

## Particle acceleration by the shock waves propagating in a non-uniform medium

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It is widely recognized that cosmic rays are accelerated in supernova remnants by the diffusive shock acceleration (DSA). Although the standard DSA predicts the energy spectrum of  $E^{-2}$ , several observations suggest a deviation from  $E^{-2}$ . To explain such spectral deviations, we introduce a density fluctuation of interstellar medium (ISM) to the DSA. Particle acceleration is examined by Monte Carlo simulations. The simulations show that the density fluctuation of ISM does change the energy spectrum of accelerated particles from  $E^{-2}$  and the effect is remarkable when the length scale of the fluctuation is comparable to the mean free path of the accelerated particle.

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## 1. Introduction

Cosmic rays (CRs) observed on the earth have a quite large range of power-law energy spectra extending from  $10^{10}$  eV to  $10^{20}$  eV. This surprising feature of CRs has been believed to be explained by a common mechanism which can yield a power-law spectrum and accelerate particles to such high energies. The *diffusive shock acceleration (DSA)* is widely recognized as one of the most convincing theories. The advantage of DSA is it contains only a few assumptions and generally predicts a single power-law spectrum with a spectral index,  $-2$ , for the strong shock limit  $M_1 \gg 1$ , where  $M_1$  is the (sound) Mach number of the shock upstream region. In particular, supernova remnants (SNRs) are believed to accelerate Galactic CRs (GCRs) because they universally contain shock waves produced by the explosion of stars, which are necessary for DSA and the released energy by the explosion is sufficient to maintain the GCRs.

Although the picture that DSA in SNRs accelerates GCRs is supported by many observations, there are some problems. We here focus on a problem for the spectral index of accelerated particles. For instance, radio observations of many SNRs suggest that the spectral indexes of accelerated electrons inside the SNRs are not just 2 [3]. Although the deviation from just 2 is not large, about 10% level, understanding the origin of the spectral deviation would help to understand other important problems.

The standard theory of DSA assumes that the upstream region of shock is uniform. However, in reality, there are density fluctuations in the interstellar medium (ISM). In this study, we investigate effects of the density fluctuation of the ISM on DSA. Particle motions are modeled as an isotropic scattering in the fluid rest frame which is given by the linear analysis of the shock structure. In section 2, the standard theory of DSA and the linear analysis of the shock structure by McKenzie and Westphal (1968) [2] are reviewed. Section 3 is devoted to the explanation of basic concept of our simulations and Section 4 shows the results. We discuss the results in Section 5.

## 2. DSA and shock structure in a fluctuated medium

### 2.1 Standard theory of DSA

The DSA theory predicts the following energy spectrum of the accelerated particle at the shock,

$$\frac{dN}{dE} \propto E^{-s}, \quad s = \frac{r+2}{r-1}, \quad (2.1)$$

where  $r = u_1/u_2$  is the compression ratio of the shock and calculated by so-called Rankine-Hugoniot relations,

$$r = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \quad (2.2)$$

The subscripts 1 and 2 indicate the quantities at the upstream or downstream region, respectively. The flow velocity,  $u_1$  and  $u_2$ , are evaluated in the shock rest frame and  $\gamma$  is the specific heat ratio and equals to 5/3 here,  $M_1$  is (sound) Mach number. It can be easily confirmed that the compression ratio  $r$  goes to 4 and the spectral index  $s$  approaches to 2 for the strong shock limit ( $M_1 \rightarrow \infty$ ).

## 2.2 Interaction between a shock wave and a density fluctuation

McKenzie and Westphal (1968) [2] analyzed the combined system of shock structure and perturbed fluid field utilizing linearized fluid equations. They showed that a sound wave and an entropy wave are generated in the downstream when the wave vector in the upstream region is parallel to the shock normal direction. Although the entropy wave would make a spacial fluctuation of the diffusion coefficient and affect the propagation of the sound wave by a nonlinear interaction, we neglect the entropy wave in this study. This is because the entropy wave does not have a velocity fluctuation, so that it cannot directly accelerate particles. The velocity fluctuation associated with the downstream sound wave is related to the upstream density fluctuation,

$$\frac{\delta u_2}{u_1} = \frac{2M_1^2 - 2}{(\gamma + 1)M_1^2} \cdot \frac{M_2}{1 + 2M_2 + M_1^{-2}} \cdot \frac{\delta \rho_1}{\rho_1}. \quad (2.3)$$

