

## QCD Phase Diagram with a Critical Point from Holography

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We use a holographic black hole approach to study the QCD phase diagram at finite temperature and baryon-chemical potential,  $\mu_B$ . The free parameters of our black hole model were engineered to mimic the equation of state of QCD obtained on the lattice at zero  $\mu_B$ . Once the parameters were fixed, the black hole model predicts the equation of state at finite  $\mu_B$ . In this contribution, we discuss the mapping of the black hole model in the QCD phase diagram and show that the crossover transition, between hadrons and the quark-gluon plasma, ends in a critical point where a first order line begins.

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## 1. Introduction

The QCD phase diagram, which maps the phases of Quantum Chromodynamics (QCD) in the plane of temperature ( $T$ ) and baryonic chemical potential ( $\mu_B$ ), is a topic of great interest and active investigation, but it is still mostly unknown. One of the biggest challenges is the precise mapping of the transition, with increasing  $T$  or  $\mu_B$ , between the hadronic state and the Quark-Gluon Plasma (QGP). The change of degrees of freedom from hadrons to quarks and gluons is a non-perturbative phenomenon that can only be addressed from first principles by means of numerical lattice simulations. Thanks to those calculations, the QCD equation of state is known with high precision at zero  $\mu_B$  [1, 2], and the transition from hadrons to deconfined quarks and gluons has been established to be a smooth crossover taking place in the temperature range  $T \simeq 145 - 165$  MeV [3].

It is expected that, as  $\mu_B$  increases, the crossover sharpens into a critical point that separates the crossover from a first order phase transition. The question of both the existence and location of the critical point is fundamental to understand QCD matter, but it is hard to determine theoretically due to the strongly coupled nature of the theory in the vicinity of the phase transition. Locating the critical point from first principles calculation is challenged due to the Fermi-sign problem that impedes lattice simulations at real  $\mu_B$ . However, the position of the critical point is of considerable interest, especially with the forthcoming second Beam Energy Scan at RHIC, scheduled for 2019-2020, the next fixed-target CBM project at FAIR, which is presently under construction at GSI in Germany, and the NICA facility operating in Russia. Those machines are dedicated to exploring an unprecedented high-density region of the QCD phase diagram, where the critical point could be located.

In the absence of lattice calculations at finite  $\mu_B$ , effective approaches must be used to guide the experimental search for the critical point in heavy-ion colliders. One alternative approach is the holographic correspondence [4], a well-known tool developed in string theory that has been employed to study properties of the strongly interacting QGP. The holographic model intrinsically contains an important feature of the strongly coupled QGP, which is the small value obtained for the ratio between the shear viscosity and the entropy density. This method was employed in [5] to construct black hole solutions of higher dimensional gravitational theories with thermodynamic properties similar to the QGP computed on the lattice at  $\mu_B = 0$ . The generalization of this type of model to nonzero  $\mu_B$  was done in [6, 7], where it was shown that these models can display a critical point at large baryon densities.

In this contribution, we use the holographic black hole model engineered in [8] to map the initial condition of the black hole model into the QCD phase diagram and study the transition between the gas of hadrons and the QGP state. Our model correctly reproduces the available lattice calculations for the equilibrium thermodynamic of QCD (for  $\mu_B/T \leq 2$ ) and predicts its behavior for larger values of  $\mu_B$  where the QCD critical point and a first-order transition line in the QCD phase diagram were found.

## 2. Holographic Model

The holographic black hole model is given by the action [6, 7]

$$S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left[ \mathcal{R} - \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) - \frac{1}{4}f(\phi)F_{\mu\nu}^2 \right], \quad (2.1)$$

where  $\kappa_5^2 \equiv 8\pi G_5$  is Newton's constant in five spacetime dimensions,  $\mathcal{R}$  is the Ricci scalar, and  $\phi$  is a dilaton field with a potential  $V(\phi)$ . The conserved baryonic charge is taken into account by including the Maxwell field  $A_\nu$  in the strength tensor  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ , which couple to the dilaton field through the function  $f(\phi)$  defining an Einstein-Maxwell-dilaton (EMD) model. The action in Eq. 2.1 is complemented by boundary terms which affect the evaluation of the free energy, but not the equations of motion from where the temperature, baryon chemical potential, and the entropy and baryon charge densities are calculated.

The EMD model becomes completely specified by fixing only two parameters,  $\kappa_5^2$ , and  $\Lambda$ , and two free functions,  $V(\phi)$  and  $f(\phi)$ . The dilaton potential  $V(\phi)$  is responsible of breaking the conformal symmetry of the theory in the infrared regime, and it will determine the behavior of the thermodynamics at  $\mu_b = 0$ , while the Maxwell-dilaton coupling  $f(\phi)$  will define the response of the system to a finite  $\mu_b$ . The characteristic energy scale  $\Lambda$  is used to convert physical observables, calculated on the gravity side in terms of inverse powers of the AdS radius  $L$ , to physical units expressed in powers of MeV.

