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The strong coupling from $e^+e^- \to$ hadrons

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We present a brief summary of our recent determination of α_s from $e^+e^- \to$ hadrons, in the region $3 \le s \le 4$ GeV^2, with s the square of the center-of-massenergy.

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In a recent paper [1], we used a new compilation of data for the R-ratio $R(s)$, measured in the process $e^+e^- \to$ hadrons, to extract a value for the strong coupling, α_s, using finite energy sum rules (FESRs). This determination can directly be compared with the determination from hadronic τ decays. Here we present a brief summary of this determination. A more extensive informal overview can be found in Ref. [2]; full details can be found in Ref. [1].

The data set we employed for our work is that of Ref. [3], and it is shown in the left panel of Fig. 1. This plot shows the R-ratio as a function of the square of the center-of-mass energy s, in GeV^2, below the threshold for charm production. In the right panel of Fig. 1 we show a blow-up of these same data, for 2 GeV$^2 \le s \le 6$ GeV^2. This plot shows more clearly that there are a lot more data in the region $s \le 4$ GeV^2, where $R(s)$ was compiled from summing exclusive-channel experiments, than in the region $s \ge 4$ GeV^2, where $R(s)$ was compiled from inclusive experiments. A detailed analysis shows that an extraction of α_s employing FESRs using all data below 4 GeV^2 will yield a value with a smaller error than an extraction of α_s from $R(s)$ by direct comparison with QCD perturbation theory.

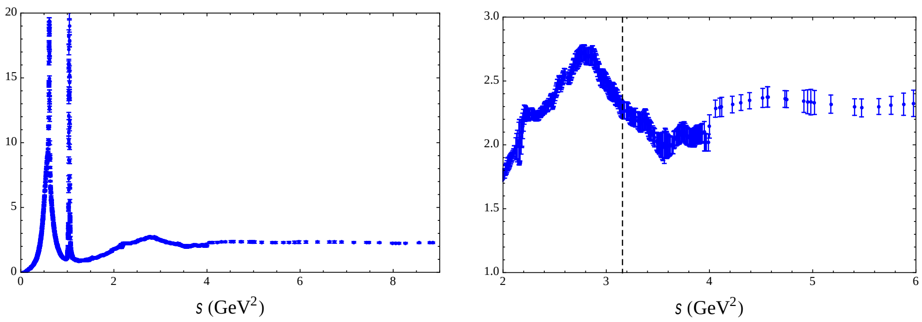

Figure 1: Left: R-ratio data from Ref. [3], as a function of s, the hadronic invariant squared mass. Right: A blow-up of the region $2 \le s \le 6$ GeV^2.

The sum rules we employ take on the form [1]

$$I^{(w)}(s_0) \equiv \frac{1}{s_0}\int_{m_\pi^2}^{s_0} ds\, w(s/s_0)\, \frac{1}{12\pi^2} R(s) = -\frac{1}{2\pi i s_0}\oint_{z=|s_0|} dz\, w(z/s_0)\, \Pi(z)\ , \tag{1}$$

with $\Pi(z)$ the usual scalar electromagnetic polarization function, and $w(y)$ one of the following analytical weight functions

$$\begin{aligned} w_0(y) &= 1\ , \\ w_2(y) &= 1 - y^2\ , \\ w_3(y) &= (1-y)^2(1+2y)\ , \\ w_4(y) &= (1-y^2)^2\ . \end{aligned} \tag{2}$$

In Eq. (1), the left-hand side represents the "data" side, and it incorporates all data between threshold and $s = s_0$. The right-hand side represents the "theory" side, and, if s_0 is large enough, we can use the theory representation

$$\Pi(z) = \Pi_{\rm pert}(z) + \Pi_{\rm OPE}^{D>0}(z) + \Pi_{\rm DV}(z)\ , \tag{3}$$

where the first term, $\Pi_{\rm pert}(z)$, represents massless perturbation theory, and is known to order α_s^4 [5, 6],[1] the second term represents mass-dependent perturbative and non-perturbative condensate contributions to the operator product expansion (OPE), while the "duality-violation" part $\Pi_{\rm DV}(z)$ represents contributions to $\Pi(z)$ manifested by the presence of resonance peaks, which are not captured by perturbation theory or the OPE. In our analysis, we also included electromagnetic (EM) corrections to perturbation theory. For details, we refer to Ref. [1]. We just point out that duality violations, represented by the term $\Pi_{\rm DV}(z)$, are expected to give a contribution which decreases exponentially with increasing s_0. In addition, their largest contribution to the integral on the right-hand side of Eq. (1) is expected to come from the part of the circle closest to the real axis, *i.e.*, $z \approx s_0$ [8]. Their contribution is thus suppressed for $w = w_2$, which has a single zero at $z = s_0$ (w_2 is "singly pinched"), and more suppressed for $w = w_{3,4}$, which both have a double zero at $z = s_0$ ($w_{3,4}$ are "doubly pinched"). Note that the integral on the right-hand side of Eq. (1) with a polynomial weight containing y^N receives a contribution from the effective condensate C_D for $D = 2N + 2$ in the OPE.

