix} \Omega^\dagger \equiv \Omega \hat{\mathscr{D}}^{-1} \Omega^\dagger
\end{aligned} \tag{3.22}$$

with $\hat{K}_0 \equiv D_\lambda^+ D_+^\lambda + M^2$ and the spatial rotation $\Omega \equiv \begin{pmatrix} 1 & & & \\ & \frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & \\ & \frac{-i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \\ & & & 1 \end{pmatrix}$ which diagonalizes $\hat{D}$. By the spatial rotation Ω, the coordinate basis $(\hat{t}, \hat{x}, \hat{y}, \hat{z})$ is transformed to the $\hat{D}$-diagonalized basis $(\hat{t}, \hat{R}, \hat{L}, \hat{z})^T = \Omega(\hat{t}, \hat{x}, \hat{y}, \hat{z})^T$. Note that only one spatial sector of $\hat{L} \equiv \frac{-i}{\sqrt{2}}(\hat{x} + i\hat{y})$-component in $\hat{\mathscr{D}}$ includes the lowest Landau level. For the spatial sector $\hat{\mathscr{D}}_s$ in the diagonalized charged-vector propagator $\hat{\mathscr{D}}$, its eigenvalue $D_s(p_t, p_z, n, s_z)$ is written by

$$D_s^{-1}(p_t, p_z, n, s_z) \equiv \langle p_t, p_z, n, s_z | \hat{\mathscr{D}}_s^{-1} | p_t, p_z, n, s_z \rangle = -p_t^2 + p_z^2 + 2eB\left(n + \frac{1}{2} - s_z\right) + M^2. \quad (3.23)$$

Then, the coordinate-space representation of the spatial charged-vector propagator $\hat{\mathscr{D}}_s$ is given by

$$\begin{aligned} D_s(x, x') &\equiv \langle x | \hat{\mathscr{D}}_s | x' \rangle = \sum_{p_t, p_z, n, s_z} \langle x | p_t, p_z, n, s_z \rangle D_s(p_t, p_z, n, s_z) \langle p_t, p_z, n, s_z | x' \rangle \\ &= \int_{-\infty}^{\infty} \frac{dp_t}{2\pi} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \sum_{n=0}^{\infty} \sum_{s_z=-1}^{1} \psi_{p_t p_z n s_z}(x) \frac{1}{-p_t^2 + p_z^2 + 2eB(n + \frac{1}{2} - s_z) + M^2} \psi^\dagger_{p_t p_z n s_z}(x') \end{aligned} \quad (3.24)$$

with the eigenstate $\psi_{p_t p_z n s_z}(x) \equiv \langle x^\mu | p_t, p_z, n, s_z \rangle$ in Eq.(3.12).

In the strong constant magnetic field, we use the lowest Landau level (LLL) approximation, keeping only the LLL ($n = 0$, $s_z = 1$) contribution:

$$\begin{aligned} D_s(x, x') &\simeq D_{LLL}(x, x') \equiv \sum_{p_t, p_z, n, s_z} \langle x | p_t, p_z, n, s_z \rangle D_s(p_t, p_z, n, s_z) \langle p_t, p_z, n, s_z | x' \rangle \delta_{n0} \delta_{s_z 1} \\ &= \int_{-\infty}^{\infty} \frac{dp_t}{2\pi} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} e^{-ip_t(t-t')} e^{ip_z(z-z')} \psi_0(x, y) \psi_0^\dagger(x', y') \frac{1}{-p_t^2 + p_z^2 - eB + M^2} \\ &= \frac{eB}{2\pi} e^{-\frac{x^2+y^2}{4l^2}} e^{-\frac{x'^2+y'^2}{4l^2}} \int_{-\infty}^{\infty} \frac{dp_t}{2\pi} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} e^{-ip_t(t-t')} e^{ip_z(z-z')} \frac{-1}{p_t^2 - p_z^2 - \mu^2}, \end{aligned} \quad (3.25)$$

with $\mu^2 \equiv M^2 - eB$ and $l \equiv \frac{1}{\sqrt{eB}}$. The factor $\frac{eB}{2\pi}$ corresponds to the Landau degeneracy.

This spatial propagator $D_s(x, x') \simeq D_{LLL}(x, x')$ of the charged vector field $A_\mu^\pm$ is similar to a 1+1 dimensional propagator with the mass μ, although $D_{LLL}(x, x')$ acts on the spatial ($\hat{L}$) component of the charge current and hence the total sign is different. In fact, in the strong constant magnetic field, the charge motion in the x, y direction is frozen, and the low-dimensionalization is realized. Actually, the correlation brought by $D_{LLL}(x, x')$ in Eq.(3.25) is localized within the order of the length $l \equiv \frac{1}{\sqrt{eB}}$ in the x, y direction.

When the strong constant magnetic field eB is near and below the critical value $eB_c \equiv M^2$, the effective mass-squared $\mu^2 \equiv M^2 - eB$ becomes small (and non-negative), and the charged vector fields $A_\mu^\pm$ behave as 1+1-dimensional quasi-massless fields, and therefore this spatial propagator $D_{LLL}(x, x')$ induces a strong correlation along the magnetic-field direction between off-diagonal charges coupled with $A_\mu^\pm$. (For $\mu^2 < 0$, i.e., $eB > M^2$, the Nielsen-Olesen instability occurs [9, 15].)

