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Overview of small-x physics and gluon TMDs

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Inclusive particle production in forward pA collisions is one of the observables that is used frequently in order to study the high-energy collision data within the Color Glass Condensate (CGC) framework. Moreover, at certain kinematics (in the so called correlation limit), one gets access to gluon TMDs from the CGC calculations of these observables. We discuss recent advances on the equivalence between the TMD and CGC frameworks focusing on multi-jet production.

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1. Introduction to small-x physics and Color Glass Condensate

High energy hadronic collisions, particularly between heavy nuclei, have been one of the most appealing but also challenging problems in physics for many years. They have been at the focus of the theoretical effort before the proposal of the Quantum Chromodynamics (QCD) as the quantum field theory to describe the strong interactions. On the other hand, the experimental studies to investigate QCD under extreme conditions via heavy ion collisions, have been going on for decades.

The energy (or equivalently rapidity) evolution of a hadronic wavefunction within the QCD framework has been considered in two different regimes: the Bjorken and the Regge-Gribov limits. Even though, they both describe scattering at high energy, they probe completely different physics. In the Bjorken limit, the increase in energy is accompanied with an increase in virtuality and thus results in a more dilute partonic system. On the other hand, in the Regge-Gribov limit, the increase in energy is due to the decrease of the longitudinal momentum fraction carried by the interacting partons. This low-x evolution results in a rapid increase in the number of gluons in the colliding objects and it is governed by the linear BFKL equation.

The BFKL equation was a milestone in the study of high energy scattering and has given tremendous insight to both theoretical and experimental works. However, it was realised that this linear equation does not tame the rapid growth of the gluon densities and is only valid until gluon densities reach sufficiently high values where the nonlinear effects become important. These nonlinear effects slow down the growth of the gluon density, eventually causing the phenomenon known as *gluon saturation*. This phenomenon is characterised by a new perturbative scale Q_s, known as the saturation momentum. Nowadays, the weak coupling but nonperturbative realization of saturation within QCD is called the Color Glass Condensate (CGC).

It was noted by McLerran and Venugopalan that a convenient approach to gluon saturation can be given by the nonlinearities of the classical Yang-Mills field theory [1, 2]. With this new development, the nonlinear generalization of the of the BFKL equation, known as the Balitsky-Kovchegov/ Jalilian-Marian-Iancu-McLerran-Wiegert-Leonidov-Kovner (BK-JIMWLK) functional evolution equation was derived (for a review see [3] and references in there).

In recent years, these developments have become the basis for phenomenological studies of saturation physics applied to high-energy colllision data. This approach is valid as long as one of the colliding objects is dilute. Typical examples for dilute-dense scatterings are Deep Inelastic Scattering (DIS) on a nuclear target, DIS on a high-energy proton, *proton-nucleus (pA) collisions* and forward particle production in proton-proton collisions.

2. Gluon TMDs from forward pA collisions in the CGC

One observable used frequently to test the compatibility of saturation physics with the proton-nucleus collision data from RHIC and the LHC experiments is particle production at forward rapidities. The state of the art calculation framework for forward production in pA collisions is called "hybrid formalism" [4]. In this approach, the wave function of the dilute projectile is calculated perturbatively, without any kinematic approximation, in the spirit of the collinear factorization, while the scattering of the projectile partons on the target fields is treated in the eikonal approximation within the CGC framework. Within the hybrid factorization framework, the production cross section of a quark with longitudinal momentum k^+ and transverse momenta $k_\perp$ is given as a convolution of parton distribution function (PDF) of the incoming quark $f_q(x_p,\mu^2)$ and the partonic cross section which is given in terms of a dipole operator $d(x_\perp,y_\perp)$:

$$\frac{d\sigma^{pA\to q+X}}{dk^+d^2k_\perp} = \int dx_p f_q(x_p,\mu^2)\int d^2x_\perp d^2y_\perp e^{ik_\perp(x_\perp-y_\perp)}d(x_\perp,y_\perp) \tag{2.1}$$

where the dipole operator is defined as

$$d(x_\perp,y_\perp) = \frac{1}{N_c}\langle \mathrm{tr}\big[U(x_\perp)U^\dagger(y_\perp)\big]\rangle \tag{2.2}$$

with $U(x_\perp)$ being the Wilson line operator which is defined as the path ordered exponential of the background target field $A^-(x^+,x_\perp)$ as

$$U(x_\perp)\equiv U(-\infty,+\infty;x_\perp) = \mathscr{P}\exp\Big[ig\int_{-\infty}^{+\infty}dx^+A^-(x^+,x_\perp)\Big] \tag{2.3}$$