Our fits of the FESRs (1) to the data were carried out on a window $s_0 \in [s_0^{\rm min}, s_0^{\rm max}]$, with $3.25\ {\rm GeV}^2 \le s_0^{\rm min} \le 3.80\ {\rm GeV}^2$ and $s_0^{\rm max} = 4\ {\rm GeV}^2$, finding good stability for these values of $s_0^{\rm min}$. In Fig. 2 we show typical fits for all four weights (2), with $s_0^{\rm min} = 3.25\ {\rm GeV}^2$. Fits were carried out neglecting the duality-violating term $\Pi_{\rm DV}$ in Eq. (3). All fits take into account all the correlations in the data set, and have p-values varying from 0.09 to 0.42.

We note that the values of s_0 used in our fits are all larger than the square of the τ mass m_τ^2, the kinematic end point for a similar analysis of spectral functions measured in hadronic τ decays. In particular, we notice that in the e^+e^- case good fits are obtained neglecting duality violations, in contrast to the τ-decay case (see below). For $w = w_0$, a remnant of integrated duality violations (the small oscillation in the upper left panel of Fig. 2) is visible, but the fit is consistent with the data. For the higher-degree weights (which are all pinched) no effect from integrated duality violations is visible.

As usual, two different resummations of the perturbative series are employed in our sum-rule analysis, FOPT (fixed-order perturbation theory) and CIPT (contour-improved perturbation theory [9]), leading to two different values for α_s. For a more detailed discussion, we refer to Refs. [1, 4, 7] and references therein, as well as Ref. [10].

In Table 1 below, we show our results for the values of $\alpha_s(m_\tau^2)$ obtained from these fits, where we quote α_s at the τ mass in order to facilitate comparison with values obtained from hadronic τ decays. Clearly, there is excellent agreement between the values obtained from different weights. This agreement is also found for the fit values for the condensate C_6, between the weights w_2, w_3 and w_4 [1]. The errors shown are a combination of the fit error and the error due to the variation of $s_0^{\rm min}$; the first error dominates the total error.

We carried out a number of additional tests. First, we did a number of fits with $s_0^{\rm max}$ or both $s_0^{\rm min}$ and $s_0^{\rm max}$ in the inclusive region $s > 4\ {\rm GeV}^2$. We found results consistent with those reported in the table above but including data in the inclusive region does not lead to a reduction of the errors shown in the table.

Second, while fits without duality violations lead to good p-values, we tested the stability

[1] We use an educated guess for the 5th order [7].