For the off-diagonal current $J_\pm^\mu$ coupled with $A_\mp^\mu$, the current-current correlation is derived as

$$S_{JJ} = \int d^4x\, d^4x'\, J_+^\mu(x) \hat{D}_{\mu\nu}(x, x') J_-^\nu(x') \simeq \int d^4x\, d^4x'\, J_+^{LLL}(x) \hat{D}_{LLL}(x, x') J_-^{LLL}(x'), \quad (3.26)$$

where $J^{LLL}_{\pm}$ is the spatial component coupled with the LLL state of $A^{\mu}_{\mp}$. The inter-charge potential $V(r)$ along $\vec{B}$ is estimated with a spiral current $J^{LLL}_{\pm} \sim Q_{\pm}\delta(x)\delta(y)\delta(z \pm \frac{r}{2})$ localized near $x = y = 0$,

$$V(r) = -Q_+Q_-\frac{eB}{4\pi}\frac{1}{\mu}e^{-\mu r} \quad \to \quad V(r) = Q_+Q_-\frac{eB}{4\pi}r \quad \text{as} \quad eB \to eB_c \equiv M^2. \tag{3.27}$$

In the limit of $eB \to eB_c - 0$, the charged vector fields $A^{\mu}_{\pm}$ become massless as $\mu^2 \equiv M^2 - eB \to +0$, and the linear potential appears along $\vec{B}$. (This strong correlation may relate to the Nilsen-Olesen instability.) In this system, the Landau degeneracy $\frac{eB}{2\pi}$ gives the dimension of the string tension.

Then, for example, for the fermion (fundamental-rep.) coupled with the SU(2) gauge field, the charged fermion makes the cyclotron motion in the strong constant magnetic field $\vec{B}$, with suffering from the strong correlation along $\vec{B}$ induced by charged vector fields $A^{\pm}_{\mu}$ (see Fig.1).

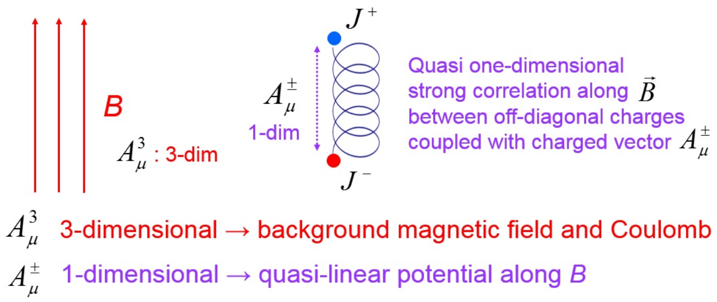

Figure 1: The SU(2) NAH theory in the strong constant magnetic field $\vec{B}$ near and below $eB_c \equiv M^2$. The 3-dimensional massless photon $A_\mu \equiv A^3_\mu$ gives the background magnetic field $\vec{B}$, and induces the cyclotron motion for charged particles. The charged vector fields $A^{\pm}_{\mu}$ become spatially quasi-one-dimensional, and induce a strong correlation along $\vec{B}$ between off-diagonal charges coupled with $A^{\pm}_{\mu}$.

4. Summary and Conclusion

We have investigated the non-abelian Higgs (NAH) theory with the Higgs triplet in a strong constant magnetic field $\vec{B}$, where the lowest-Landau-level (LLL) approximation can be used. We have found that, near and below the critical magnetic field of $eB_c \equiv M^2$, the charged vector fields $A^{\pm}_{\mu}$ behave as spatially one-dimensional quasi-massless fields, and give a strong correlation along $\vec{B}$ and eventually a linear potential $V(r) \propto eBr$ at $eB = eB_c$ between off-diagonal charges coupled with $A^{\pm}_{\mu}$. This may lead a new type of confinement caused by charged vector fields $A^{\pm}_{\mu}$.

For this confinement, although the external photon field A_μ is important for the formation the LLL state of $A^{\pm}_{\mu}$, off-diagonal charged-vector fields $A^{\pm}_{\mu}$ plays an essential role to the linear confinement potential $V(r) \propto eBr$, so that this can be called "off-diagonal (charged) dominance" (see Fig.2), in contrast to "Abelian dominance" for quark confinement in QCD in the MA gauge.

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PoS(Confinement2018)064

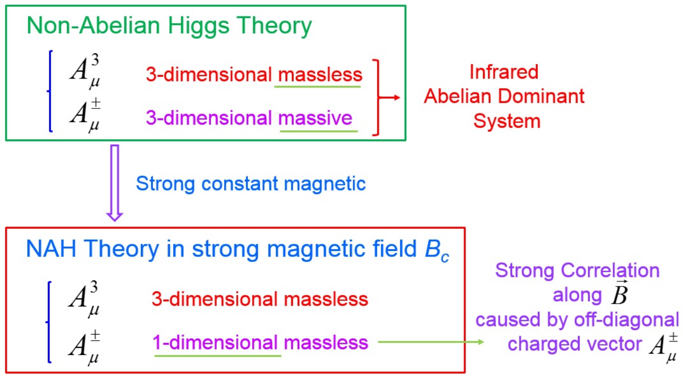

Figure 2: The SU(2) NAH theory with and without the magnetic field. At $B = 0$, the Higgs mechanism gives a large mass M for $A_\mu^\pm$ and infrared Abelian dominance. At the critical magnetic field of $eB_c = M^2$, however, off-diagonal charged-vector fields $A_\mu^\pm$ become spatially one-dimensional and massless, and play an essential role to the linear potential $V(r) \propto eBr$ between off-diagonal charges coupled with $A_\mu^\pm$.

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