2.1 From dipole operator to gluon TMDs

It has been shown in [5]-[7], that the operator definition of a transverse momentum dependent PDF (which is refereed to as TMD) is given by the Fourier transform of forward matrix elements of bilocal products of gluon field strength tensor:

$$F(x,k_\perp) = \int dz^+d^2z_\perp e^{ixp_A^-z^+-ik_\perp z_\perp}\langle p_A|\mathrm{tr}\big[F_0^{i-}U^{[C]}_{(0,z)}F_z^{i-}U^{[C']}_{(z,0)}\big]|p_A\rangle \tag{2.4}$$

with $U^{[C]}_{0,z}$ being the gauge staples connecting the points $(0^+,0_\perp)$ and $(z^+,z_\perp)$ to ensure the gauge invariance.

It was realized in [8], that one can get the gauge staple structure together with the field strength tensor by considering the derivative of the Wilson line operators defined in Eq. (2.3). Therefore, the small-x limit of different gluon TMDs, can be written in terms of the derivates of the Wilson line operators. For example, the small-x limit of the dipole TMD reads

$$F^{(1)}_{qg}(x,k_\perp)\Big|_{x\to 0}\to\int d^2z_\perp e^{ik_\perp z_\perp}\Big\langle\mathrm{tr}\Big\{\Big[\partial^iU^\dagger\big(\frac{z}{2}\big)\Big]\Big[\partial^iU\big(-\frac{z}{2}\big)\Big]\Big\}\Big\rangle_x \tag{2.5}$$

It has been discussed in [9]-[10], that in a certain kinematic limit (called as the *correlation limit*), one can actually probe the small-x limit of the gluon TMDs. Consider the production of two hard jets with transverse momenta $|p_1|\sim|p_2|\gg Q_s$. In this case, there are two typical transverse scale: total momentum of the produced jets ($k_T = p_1+p_2$) and the momentum imbalance of the two jets ($Q_T = p_1-p_2$). In the limit where $k_T\ll Q_T$, the two jets fly almost back-to-back in momentum space which is referred to as the correlation limit. This situation corresponds to small transverse size of the jets in coordinate space which allows us to perform a Taylor expansion of the Wilson line operators and get access to the gluon TMDs:

$$U_{b+\frac{r}{2}}U_{b-\frac{r}{2}}-1 = \frac{r^i}{2}\Big[\big(\partial^iU_b\big)U_b-U_b\big(\partial^iU_b\big)\Big]+O(r^2) \tag{2.6}$$

To sum up, in the small-x limit of the gluon TMDs, the phase drops and the staple gauge links depends only on longitudinal coordinates. On the other hand, in the correlation limit of the CGC, one can perform a small dipole size expansion and get access to the derivatives of the Wilson line operators. Therefore, the small-x limit of the TMD factorization and the correlation limit of the CGC overlaps.

2.2 Beyond the correlation limit in the CGC and iTMD framework

In [11]-[12], it has been shown that one can indeed go beyond the correlation limit and prove an all order equivalence between the small-x limit of the TMD factorization and the CGC framework.

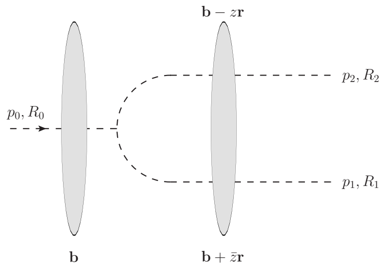

Figure 1: A generic $1 \to 2$ CGC amplitude. Each gray blob represents the dressing of the partons with the Wilson line operators. R_i stands for the color representation of each parton. z and $\bar{z} = 1-z$ are the longitudinal momentum fractions carried by the partons after the splitting.