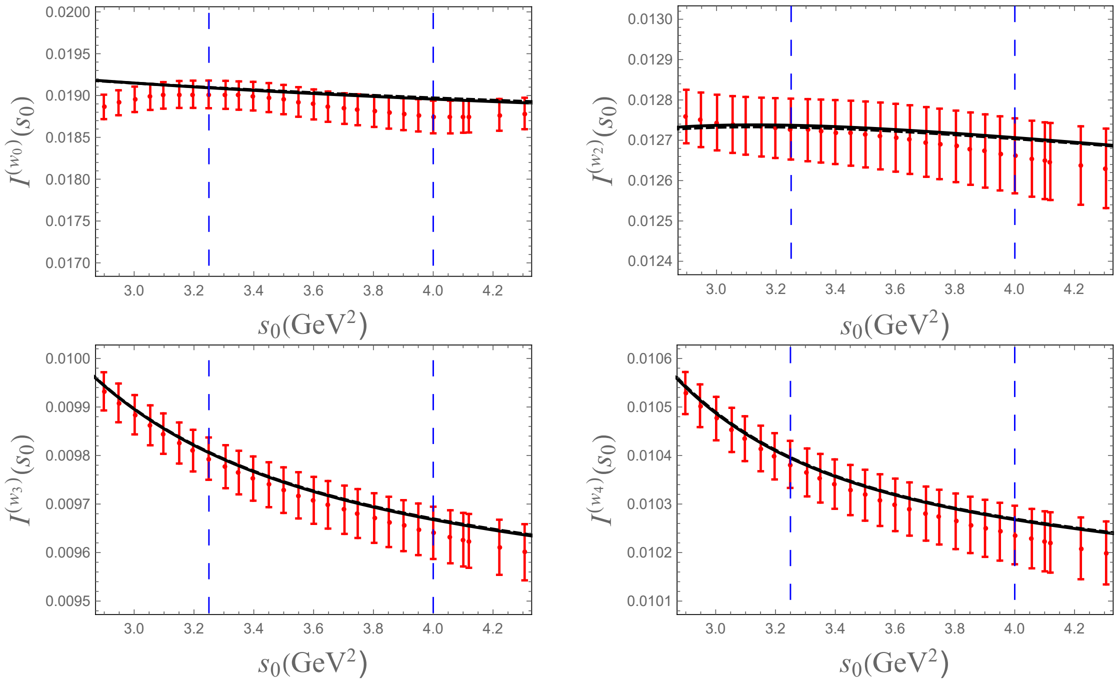

Figure 2: Comparison of the data for $I^{(w)}(s_0)$ with the fits on the interval $s_0^{\rm min} = 3.25$ to 4 GeV^2, for $w = w_0$ (upper left panel), $w = w_2$ (upper right panel), $w = w_3$ (lower left panel), and $w = w_4$ (lower right panel). Solid black curves indicate FOPT fits, dashed curves CIPT. The fit window is indicated by the dashed vertical lines.

Table 1: Values for $\alpha_s(m_\tau^2)$ obtained from the various weights, with FOPT values in the second column, and CIPT values in the third.

weight	$\alpha_s(m_\tau^2)$ (FOPT)	$\alpha_s(m_\tau^2)$ (CIPT)
w_0	0.299(16)	0.308(19)
w_2	0.298(17)	0.305(19)
w_3	0.298(18)	0.303(20)
w_4	0.297(18)	0.303(20)

of the fits with weight w_0 against the inclusion of a model for duality violations. For a detailed discussion of this test, we refer to Ref. [1]. The upshot is that our fits are stable with respect to the inclusion of duality violations, and that duality violations can be ignored within current errors. The basic reason is that the analysis based on the R-ratio allows us to restrict our attention to values of s_0 large enough compared to m_τ^2 that the exponentially decreasing duality violations are sufficiently suppressed.

Before coming to our final results, we present a brief comparison between FESR fits of moments of the non-strange $I = 1$ vector spectral function obtained from hadronic τ decays [11], and FESR fits of the EM spectral function proportional to $R(s)$. Figure 3 shows fits of the moments $I^{(w_0)}(s_0)$ (upper panels) and $I^{(w_2)}(s_0)$ (lower panels), comparing fits based on the τ data