The density and velocity fields in our simulations are shown in the left and right panel of figure 1 respectively. For the strong shock limit, the downstream wavelength of each wave is given by

$$\lambda_{\text{sound}} \simeq 0.81 \lambda_{\delta \rho_1} \quad (2.4)$$

$$\lambda_{\text{entropy}} = (1/4) \lambda_{\delta \rho_1}, \quad (2.5)$$

where  $\lambda_{\delta \rho_1}$  is the wavelength of the upstream density fluctuation. The horizontal axes are normalized by the wavelength of the upstream density fluctuation. The blue curves show the fields when the perturbations are absent while the orange curves are fluctuated fields by sound waves. The sound wave propagates with the sound speed of the unperturbed fluid in the downstream region.

In addition to the generation of the downstream waves, the upstream density fluctuation shakes the shock front. The displacement of the shock position,  $\delta x_{\text{sh}}$ , is given by

$$\delta x_{\text{sh}} \simeq 0.24 \lambda_{\delta \rho_1} \frac{\delta \rho_1}{\rho_1}. \quad (2.6)$$

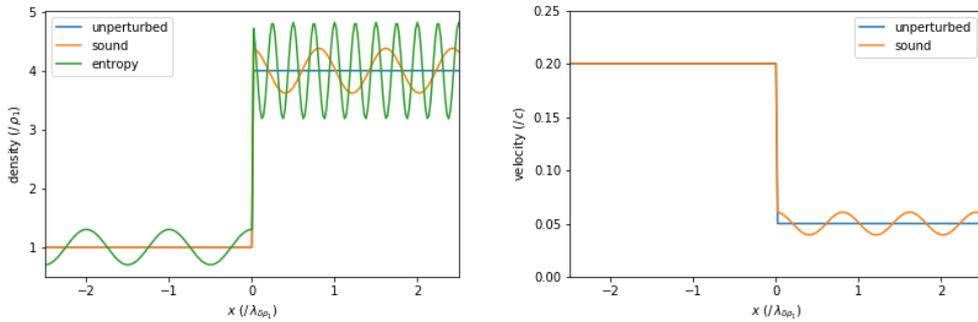


Figure 1: Left : density field, Right : velocity field

### 3. Monte Carlo simulations

In this work, we describe particle motions as the random walk by isotropic scattering in the local fluid rest frame. Particles are initially put on the shock front isotropically in the shock rest frame. The initial particle energy is  $E_0$ . The positions of the particles are displaced in this frame by  $\vec{v}\Delta t$  at each time step, where  $\vec{v}$  is the particle velocity and  $\Delta t$  is the simulation time step. Whether the scattering occurs or not in each time step is decided by random numbers, where the probability of scattering is assumed to be proportional to the inverse of energy of particles, that is, the Bohm type scattering is assumed. When scattering happens, the particles are isotropically scattered by the local rest frame. Because the number of particles decreases as their energy increase, we introduced the so-called particle splitting method to guarantee statistical precision even for high energy, which can easily reduce the computational cost. When the particle energy reach some thresholds, we divide the particles to ten and adjust the 'weight' of them so as to preserve the total number of particles.

### 4. Results

Figure 2 shows energy spectra in our simulations. The parameters for our simulations are listed in table 1. The blue line is the energy spectrum for a test simulation in which density fluctuation is absent. This horizontal structure indicates that our treatment of simulation is consistent with the standard DSA theory because the standard DSA predicts  $dN/dE \propto E^{-2}$ . As shown in figure 2, the influence of upstream fluctuations on the energy spectrum is most remarkable when their wavelength is comparable to the mean free path of accelerated particles, while fluctuation much larger than the mean free path hardly affects on the spectrum.

To clarify the effect of the upstream density fluctuation, trajectories of two accelerated particle are shown in figure 3, where the left and right panels show  $t - E$  and  $x - E$  diagrams, respectively. The blue lines shows the trajectory of a particle which finally reaches relatively higher energy  $E \simeq 10^3 E_0$ . This particle is accelerated in the vicinity of the shock front  $x = 0$  and it coincides with the picture of DSA. On the other hand, the particle accelerated to  $E \simeq 10^2 E_0$ , indicated by the orange line in figure 3, is almost not accelerated near the shock. It loses and gains energy in the downstream region but finally acquires energy on average. This process is *second order Fermi acceleration* that is the acceleration by the motion of the sound wave in the downstream region. This additional acceleration plays a role to modify the energy spectra.

