

## Determination of $\alpha_s(m_Z)$ from the Z-boson transverse momentum distribution

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**Stefano Camarda\***

*CERN, EP Department, CH-1211 Geneva 23, Switzerland*

*E-mail: [stefano.camarda@cern.ch](mailto:stefano.camarda@cern.ch)*

The strong-coupling constant  $\alpha_s(m_Z)$  is measured from the transverse-momentum distribution of Z bosons measured at  $\sqrt{s} = 1.96$  TeV with the CDF experiment, using predictions based on  $q_T$ -resummation at NNLO+NNLL, as implemented in the `DYTurbo` program. The measurement is performed through a simultaneous fit of  $\alpha_s(m_Z)$  and the non-perturbative Sudakov form factor.

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\*Speaker.

The strong-coupling constant has been measured at hadron colliders in final states with jets [1, 2, 3], and more recently from top-antitop production cross sections [4]. Such measurements allow probing the strong coupling at high values of momentum transfer. However, they generally suffer from large uncertainties, and do not provide a competitive determination of the strong coupling at the scale of the Z-boson mass,  $\alpha_s(m_Z)$ . This contribution aims at discussing a new technique for precisely measuring  $\alpha_s(m_Z)$  at hadron colliders from a semi-inclusive (i.e. radiation inhibited) observable: the Z-boson transverse-momentum distribution. This measurement has all the desirable features for a precise determination of  $\alpha_s(m_Z)$  [5]: large observable's sensitivity to  $\alpha_s(m_Z)$  compared to the experimental precision; high accuracy of the theoretical prediction; small size of non-perturbative QCD effects.

Measuring  $\alpha_s(m_Z)$ , or equivalently  $\Lambda_{\text{QCD}}^{\overline{\text{MS}}}$ , from semi-inclusive Drell-Yan cross sections was first proposed in Ref. [6], by using Monte Carlo parton showers to determine  $\Lambda_{\text{QCD}}^{\text{MC}}$  and later convert it to  $\Lambda_{\text{QCD}}^{\overline{\text{MS}}}$ . The conversion is based on resummation arguments showing that a set of universal QCD corrections can be absorbed in coherent parton showers by applying a simple rescaling, the so-called Catani-Marchesini-Webber (CMW) rescaling.

The Z-boson transverse-momentum distribution at small transverse momentum is one of such semi-inclusive observables. The recoil of Z bosons produced in hadron collisions is mainly due to QCD initial-state radiation, and the Sudakov form factor is responsible for the existence of a Sudakov peak in the distribution, at transverse-momentum values of approximately 4 GeV. The position of the peak is sensitive to the value of the strong-coupling constant. The arguments of Ref. [6] can be used to interpret the ATLAS result of a PYTHIA 8 Monte Carlo tuning to the Z-boson transverse-momentum distribution [7] as a measurement of  $\alpha_s(m_Z)$ . Table 1 shows the results of the ATLAS tune of PYTHIA 8, named AZ, where the Monte Carlo parameter  $\alpha_s^{\text{ISR}}(m_Z)$  was determined simultaneously with primordial  $k_T$  and the parton shower infrared cut-off from a fit to the ATLAS measurement of the Z-boson transverse-momentum distribution. The CMW conversion leads to:  $\alpha_s^{\text{ISR}}(m_Z) = 0.124 \rightarrow \alpha_s^{\overline{\text{MS}}}(m_Z) = 0.116$ .

**Table 1:** Results of the AZ tune of the PYTHIA 8 Monte Carlo to the ATLAS measurement of the Z-boson transverse-momentum distribution [7].

ISR $\alpha_s^{\text{ISR}}(m_Z)$	$0.1237 \pm 0.0002$
primordial $k_T$ [GeV]	$1.71 \pm 0.03$
ISR cut-off [GeV]	$0.59 \pm 0.08$

The relative uncertainty on  $\alpha_s(m_Z)$  of the ATLAS tune is 0.2%, which includes the experimental uncertainties, as well as the non-perturbative QCD uncertainties, since the non-perturbative QCD parameters are fitted simultaneously with  $\alpha_s^{\text{ISR}}(m_Z)$ . This naive result is missing important theory uncertainties as PDFs and missing higher order corrections. However, this simple exercise already shows a great experimental sensitivity and relatively small non-perturbative QCD uncertainties.