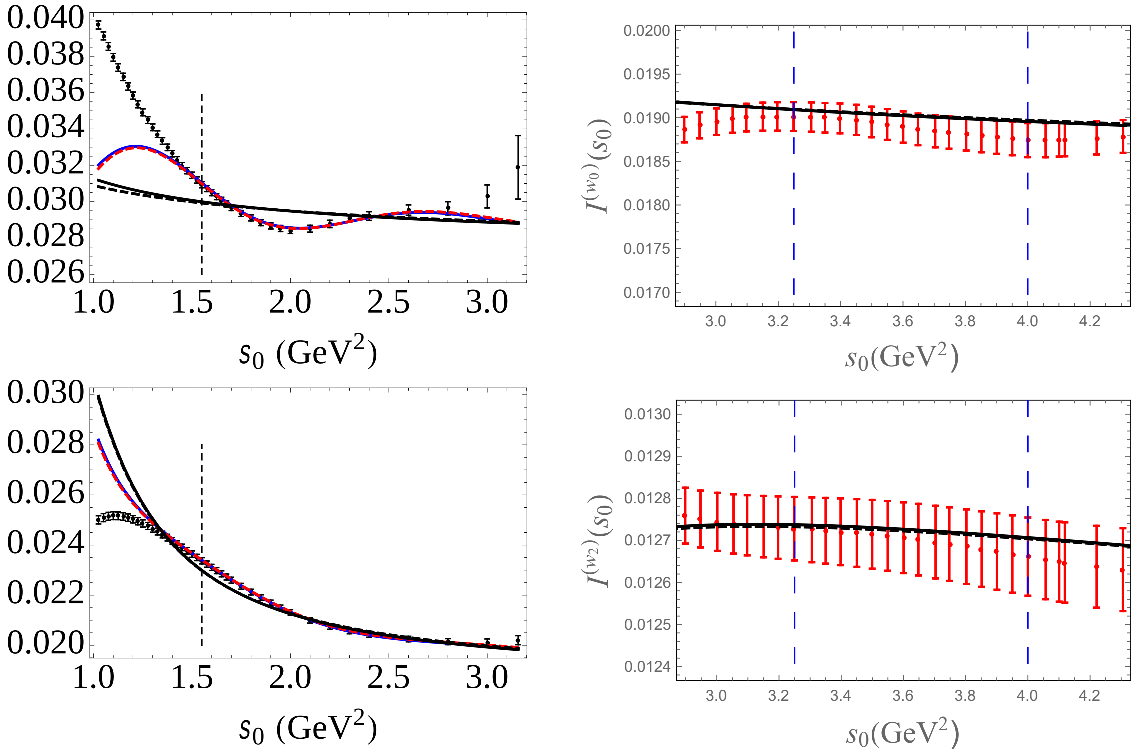

Figure 3: Comparison of FESR fits extracting α_s from hadronic τ-decay data (left panels) *vs.* $e^+e^- \to$ hadrons(γ) (right panels). Top panels show fits with weight w_0, bottom panels show fits with weight w_2. Because of the comparison between τ-based moments and e^+e^--based moments, we show those obtained from the vector channel in the plots on the left.

(left panels) with fits based on the e^+e^- data (right panels). The τ-based fits have $s_0^{\rm max} = m_\tau^2$ and $s_0^{\rm min} = 1.55$ GeV^2; the e^+e^--based fits have $s_0^{\rm max} = 4$ GeV^2 and $s_0^{\rm min} = 3.25$ GeV^2. In the τ panels, the blue curve represents FOPT fits with duality violations and the red dashed curve CIPT fits with duality violations. The black curves represent the perturbation theory plus OPE parts of these fits, omitting the duality-violating part. In the e^+e^- panels, which just reproduce the top panels already shown in Fig. 2, the black curves represent FOPT (solid) and CIPT (dashed) fits, with no duality violations.

Duality violations show up in the data points as oscillations around the perturbation theory plus OPE curves (black solid and dashed curves in all panels). Clearly, duality violations are very visible in the left panels. In contrast, they are barely visible in the upper right panel, and not visible in the lower right panel. These comparisons of theory with data show that duality violations cannot be ignored in the τ-based results, while fits of moments of $R(s)$ at sufficiently higher s_0 are consistent with integrated duality violations being small enough at these higher values to be neglected, within current errors. This is consistent with the expected exponential decay of the duality-violating part of the spectral function with increasing s, as discussed in more detail in Refs. [12, 13].

Our final results for $\alpha_s(m_\tau^2)$ from the FESR-analysis of $R(s)$ are

$$\begin{aligned} \alpha_s(m_\tau^2) &= 0.298(17) \qquad \text{(FOPT)}\,, \\ &= 0.304(19) \qquad \text{(CIPT)}\,. \end{aligned} \tag{4}$$

We note that the error is dominated by the fit errors, obtained by propagating the errors on the data compilation of Ref. [3]. These results can be directly compared with the values obtained from the τ-based analysis [11]:

$$\begin{aligned} \alpha_s(m_\tau^2) &= 0.303(9) \qquad \text{(FOPT)}\,, \\ &= 0.319(12) \qquad \text{(CIPT)}\,. \end{aligned} \tag{5}$$

There is excellent agreement between the results obtained from e^+e^-, and those obtained from τ decays. We note the much reduced difference between the FOPT and CIPT central values in the e^+e^- analysis, which we believe can be partially ascribed to the fact that these values are extracted from spectral-weight moments at larger s_0, where the convergence properties of perturbation theory are expected to be better.

We also quote the e^+e^--based values after running the values of Eq. (4) to the Z-mass, converting from three to five flavors:

$$\begin{aligned} \alpha_s(m_Z^2) &= 0.1158(22) \qquad \text{(FOPT)}\,, \\ &= 0.1166(25) \qquad \text{(CIPT)}\,. \end{aligned} \tag{6}$$

These values are both consistent, within errors, with the world average as reported in Ref. [14], confirming the running predicted by QCD between the scale of the e^+e^- analysis and m_Z [15].

Finally, we point out that the R-ratio data can be used to test results obtained in the τ-based approach, as explained in the contribution by Peris to these proceedings [16].

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[16] Contribution by S. Peris to these proceedings.

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