The metric for the charged black holes, spatially isotropic and translationally invariant, that we are considering is described by the ansatz [6]

$$ds^2 = e^{2A(r)} [-h(r)dt^2 + d\vec{x}^2] + \frac{dr^2}{h(r)}. \quad (2.2)$$

The EMD model produces four coupled second order differential equations of motion given in terms of the fields  $A=A(r)$ ,  $h=h(r)$ ,  $\phi=\phi(r)$ , and  $\Phi=\Phi(r)$ , where  $\Phi(r)$  is defined from the zero component of the Maxwell field ( $A_\nu dx^\nu = \Phi(r)dt$ ). A black hole solution is obtained by integrating the equations of motion from its horizon, at  $r=r_H$ , to the asymptotic AdS boundary at  $r \rightarrow \infty$  where, according to the holographic dictionary, the thermodynamics of the four-dimensional gauge theory can be computed. Thus, it is necessary to define boundary conditions at the horizon of the EMD fields. These boundaries are parametrized in terms of two initial values:  $\phi_0$  (the value of the field  $\phi$  at the horizon) and  $\Phi_1$  (the value of the electric field  $\Phi$  perpendicular to the horizon) [6, 7, 8]. The pair  $(\phi_0, \Phi_1)$  defines a black hole solution that is mapped to the QCD phase diagram from where the entropy and charge density of the system are computed.

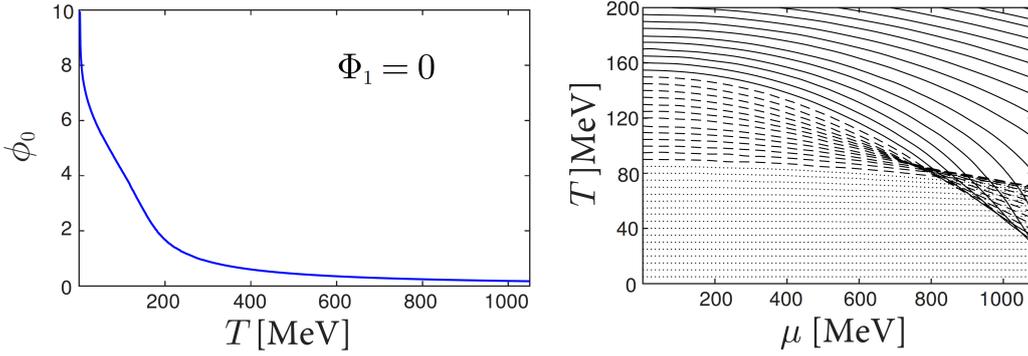
The model parameters and free functions of our model were engineered in [8] by matching the black hole solutions with lattice QCD results calculated at  $\mu_B = 0$ . In particular, the scalar potential  $V(\phi)$  is constructed to reproduce the entropy density in [1], and the Maxwell-dilaton coupling  $f(\phi)$  is constructed to reproduce the second baryon susceptibility  $\chi_2$  in [9] to ensure that the charge in our model is baryonic.

### 2.1 Mapping the QCD Phase Diagram

The mapping from the holographic initial conditions  $(\phi_0, \Phi_1)$  of a black hole to the QCD phase diagram  $(T, \mu_b)$  is non-linear. The left panel of Figure 1 shows the black hole solutions for  $\Phi_1 = 0$ ,

which maps to the  $T$ -axis of the QCD phase diagram. To populate the QCD phase diagram with black hole solutions, we fix  $\phi_0$  to produce equally spaced intervals of  $T$ . Then, we vary  $\Phi_1$  from zero to the maximum bound imposed by requiring an asymptotically AdS black hole [6]. The right panel in Figure 1 shows the lines of constant  $\phi_0$  evolving in the  $\mu_B$  direction as  $\Phi_1$  increases.

One can distinguish between three kinds of lines: the dotted lines that do not cross each other; the dashed lines in the middle of the plane that cross some of the dotted lines; and the solid lines on the top of the plane that cross some of the dotted and dashed lines. As it was explained in [10], the dashed lines in the overlapping region are thermodynamic unstable producing a negative constant heat or negative second baryon susceptibility. From the region where the dashed lines and the solid lines coexist, it was found that the entropy  $S$  is higher in the dotted lines than in the dashed lines. Therefore, the system will maximize the entropy by moving from the lower plane (dotted lines on the right panel in Figure 1) to the higher plane (solid lines on the right panel in Figure 1), and when the overlapping between those two regions begin, a discontinuity in the entropy and baryon density is developed producing a first order phase transition. This transition extends along the line defined by the boundary of those two regions (dotted and solid lines). The point where the overlapping of the planes begins ( $T^c = 89$  MeV,  $\mu_B^c = 724$  MeV) was identify as the location of the QCD critical point in our holographic model.