Lorentz factor of injection particles	$\Gamma_0$	10
Shock velocity	$u_{\text{sh}}/c$	0.20
Upstream Mach number	$M_1$	100
Amplitude of the upstream density fluctuation	$\delta\rho_1/\rho_1$	0.30
Wavelength of the upstream density fluctuation	$\lambda_{\delta\rho_1}/\lambda_{\text{mfp}}$	10, 100, 1000

**Table 1:** Parameter sets

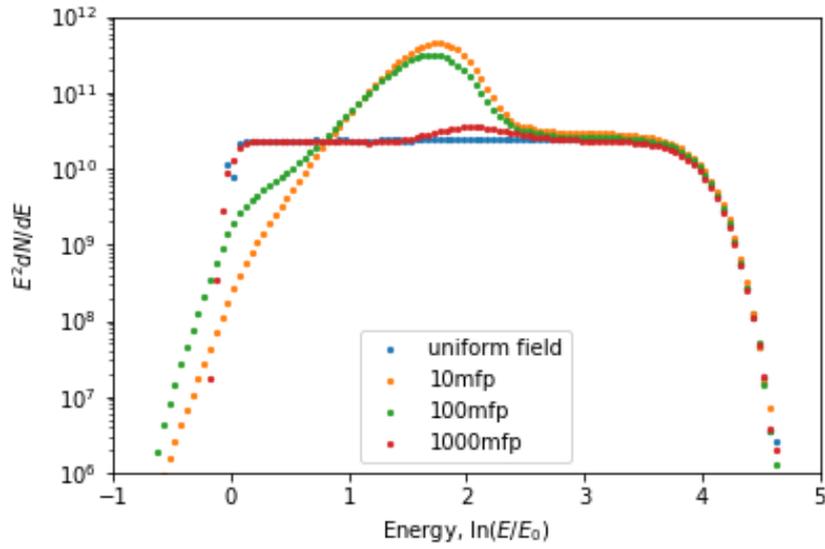


Figure 2: Energy spectra,  $E^2 dN/dE$ .

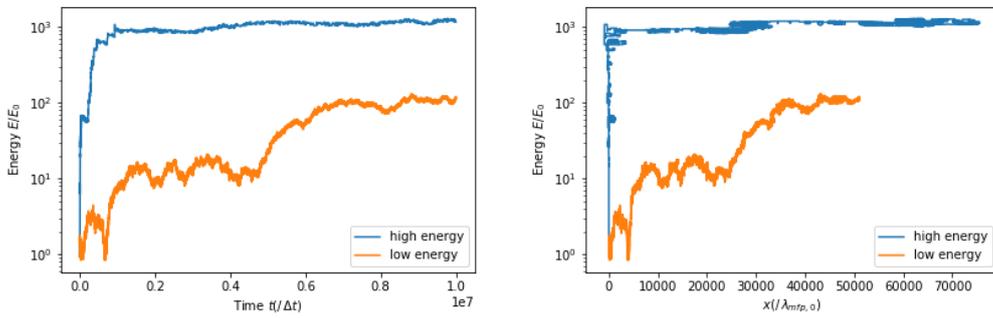


Figure 3: Left :  $t - E$  diagram, Right :  $x - E$  diagram

## 5. Discussion

Our treatment contains a lot of idealizations in this work. For example, we neglected dissipation of the sound wave and restricted our discussion to a one-dimensional fluctuation propagating parallel to the shock normal. Moreover, as mentioned in section 2.2, we adopted the effect of fluctuation only through the variation of velocity field. In spite of these constrains, it has been revealed that the upstream density fluctuation *does* modify the energy spectrum of the accelerated particles from  $E^{-2}$ . This is due to the additional second order-Fermi acceleration by the motion of the downstream sound wave. Therefore, the large-scale turbulence ( $\lambda > \lambda_{\text{mfp}}$ ) is important for the CR acceleration [4]. The quantitative estimate of this effect and comparison with observations will be addressed in future work.

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