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From saturation to high p_t jets: toward a unified picture of particle production at all transverse momenta

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We propose a new formalism that aims to unify particle production in high energy hadron/nucleus collisions; from collinear factorization approach in pQCD at high p_t (large x) to Color Glass Condensate framework which describes multi-particle production at low p_t (small x). To do so we consider scattering of a quark projectile from a classical color field representing small x gluons of the target to all order in the field and a dynamical gluon field representing large x gluons of the target proton or nucleus. We then compute the "tree-level" scattering cross section. We discuss the effects of the one-loop corrections to our tree-level cross section which is expected to lead to leading twist pQCD and DGLAP evolution equation of the cross section at high p_t and the JIMWLK evolution equation of Color Glass Condensate at low x.

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1. Introduction

Perturbative QCD has been extremely successful in describing high p_t particle production in hadronic collisions at high energies. It is based on twist expansion of non-local operators and their renormalization, typically referred to as collinear factorization [1]. In this formalism the single inclusive particle (here, a hadron to be specific) production cross section is written as a convolution of distribution functions $f(x,Q^2)$, the probability a find a parton (quark or gluon) inside a hadron carrying a momentum fraction x of the hadron energy, with parton-parton scattering cross section $\frac{d\sigma}{dt}$, and a fragmentation function $D(z,Q^2)$ which describes how a parton becomes a hadron which carries fraction z of the parton energy. Symbolically this is written as

$$E\frac{d\sigma}{d^3p} = f_1(x_1,Q^2)\otimes f_2(x_2,Q^2)\otimes \frac{d\sigma}{dt}\otimes D(z,Q^2) \tag{1.1}$$

The distribution and fragmentation functions are expectation values of bi-local operators involving quark and gluon fields between the appropriate hadronic states. These operators are in principle divergent and their renormalization leads to the scale (Q^2) dependence of the distribution and fragmentation functions.

The essence of this approach is factorization of short distance physics, computable in perturbation theory, and long distance physics which is non-perturbative but universal. The non-perturbative ingredients of the production cross section are the parton distribution and fragmentation functions which then evolve (depend on the scale) as prescribed by QCD evolution equations, commonly known as the DGLAP evolution equation [2]. The hard part, parton-parton scattering cross section $\frac{d\sigma}{dt}$ is process-dependent but can be computed, in principle, to any order in QCD coupling constant α_s.

The collinear factorization formalism is valid in the limit when $p_t \to \infty$ and receives corrections at any finite p_t. These corrections are power suppressed but generically break the factorization signified by appearance of new operators in the cross section. This is also typically the case when one considers more exclusive observables (rather than single inclusive particle production). Furthermore, in the limit of very high energy new kinematic logs of the form $\log 1/x$ appear which lead to a rapid rise of parton distribution functions with $1/x$ which results in hadron being a highly occupied/dense state of partons (mainly gluons) at small x [3]. This signifies the breakdown of QCD-improved parton model, essential for collinear factorization.

An alternative to collinear factorization approach, known as the Color Glass Condensate (CGC) formalism [4], aims to include the leading high energy effects while at fixed and not so large transverse momentum. This is the so-called Regge limit where $x = \frac{Q^2}{S} \to 0$. In this formalism a target hadron (or nucleus) is treated as a classical color field generated coherently by many small x gluons. To compute a cross section one then considers scattering of two classical color fields which then produce a color field in the forward light cone which subsequently decays into many partons which eventually hadronize. Quantum corrections of the type $\alpha_s \log 1/x$ are then incorporated into the cross section via JIMWLK evolution [5] of the ingredients of the cross section. The main drawback of this formalism is that one can not analytically solve for the classical field equations in the forward light cone and one has to resort to numerical methods. Nevertheless this formalism has been very useful in understanding many-body aspects of parton production in high

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energy heavy ion collisions, for example at RHIC and the LHC. It is quite common now to use the results of CGC calculations for number and energy density of produced partons as the initial conditions for subsequent evolution of the produced system, a Quark Gluon Plasma, as described by hydrodynamic models.

However, there are several processes for which analytical results for particle production in the CGC formalism can be worked out. These are generically referred to as dilute-dense collisions, of which high energy proton-nucleus collisions is a prime example. In this case one treats the classical color field of the proton as weak and expands it to the first order while keeping the field of the target nucleus to all orders. The ingredients of the cross section in this case are known as intrinsic gluon distribution functions and are related to the standard gluon distribution function via an integral over the transverse momentum. Here one does not have a strict factorization in the sense that different operators appear in various processes, nevertheless, the situation is still manageable since all these operators satisfy the JIMWLK equation.

