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On correlators of Reggeon fields in high energy QCD

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We discuss Dyson-Schwinger hierarchy of the equations for the correlators of reggeized gluon fields in the framework of Lipatov's high energy QCD effective action formalism and RFT corrections to the propagator of reggeized gluons.

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1. Lipatov's effective action and correlators of Reggeon fields

The Lipatov's effective action for reggeized gluons $\mathscr{B}_{\pm}$, formulated as Regge Field Theory (RFT), can be obtained by an integration out the gluon fields v in the generating functional for the $S_{eff}[v,\mathscr{B}]$:

$$e^{\imath\Gamma[\mathscr{B}]} = \int Dv e^{\imath S_{eff}[v,\mathscr{B}]} \tag{1.1}$$

where

$$S_{eff} = -\int d^4x \left(\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + tr\left[\left(\mathscr{T}_+(v_+) - \mathscr{B}_+ \right) j^+_{reg} + \left(\mathscr{T}_-(v_-) - \mathscr{B}_- \right) j^-_{reg} \right] \right), \tag{1.2}$$

with

$$\mathscr{T}_{\pm}(v_{\pm}) = \frac{1}{g}\partial_{\pm} O(v_{\pm}) = v_{\pm} O(v_{\pm}), \;\; j^{\pm}_{reg\,a} = \frac{1}{C(R)}\partial_i^2 \mathscr{B}^{\pm}_a , \tag{1.3}$$

here $C(R)$ is eigenvalue of Casimir operator in the representation R, $tr(T^aT^b) = C(R)\,\delta^{ab}$ see [1, 2]. The form of the Lipatov's operator O (and correspondingly $\mathscr{T}$) depends on the particular process of interests, in the simplest case it has th eform of the P ordered exponential for the longitudinal gluon fields in an arbitrary representation:

$$O(v_{\pm}) = P e^{g\int_{-\infty}^{x^{\pm}} dx^{\pm} v_{\pm}(x^+,x^-,x_{\perp})}, \;\; v_{\pm} = \imath T^a v^a_{\pm} . \tag{1.4}$$

The effective action Γ of the interactions of reggeized gluons, calculated to one-loop precision in [2], has the following form:

$$\Gamma = \sum_{n,m=1} \left(\mathscr{B}^{a_1}_+ \cdots \mathscr{B}^{a_n}_+ \left(K^{+\cdots+}_{-\cdots-} \right)^{a_1\cdots a_n}_{b_1\cdots b_m} \mathscr{B}^{b_1}_- \cdots \mathscr{B}^{b_m}_- \right) = -\mathscr{B}^a_{+x}\,\partial_i^2\,\mathscr{B}^a_{-x} + \mathscr{B}^a_{+x}\left(K^{ab}_{xy} \right)^+_- \mathscr{B}^b_{-y} + \cdots , \tag{1.5}$$

where $\mathscr{B}_{\pm}$ are the reggeized gluon fields and shorthand notations for the integration over the variables in the action were used. The vertices in the action depend on some rapidity interval η which is an analog of the ultraviolet cut-off in the relative longitudinal momenta arising in the regularization of teh corresponding integrals in Eq. (1.2). Physically it determines the value of the cluster of the particles in the Lipatov's effective action approach, see [1]. The effective vertices (kernels[1]) K in Eq. (1.5) represents the processes of multi-Reggeon interaction in t-channel of the high energy scattering amplitude. For the calculation of the correlators of the Reggeon $\mathscr{B}_{\pm}$ fields, we will use the following generating functional for Reggeon fields:

$$Z[J] = \int D\mathscr{B} \exp\left(\imath\Gamma[\mathscr{B}] - \imath\int d^4x J^a_- \mathscr{B}^a_+ - \imath\int d^4x J^a_+ \mathscr{B}^a_- \right), \tag{1.6}$$

see [2], with some auxiliary currents $J_{\pm}$ introduced. The Schwinger-Dyson equations for the correlators we obtain now taking derivative of the field's variation of $Z[J]$ in respect to the currents and taking them equal to zero at the end. In doing so we obtain the following general equation for the arbitrary correlators of the theory:

$$< \mathscr{B}^a_{\pm}\,\mathscr{B}^{a_1}_{\pm} \cdots \mathscr{B}^{a_m}_{\pm} > = \sum_{n=1} \left(\hat{K}(\eta) \right)^{a\,b_1\cdots b_n} < \mathscr{B}^{b_1}_{\pm} \cdots \mathscr{B}^{b_n}_{\pm}\,\mathscr{B}^{a_1}_{\pm} \cdots \mathscr{B}^{a_m}_{\pm} > . \tag{1.7}$$

[1] In the language of high-energy perturbative QCD these vertices are BFKL-like kernels of the integro-differential equations for the objects of interests or, equivalently, they can be considered as analog of the different parts of a Hamiltonian in Balitsky-JIMWLK approach, see in [3, 4, 5, 6].