Turning this idea into an actual measurement poses several challenges. In order to achieve higher precision, it is highly desirable to employ for the measurement analytic predictions of the Z-boson transverse-momentum distribution including resummation of large logarithmic corrections

of the form  $\log(p_T/m)$ . Such  $q_T$ -resummed predictions are available since long time, and they have recently reached N<sup>3</sup>LL logarithmic accuracy [8]. The measurements of Z-boson transverse-momentum distribution have small experimental uncertainties, at the level of 2% at the Tevatron and 0.5% at the LHC. High numerical precision of the theory predictions is required to match such small uncertainties, which is a great challenge for these complicated high-order QCD calculations. Large correlations between  $\alpha_s(m_Z)$  and non the perturbative QCD effects would spoil the measurement. Small correlations were observed with the PYTHIA 8 model, but they need to be studied also in the case of analytic predictions. At the LHC, significant heavy-flavour initiated production, at the level of 6% for  $c\bar{c} \rightarrow Z$  and and 3% for  $b\bar{b} \rightarrow Z$  introduce additional uncertainties.

For the measurement of  $\alpha_s(m_Z)$  from the Z-boson transverse-momentum distribution it is necessary to rely on fast computing codes which allow the calculation of variations in the input parameters with small numerical uncertainties. To this end, the `DYTurbo` program has been created. It aims to provide fast and numerically precise predictions of fully-differential Drell–Yan production cross sections, for phenomenological applications such as QCD analyses and extraction of fundamental parameters of the Standard Model. The enhancement in performance over previous programs is achieved by overhauling pre-existing code, by factorising the fully-differential cross section into production and decay variables, and by introducing the usage of one-dimensional and multi-dimensional numerical integration based on interpolating functions. The `DYTurbo` program is a reimplementation of the `DYRes` [9] program for the small- $q_T$  resummed cross sections at up to next-to-next-to-leading-logarithmic (NNLL) accuracy. As an example of fast and numerically precise predictions, `DYTurbo` can compute the Z-boson transverse-momentum distribution at 13 TeV in full-lepton phase space, in 100 equally-spaced bins from zero to 25 GeV, with a target relative numerical uncertainty of  $10^{-4}$ , in 4 min. at NLO+NLL and 3.4 h at NNLO+NNLL, using simultaneously 20 parallel threads. The great majority of the computation time is spent in evaluating the LO or NLO  $V$ +jet term. However, it is possible to use `ApplGrid` [10] interfaced to `MCFM` for this term. Once this is done, the computation requires 6 s (10 s) at NLO+NLL (NNLO+NNLL).

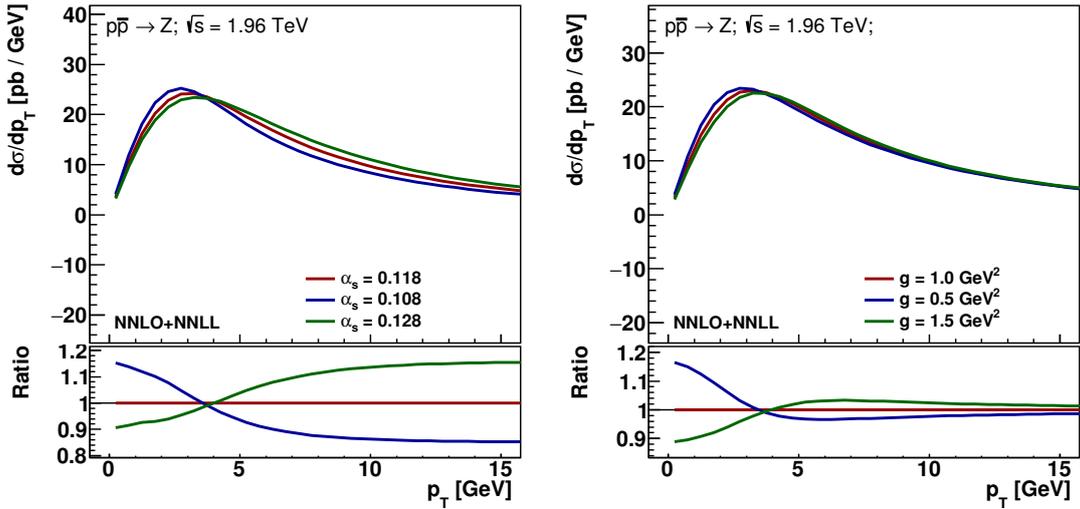
The CDF measurement of Z-boson transverse-momentum distribution [11] at the Tevatron collider is ideal for testing the extraction of  $\alpha_s(m_Z)$  with `DYTurbo` predictions. This measurement was performed with the angular coefficients technique, which allows extrapolating the cross section to full-lepton phase space with small theoretical uncertainties. The full-lepton phase space cross section allows fast predictions and avoid any theoretical uncertainties on the modelling of the Z-boson polarisation. Another advantage of this measurement with respect to similar measurements performed at the LHC is the fact that Tevatron is a proton-antiproton collider, and the Z-boson production has reduced contribution from heavy-flavour-initiated processes compared to proton-proton collisions at the LHC.