On the other hand and in the kinematic region where the proton projectile wave function is probed at not very small x, then it is more appropriate to use another formalism known as the hybrid approach [6]. In this formalism one treats the proton as a collection of quasi-free partons, as in the QCD improved parton model while treating the target nucleus as a strong color field. One then considers scattering of partons of the proton on the strong color field of the target nucleus. At the leading order in the coupling constant, this is just the standard eikonal approximation in Quantum Mechanics, generalized to include color degree of freedom. This approach is most suitable for particle production in the forward rapidity region where one probes large x degrees of freedom of the proton and small x fields of the target. The scattering amplitude (for quark scattering) can be written as

$$i\mathcal{M}_{eik}(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\not{n}\int d^2x_t\, e^{-i(q_t-p_t)\cdot x_t}\,[V(x_t)-1]\,u(p) \tag{1.2}$$

where

$$V(x_t) \equiv \hat{P}\exp\left\{ig\int_{-\infty}^{+\infty} dx^+ A^-_a(x^+,x_t)\,t_a\right\} \tag{1.3}$$

is an infinite Wilson line which describes successive multiple scatterings of the projectile quark with light cone energy p^+ on the color field A^- of the target field in the light cone gauge $A^+ = 0$. Due to the high energy approximation the transverse coordinate of the projectile quark remains fixed, allowing only small angle deflection of the projectile. As such, eikonal approximation is valid only when the scattered projectile has small transverse momentum. Here $\not{n} = n^\mu\,\gamma_\mu$ with n^μ a light like vector which points in the negative light cone direction. The scattering cross section can then be written in terms of a quark anti-quark dipole cross section S

$$S(x_t,y_t) = \frac{1}{N_c}\,Tr V(x_t)\,V^\dagger(y_t) \tag{1.4}$$

We note that this scattering cross section is energy (equivalently x or rapidity) independent at this level, in analogy with tree level parton-parton scattering cross section in pQCD which is independent of the hard scale Q^2. Here the energy dependence arise from considering one loop corrections to S due to gluon radiation. The equation that describes this energy or x dependence is known as the JIMWLK/BK evolution equation and can be symbolically written as

$$\frac{d}{d\log 1/x} S(x_t,y_t) \sim \int d^2z_t\, \mathcal{K}(x_t,y_t,z_t) \Big[S(x_t,z_t) + S(z_t,y_t) - S(x_t,y_t) - S(x_t,z_t)\, S(z_t,y_t)\Big] \quad (1.5)$$

It describes "evolution" of a quark anti-quark dipole by radiating a gluon, real or virtual, that in the large N_c can be itself thought of a quark anti-quark dipole which hence results in having two dipoles. The various terms above describe the scattering of one, both or none of the dipoles from the target. This equation can be solved analytically in some limits but most phenomenological applications use a numerical solution to this equation, now available at the Next to Leading Log accuracy.

Since the underlying physics of the scattering above is eikonal approximation, this approach is limited in the kinematics where it can be applied. For instance, this approximation breaks down when the scattering is at large angle, or equivalently at high transverse momentum. Therefore one needs to go beyond eikonal approximation if one wants to incorporate the physics of hard scattering and DGLAP evolution into the Color Glass Condensate formalism. In [7] we proposed a formalism that aims to accomplish that by including scattering not only from small x gluons of the target described by the A^- background field, but also from large x gluons. To do this we consider the case when the projectile quark can undergo multiple scatterings from the soft fields before, after or both before and after scattering from a large x gluon field of the target. This amplitude is given by

$$\begin{aligned} i\mathcal{M}_1 &= \int d^4x\, d^2z_t\, d^2\bar{z}_t \int \frac{d^2k_t}{(2\pi)^2}\frac{d^2\bar{k}_t}{(2\pi)^2}\, e^{i(\bar{k}-k)x}\, e^{-i(\bar{q}_t-\bar{k}_t)\cdot\bar{z}_t}\, e^{-i(k_t-p_t)\cdot z_t} \\ &\quad \bar{u}(\bar{q}) \left[\overline{V}_{AP}(x^+,\bar{z}_t)\, \not{n}\, \frac{\not{\bar{k}}}{2\bar{k}^+} \left[ig\not{A}(x)\right] \frac{\not{k}}{2k^+} \not{n}\, V_{AP}(z_t,x^+)\right] u(p) \end{aligned} \quad (1.6)$$

with $k^+ = p^+, k^- = \frac{k_t^2}{2k^+}$, $\bar{k}^+ = \bar{q}^+, \bar{k}^- = \frac{\bar{k}_t^2}{2\bar{k}^+}$ and similarly for $l,\bar{l}$. Here $V,\bar{V}$ are anti path-ordered semi-infinite Wilson lines resumming multiple soft scatterings from the target in the eikonal approximation while the scattering from the hard field $A(x)$ is exact.