Taking the derivative on rapidity from the both sides of the equation we will obtain the BFKL like equations for the correlators, this is the way how the BFKL calculus is arising in the formalism of Lipatov's effective action.

2. On correlator of two reggeized gluons

The correlator of two reggeized gluons to leading RFT order was obtained in [2]:

$$\partial^2_{\perp x} < A^{a}_{+}(x^{+},x_{\perp}) A^{a_1}_{-}(y^{-},y_{\perp}) >= -\imath\, \delta^{a a_1}\, \delta(x^{+})\, \delta(y^{-})\, \delta^{2}(x_{\perp}-y_{\perp}) +$$
$$+ \int d^2 z_{\perp} \int dz^{+}_{1} d^2 z_{1\perp} K^{b_1 b_2 a}_{++-}(x^{+},z_{\perp};z^{+}_{1},z_{1\perp};x_{\perp}) < A^{b_1}_{+}(x^{+},z_{\perp}) A^{b_2}_{+}(z^{+}_{1},z_{1\perp}) A^{a_1}_{-}(y^{-},y_{\perp}) > +$$
$$+ \int d^2 z_{\perp} \int dz^{-}_{1} d^2 z_{1\perp} K^{b_1 b_2 a}_{+--}(z_{\perp};z^{-}_{1},z_{1\perp};x^{-},x_{\perp}) < A^{b_1}_{+}(x^{+},z_{\perp}) A^{b_2}_{-}(z^{-}_{1},z_{1\perp}) A^{a_1}_{-}(y^{-},y_{\perp}) >, \tag{2.1}$$

where the following representation of the Reggeon fields was applied:

$$\mathscr{B}_{+}(x^{+},x^{-},x_{\perp}) = A_{+}(x^{+},x_{\perp}) + \mathscr{D}_{+}(x^{+},x^{-},x_{\perp}),\;\; \mathscr{D}_{+}(x^{+},x^{-}=0,x_{\perp}) = 0 \tag{2.2}$$

and

$$\mathscr{B}_{-}(x^{+},x^{-},x_{\perp}) = A_{-}(x^{-},x_{\perp}) + \mathscr{D}_{-}(x^{+},x^{-},x_{\perp}),\;\; \mathscr{D}_{-}(x^{+}=0,x^{-},x_{\perp}) = 0\,. \tag{2.3}$$

Using expressions for the three fields correlators:

$$\partial^2_{\perp x} < A^{a}_{+}(x^{+},x_{\perp}) A^{a_1}_{+}(y^{+},y_{\perp}) A^{a_2}_{-}(z^{-},z_{\perp}) >= \tag{2.4}$$
$$= \int d^2 w_{\perp} dw^{-}_{1} d^2 w_{1\perp} K^{a_4 a a_3}_{+--}(w_{\perp};x^{-},x_{\perp};w^{-}_{1},w_{1\perp}) < A^{a_4}_{+}(x^{+},w_{\perp}) A^{a_1}_{+}(y^{+},y_{\perp}) A^{a_2}_{-}(z^{-},z_{\perp}) A^{a_3}_{-}(w^{-}_{1},w_{1\perp}) >$$

and

$$\partial^2_{\perp x} < A^{a}_{+}(x^{+},x_{\perp}) A^{a_1}_{-}(y^{-},y_{\perp}) A^{a_2}_{-}(z^{-},z_{\perp}) >= \tag{2.5}$$
$$= \int d^2 w_{\perp} dw^{-}_{1} d^2 w_{1\perp} K^{a_3 a_4 a}_{++-}(x^{+},w_{\perp};w^{+}_{1},w_{1\perp};x_{\perp}) < A^{a_3}_{+}(x^{+},w_{\perp}) A^{a_4}_{+}(w^{+}_{1},w_{1\perp}) A^{a_1}_{-}(y^{-},y_{\perp}) A^{a_2}_{-}(z^{-},z_{\perp}) >$$

and different expressions for the correlator of four Reggeon fields:

$$< A^{a_4}_{+}(x^{+},w_{\perp}) A^{a_1}_{+}(y^{+},y_{\perp}) A^{a_2}_{-}(z^{-},z_{\perp}) A^{a_3}_{-}(w^{-}_{1},w_{1\perp}) >= \tag{2.6}$$
$$= \imath \delta^{a_4 a_3} \delta(x^{+}) \delta(w^{-}_{1}) D_0(w_{\perp},w_{1\perp}) < A^{a_1}_{+} A^{a_2}_{-} > + \imath \delta^{a_4 a_2} \delta(x^{+}) \delta(z^{-}) D_0(w_{\perp},z_{\perp}) < A^{a_1}_{+} A^{a_3}_{-} >$$