The CDF measurement is performed in the electron channel, with central ( $|\eta^e| < 1.1$ ) and forward ( $1.2 < |\eta^e| < 2.8$ ) electrons, allowing a coverage up to Z-boson rapidity of  $|y| = 2.8$ , and a small extrapolation to the full rapidity range  $|y_{\max}| \approx 3.1$  of Z-boson production at  $\sqrt{s} = 1.96$  TeV. The data sample is characterised by low values of the average number of interactions per bunch crossing, and by good electron resolution, at the level of 0.9 GeV for central electrons, and 1.1 GeV for forward electrons. The good resolution allows fine transverse-momentum binning (0.5 GeV) while keeping the bin-to-bin correlations smaller than 20%.

The non-perturbative QCD corrections to the Z-boson transverse-momentum distribution are

modelled by including a non-perturbative term in the Sudakov form factor:  $S(b) \rightarrow S(b) \cdot S_{\text{NP}}(b)$ . The general form of  $S_{\text{NP}}(b)$  is mass and centre-of-mass energy dependent [12]. However, at fixed invariant mass  $q = m_Z$ , and for one value of centre-of-mass energy, the form of  $S_{\text{NP}}(b)$  can be simplified to depend on a single parameter  $g$ :  $S_{\text{NP}}(b) = \exp(-g \cdot b^2)$ . The non-perturbative parameter  $g$  is generally determined from the data, and its value depends on the chosen prescription to avoid the Landau pole in the impact-parameter  $b$ -space, which corresponds to a divergence of the Sudakov form factor. The divergence is avoided by using the so-called  $b_*$  prescription, which freezes  $b$  at a given value  $b_{\text{lim}}$ :  $b \rightarrow b_* = \frac{b}{1+b^2/b_{\text{lim}}^2}$ . In this analysis  $b_{\text{lim}}$  is set to the value of the Landau pole, and a variation to half its value is considered as a systematic uncertainty.

The sensitivity of the Z-boson transverse-momentum distribution to  $\alpha_s(m_Z)$  mainly comes from the position of the Sudakov peak, and is related to the average recoil scale  $\langle p_T \rangle \approx 10$  GeV. The sensitivity of the Z-boson transverse-momentum distribution to  $g$  also comes from the position of the Sudakov peak. However, the scale of the non-perturbative smearing governed by  $g$  corresponds to the value of primordial  $k_T$ . Typical values of  $g \approx 0.8$  GeV<sup>2</sup> corresponds to a primordial  $k_T$  of approximately 1.8 GeV. It is possible to disentangle the perturbative contribution to the Sudakov form factor, governed by  $\alpha_s(m_Z)$ , from the non-perturbative one, determined by  $g$ , thanks to their different scale, as shown in Figure 1.

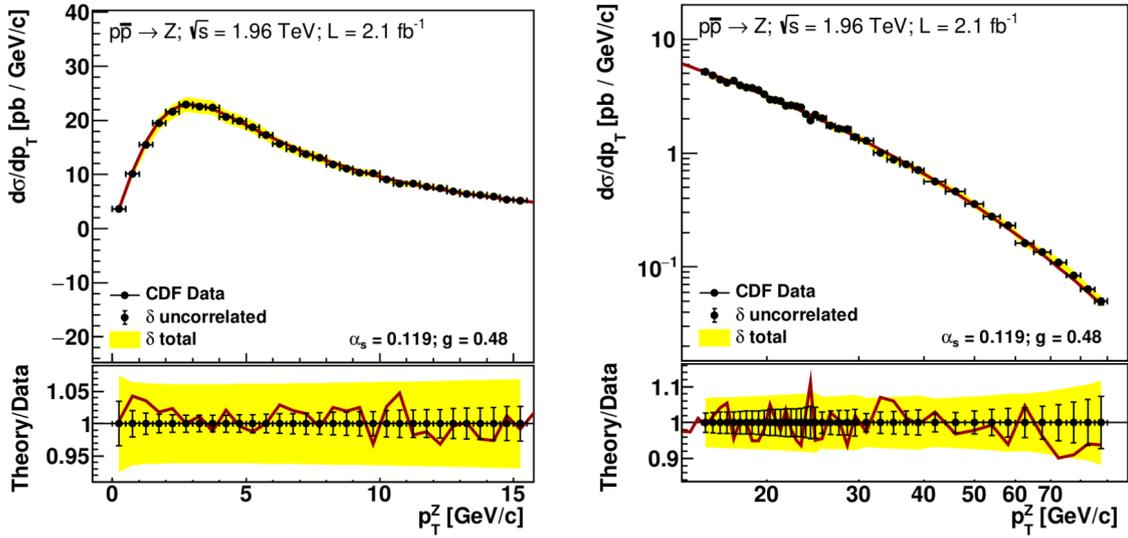


**Figure 1:** Sensitivity of the Z-boson transverse-momentum distribution to  $\alpha_s(m_Z)$  (left) and to the non-perturbative QCD parameter  $g$  (right).