Furthermore and since the hard field $A^\mu(x)$ representing large x gluons of the target is a dynamical field it can itself scatter from the soft background field. This amplitude is given by

$$\begin{aligned} i\mathcal{M}_2 &= \frac{2i}{(p-\bar{q})^2} \int d^4x\, e^{i(\bar{q}^+-p^+)x^- - i(\bar{q}_t-p_t)\cdot x_t}\, \bar{u}(\bar{q}) \Big[(ig t^a) \left[\partial_{x^+} U^\dagger_{AP}(x_t,x^+)\right]^{ab} \\ &\quad \left[n\cdot(p-\bar{q})\not{A}_b(x) - (p-\bar{q})\cdot A_b(x)\not{n}\right] \Big] u(p) \end{aligned} \quad (1.7)$$

where

$$\begin{aligned} \partial_{x^+}\left[U^\dagger_{AP}(x_t,x^+)\right]^{ab} &= (if^{bca})\,[igS_c(x^+,x_t)] \\ &+ (if^{bce})\,(if^{eda}) \int dx_1^+\, \theta(x^+-x_1^+) [[igS_c(x^+,x_t)]\, [igS_d(x_1^+,x_t)] \\ &+ (if^{bch})\,(if^{gdf})\,(if^{fea}) \int dx_1^+\, dx_2^+\, \theta(x^+-x_1^+)\, \theta(x_1^+-x_2^+) \\ &\quad [[igS_c(x^+,x_t)]\, [igS_d(x_1^+,x_t)]\, [[igS_c(x_2^+,x_t)] + \cdots\cdots, \end{aligned} \quad (1.8)$$

and the soft color field S_a is defined via $A_a^- \equiv n^- S_a$. To proceed further one also needs to include the case when both the final (or initial) state quark and the dynamical hard gluon interact with the soft background field. It can be shown that the amplitude when both the initial state quark and the hard dynamical gluon interact with the soft background field is zero. On the other hand one gets a contribution from the case when the final state quark and the hard gluon both multiply scatter from the soft background field. This contribution is given by

$$
\begin{aligned}
i\mathcal{M}_3 = -2\,i \int d^4x\, d^2\bar{x}_t\, d\bar{x}^+ \frac{d^2\bar{p}_{1t}}{(2\pi)^2}\, e^{i(\bar{q}^+ - p^+)x^-}\, e^{-i(\bar{p}_{1t}-p_t)\cdot x_t}\, e^{-i(\bar{q}_t - \bar{p}_{1t})\cdot \bar{x}_t} \\
\bar{u}(\bar{q}) \left[\left[\partial_{\bar{x}^+} \overline{V}_{AP}(\bar{x}^+,\bar{x}_t)\right] \not{n} \not{\bar{p}}_1 \,(igt^a)\, \left[\partial_{x^+} U^\dagger_{AP}(x_t,x^+)\right]^{ab} \right. \\
\left. \frac{\left[n\cdot(p-\bar{q})\not{A}^b(x) - (p-\bar{p}_1)\cdot A^b(x)\not{n}\right]}{\left[2n\cdot\bar{q}\, 2n\cdot(p-\bar{q})\, p^- - 2n\cdot(p-\bar{q})\,\bar{p}_{1t}^2 - 2n\cdot\bar{q}\,(\bar{p}_{1t}-p_t)^2\right]} \right] u(p)
\end{aligned}
\tag{1.9}
$$

The full amplitude is then given by

$$
i\mathcal{M} = i\mathcal{M}_{eik} + i\mathcal{M}_1 + i\mathcal{M}_2 + i\mathcal{M}_3 \tag{1.10}
$$

which can be used to obtain the cross section for scattering of a quark in a proton projectile from both small and large x gluons of the target nucleus (or proton).

The next step forward is to compute the one-loop correction to this "tree-level" cross section. This would bring in the standard logarithmic divergences present in DGLAP (Q^2) and JIMWLK ($1/x$) and hence result in a more general expression for the production cross section which would reduce to collinear factorized ones in the high Q^2 limit and to the CGC expression in the small x limit. Toward this one first needs to repeat this calculation for the case of a gluon scattering from the target [8] which should be a straightforward continuation of this work. As a warm up one can start with radiation of a photon from the quark lines which is a much simpler process. This work is in progress and will be reported on elsewhere.

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