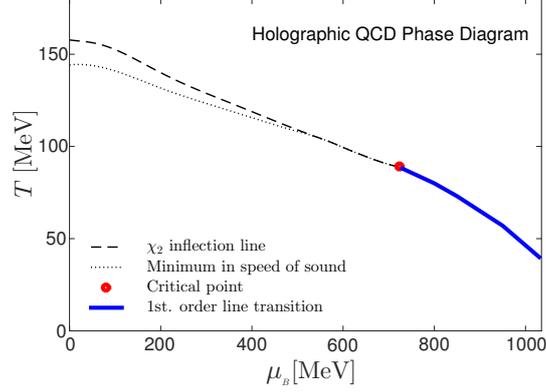


**Figure 1:** Mapping of the initial values of the black holes ( $\phi_0, \Phi_1$ ) to the QCD phase diagram ( $T, \mu_B$ ). Left panel:  $T$ -dependence of  $\phi_0$  when  $\Phi_1 = 0$  ( $\mu_B = 0$ ). Right panel: lines of constant  $\phi_0$  evolving in the  $\mu_B$  direction as  $\Phi_1$  increases.

Our holographic QCD phase diagram is shown in Figure 2. There, the crossover is illustrated by two observables that are sensitives to the change on degrees of freedom from hadrons to quarks and gluons: the inflection point of  $\chi_2$  (dashed curve) and the minimum in the speed of sound squared  $c_s^2$  (dotted curve). The critical point on the QCD phase diagram, which is shown with a dot, is located at  $T_{\text{CEP}} = 89$  MeV and  $\mu_B^{\text{CEP}} = 724$  MeV. Passing the critical point, the first order phase transition line is shown with a blue curve, which extends to higher  $\mu_B$ -values with a negative curvature.

### 3. Conclusions

We have shown the mapping of the holographic black hole model studied in [8] into the QCD phase diagram. The solutions of the black hole model were parametrized by two initial values



**Figure 2:** QCD Phase diagram obtained from the Black Hole Model. The crossover is signaled by two observables: the inflection point of  $\chi_2$  (dashed curve) and the minimum in the speed of sound squared  $c_s^2$  (dotted curve). The CEP is shown with a red dot. The first order phase transition is shown as a blue curve.

( $\phi_0, \Phi_1$ ). The lines of constant  $\phi_0$  evolve to higher values of  $\mu_B$  as  $\Phi_1$  increases. Some of those lines bent to lower values of  $T$  producing an overlap of black hole solutions. By selection the thermodynamic stable solutions that maximizes the entropy (see [10]), we found that the beginning of the overlapping region, at the point ( $T^c = 89$  MeV,  $\mu_B^c = 724$  MeV), marks the location of the critical point of QCD, and the onset of a first order phase transition line, which extends along the lower- $T$  boundary of the overlapping region (see left panel in Figure 1). Once we identified the critical point and first order transition line, we showed the holographic QCD phase diagram in Figure 2. The crossover region between hadrons and the QGP was indicated by the inflection point of  $\chi_2$  and the minimum in the speed of sound squared  $c_s^2$ .

Finally, we remark that our black hole model (Ref. [8]) has provided a prediction of the thermodynamics of QCD at finite  $\mu_B$  that agrees with the available lattice calculations, and also predicts the existence and location of a critical point in the QCD phase diagram. Having a critical behavior, this model can be used as a theoretical tool to guide the experimental search of the critical point in heavy-ion colliders.

## References

- [1] S. Borsanyi, et al. “Full result for the QCD equation of state with 2+1 flavors”, *Phys. Lett.* B730 (2014), 99-104.
- [2] A. Bazavov, et al. “Equation of state in ( 2+1 )-flavor QCD”, *Phys. Rev.* D90 (2014), 094503.
- [3] Y. Aoki, et al. “The Order of the quantum chromodynamics transition predicted by the standard model of particle physics”, *Nature* 443 (2006), 675-678.
- [4] J. M. Maldacena “The Large N limit of superconformal field theories and supergravity”, *Int. J. Theor. Phys.* 38 (1999), 1113-1133.
- [5] S. S. Gubser and A. Nellore. “Mimicking the QCD equation of state with a dual black hole”, *Phys. Rev.* D78, (2008), 086007.

- [6] O. DeWolfe, S. S. Gubser, and C. Rosen. "A holographic critical point", *Phys. Rev. D*83 (2011), 086005.
- [7] O. DeWolfe, S. S. Gubser, and C. Rosen. "Dynamic critical phenomena at a holographic critical point", *Phys. Rev. D*84 (2011), 126014.
- [8] R. Critelli, et al. "Critical point in the phase diagram of primordial quark-gluon matter from black hole physics", *Phys. Rev. D*96 (2017), 096026.
- [9] S. Borsanyi, et al. "Fluctuations of conserved charges at finite temperature from lattice QCD", *JHEP* 01 (2012), 138.
- [10] I. Portillo "Locating the QCD critical point using holographic black holes." *Proceedings* 10, (2019) 40.