The statistical analysis leading to the determination of  $\alpha_s(m_Z)$  is performed by interfacing DYTURBO to XFITTER [13]. The agreement between data and predictions is assessed by means of a  $\chi^2$  function, which includes experimental and PDFs theoretical uncertainties. The non-perturbative form factor is added as unconstrained nuisance parameter in the  $\chi^2$  definition, i.e. it is left free in the fit. The fit to the data is performed in the region of transverse momentum  $p_T < m_Z$  by minimising the  $\chi^2$  as a function of  $\alpha_s(m_Z)$ , with  $\alpha_s$  variations as provided in LHAPDF. The corrections to the Z-boson transverse-momentum distribution due to QED initial-state radiation are estimated with PYTHIA 8, and applied as multiplicative corrections. They are the level of 1%, and are responsible for a shift in the measured value of  $\alpha_s(m_Z)$  of  $\Delta\alpha_s = 0.0004$ .

**Table 2:** Results of the fit of  $\alpha_s(m_Z)$  to the CDF measurement of Z-boson transverse-momentum distribution.

	MMHT	CT14	NNPDF3.1
$\alpha_s(m_Z)$	0.1202	0.1193	0.1198
Stat. unc.	0.0008	0.0008	0.0007
Syst. unc.	0.0002	0.0003	0.0002
PDF unc.	0.0006	0.0007	0.0006
$g$ [GeV <sup>2</sup> ]	$0.48 \pm 0.07$	$0.51 \pm 0.07$	$0.35 \pm 0.08$
$\chi^2/\text{dof}$	56/71	54/71	57/71



**Figure 2:** Results of the fit of  $\alpha_s(m_Z)$  to the CDF measurement of Z-boson transverse-momentum distribution. Post-fit predictions are compared to the measured distributions.

The results of the fit of  $\alpha_s(m_Z)$  to the CDF measurement of Z-boson transverse-momentum distribution are shown in Table 2 for three different PDF sets, MMHT2014, CT14, and NNPDF3.1. The fit with the CT14 PDF set has the smallest  $\chi^2$  and is considered as central result. The post-fit predictions are compared with data in Figure 2. Additional sources of theoretical uncertainties are considered in the analysis. The predictions depend on the choice of the renormalisation, factorisation, and resummation scales. The central values of these scales are set to  $\mu_R = \mu_F = \mu_{\text{res}} = m_Z/2$ . Uncertainties arising from missing higher order corrections are estimated from the envelope of all possible combinations of factor of two variations, excluding variations where any pair of scales differ by a factor of four. The resulting uncertainty are  $+0.0035, -0.0027$  for  $\alpha_s(m_Z)$  and  $+0.61, -0.28$  GeV<sup>2</sup> for  $g$ . An uncertainty related to the matching between resummation and fixed order prediction is estimated by switching off the resummation corrections above  $p_T = m_Z/2$ . The resulting shift of  $\alpha_s(m_Z)$  is  $\Delta\alpha_s = +0.0001$ . An uncertainty related to the particular prescription used to avoid the Landau pole is estimated by setting  $b_{\text{lim}}$  to half the value of

the Landau pole. The resulting shifts of  $\alpha_s(m_Z)$  and  $g$  are  $\Delta\alpha_s = -0.0008$  and  $\Delta g = +0.18 \text{ GeV}^2$ . These values are considered as additional uncertainties.

The final result for the measurement of  $\alpha_s(m_Z)$  and the simultaneous determination of the non-perturbative parameter  $g$  is:

$$\begin{aligned}\alpha_s(m_Z) &= 0.119_{-0.003}^{+0.004} \\ g &= 0.51_{-0.34}^{+0.64} \text{ GeV}^2\end{aligned}$$

The result is dominated by missing higher order uncertainties, estimated with scale variations, which are at the level of 3%. Predictions at higher logarithmic accuracy, namely N<sup>3</sup>LL, are now available, as well as O( $\alpha_s^3$ ) corrections to the Z-boson transverse-momentum distribution, which are expected to lead to a factor of 3–5 reduction in the uncertainty [8]. Measurements of Z-boson transverse-momentum at the LHC are significantly more precise than at the Tevatron. The ATLAS measurement at  $\sqrt{s} = 7 \text{ TeV}$  yields 0.2% of relative experimental uncertainty on  $\alpha_s(m_Z)$ . Three times smaller uncertainties are expected with ATLAS and CMS measurements at  $\sqrt{s} = 8 \text{ TeV}$ , and it is likely to reach a few  $10^{-4}$  with measurements based on the full Run 2 data sample. However, in order to perform this  $\alpha_s(m_Z)$  determination at the LHC, it is primordial to improve the modelling of heavy-flavour-initiated Z-boson production.

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