or

$$< A^{a_4}_{+}(x^{+},w_{\perp}) A^{a_1}_{+}(y^{+},y_{\perp}) A^{a_2}_{-}(z^{-},z_{\perp}) A^{a_3}_{-}(w^{-}_{1},w_{1\perp}) >= \tag{2.7}$$
$$= \imath \delta(x^{+}) \delta(w^{-}_{1}) D^{a_4 a_3}(w_{\perp},w_{1\perp}) < A^{a_1}_{+} A^{a_2}_{-} > + \imath \delta(x^{+}) \delta(z^{-}) D^{a_4 a_2}(w_{\perp},z_{\perp}) < A^{a_1}_{+} A^{a_3}_{-} >,$$

we obtain different answers for the correction to the usual Reggeon propagator. The functions D_0 and D in the expressions are usual bare and full propagators of reggeized gluons:

$$< A^{a}_{+}(x^{+},x_{\perp}) A^{b}_{-}(y^{-},y_{\perp}) >= \imath\, \delta(x^{+})\, \delta(y^{-})\, \delta^{ab}\, D(x_{\perp},y_{\perp}) \tag{2.8}$$

whereas the following correlator can be considered as correction to the correlator above:

$$< \mathscr{D}^{a}_{+}(x^{+},x^{-},x_{\perp}) A^{a_1}_{-}(y^{-},y_{\perp}) >= \imath \mathscr{G}^{a a_1}_{+}(x^{+},x^{-},x_{\perp};y^{-},y_{\perp})\,. \tag{2.9}$$

Now, for the different representations of the four reggeon correlator, Eq. (2.6) and Eq. (2.7), the different answers for the $\mathscr{G}_{+}^{aa_1}$ correction are obtained. We have correspondingly:

$$\tilde{\mathscr{G}}_{+}^{aa_1}(p_+,p_\perp,p_-;x^-;Y) = \frac{2\,\delta^{aa_1}}{p_\perp^2}\left(1 - e^{\varepsilon(p_\perp^2)Y} - \int_0^{-\varepsilon(p_\perp^2)Y}\frac{dy}{y}\left(e^{-y}-1\right)\right)\left(\tilde{G}^{-0}_{x^-0} - \tilde{G}^{-0}_{0x^-}\right) \tag{2.10}$$

and

$$\tilde{\mathscr{G}}_{+}^{aa_1}(p_+,p_\perp,p_-;x^-;Y) = \frac{2\,\delta^{aa_1}}{p_\perp^2}\left(e^{\varepsilon(p_\perp^2)Y} - e^{2\,\varepsilon(p_\perp^2)Y} - \int_{-\varepsilon(p_\perp^2)Y}^{-2\,\varepsilon(p_\perp^2)Y}\frac{dy}{y}\left(e^{-y}-1\right)\right)\left(\tilde{G}^{-0}_{x^-0} - \tilde{G}^{-0}_{0x^-}\right) \tag{2.11}$$

where

$$\varepsilon(p_\perp^2) = -\frac{\alpha_s N}{4\pi^2}\int d^2k_\perp\,\frac{p_\perp^2}{k_\perp^2\,(p_\perp - k_\perp)^2} \tag{2.12}$$

is the trajectory of the propagator of reggeized gluons. The new propagator of reggeized gluons now acquires the following form:

$$D^{ab}(x^+,x^-,x_\perp;y^+,y^-,y_\perp;Y) = D^{ab}(x_\perp,y_\perp;Y) + \mathscr{G}_{+}^{aa_1}(x^+,x^-,x_\perp;y^-,y_\perp;Y) + \mathscr{G}_{-}^{aa_1}(x^+,x_\perp;y^+,y^-,y_\perp;Y)\,. \tag{2.13}$$

with $\mathscr{G}_{-}^{aa_1}$ as a correction corresponding to $\mathscr{D}_-$ field in Eq. (2.3).

3. Conclusion

The formalism of the Lipatov's effective action, based on the notion of the reggeization of the main QCD degrees of freedom, is a powerful tool applied for the calculations of amplitudes of different physical processes in high-energy QCD, see [7, 8]. The main "brick" of the approach is the Eq. (2.8) propagator of reggeized gluons, which form was obtained many years ago, [9]. In the contribution we demonstrate that the approach formulated as RFT provides non-linear unitarity corrections to the propagator which are not related to the propagator's trajectory, Eq. (2.12), but which are non-linear in their origin. Namely, the important future of the both expressions Eq. (2.10) and Eq. (2.11) is that they contribute to the final propagator with the sign opposite to the sign of perturbative terms obtained from the expansion of the exponential with the trajectory function at the each perturbative order. It means that these corrections are unitary ones but the unitarity of the propagator is achieved by adding of the non-linear terms to the usual propagator and not by the perturbative corrections to the gluon's trajectory, it is also a new result of the formalism. It turns out also that the expressions obtained are depend as well on the longitudinal coordinates that violates the only transverse coordinates dependence of the usual Eq. (2.8) propagator. The change of the propagator's form, in general, depends on the way of truncaton of the hierarchy of the correlators, it is more drastic if we account the one loop QCD correction in the expressions for the three and four Reggeon correlators, see Eq. (2.10) in comparison to Eq. (2.11).

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