A generic $1 \to 2$ CGC amplitude (see Fig. 1) can be written as

$$\mathscr{A} = 2\pi\,\delta(p_1^+ + p_2^+ - p_0^+)\int d^2r\,d^2b\,e^{-iqr-ikb}\,\mathscr{H}(r)\left[\left(U^{R_1}_{b+\bar{z}r}T^{R_0}U^{R_2}_{b-zr}\right) - \left(U^{R_1}_{b}T^{R_0}U^{R_2}_{b}\right)\right] \tag{2.7}$$

with k being the total transverse momenta ($k = p_1 + p_2$), q being the transverse boost invariant momentum ($q = \frac{p_2^+ p_1 - p_1^+ p_2}{p_1^+ + p_2^+}$) and Q being the invariant mass of the outgoing pair ($Q^2 = \frac{q^2}{2z\bar{z}}$) which is the hard scale of the process. One can actually expand this amplitude for small r and $O(r)$ terms in this expansion corresponds to the correlation limit. As shown in detail in [11] and [12], one can keep expanding in powers of r, use integration by parts to cast terms in 1-body or 2-body contributions and then resum those terms. After all said and done, the generic $1 \to 2$ CGC amplitude can be written as sum of the 1-body and 2-body amplitudes:

$$\mathscr{A} = \mathscr{A}_1 + \mathscr{A}_2 \tag{2.8}$$

where 1-body amplitude and 2-body amplitudes have the following structures (for the full expressions see [12])

$$\mathscr{A}_1 \propto \int d^2b\,e^{-ikb}\left(\partial_\alpha U^{R_1}_b\right)T^{R_0}U^{R_2}_b \tag{2.9}$$

$$\mathscr{A}_2 \propto \int d^2b_1\,d^2b_2\,e^{-ik_1b_1-ik_2b_2}\left(\partial^i U^{R_1}_{b_1}\right)T^{R_0}\left(\partial^j U^{R_2}_{b_2}\right) \tag{2.10}$$

On the other hand, in [13] a new framework that is referred to as "small-x improved TMD framework" (iTMD) is constructed. iTMD framework is constructed to interpolate between TMD regime ($k_t \ll Q$) and the BFKL regime $k_t \sim Q$ regime. In this framework, dijet production cross section can be written as a convolution of the hard factors that are constructed from off-shell gauge invariant matrix elements and linear combinations of the unpolarized gluon TMDs. In [11] it was shown that, CGC cross section that is calculated with the resummed 1-body amplitude given in Eq. (2.9) matches exactly the iTMD cross section.

2.3 Linearly polarized gluon TMDs

The Fourier transform of the correlators Wilson line operators can be decomposed into two parts, a part that is linear in Lorentz indices and a part that is is traceless. For example:

$$\int_{zy} e^{ik_t(y-z)} \Big\langle \mathrm{tr}\big[U_z^\dagger(\partial^i U_z)U_y^\dagger(\partial^j U_y)\big]\Big\rangle_{x_A} = -g_s^2(2\pi)^3\frac{1}{4} \times\left[\frac{1}{2}\delta^{ij}\mathscr{F}_{gg}^{(3)}(x_A,k_t)-\frac{1}{2}\Big(\delta^{ij}-2\frac{k_t^ik_t^j}{k_t^2}\Big)\mathscr{H}_{gg}^{(3)}(x_A,k_t)\right] \qquad (2.11)$$

where $\mathscr{F}_{gg}^{(3)}(x_A,k_t)$ is the linearly polarized Weizsäcker-Williams gluon TMD and $\mathscr{H}_{gg}^{(3)}(x_A,k_t)$ is its linearly polarized partner in an unpolarized target. Polarized gluon TMDs have been computed in [14] within McLerran-Venugopalan model. It has been shown that linearly polarized gluon TMDs can be probed in forward dijet production with massive quarks [15] and also in processes with three final state particles [16, 17]. In the processes with three final state particles, the intermediate partons behave as an effective mass and therefore give the possibility to probe the linearly polarized gluon TMDs. In [16], forward production of dijet and photon was studied both in quark and gluon initiated channels. It was shown that in the correlation limit, one gets access to many different unpolarized and linearly polarized gluon TMDs. Recently, photoproduction of three jets (quark, antiquark and gluon) have been studied in [17]. There it was shown that this process is only sensitive to unpolarized and linearly polarized Weizsäcker-Williams gluon TMDs and rapidity evolution of these TMDs have been studied.

3. Summary and outlook

Even though the equivalence between the CGC and the TMD frameworks have been first established in the correlation limit [8, 9, 10], it is shown in [11, 12] this equivalence can be extended beyond the correlation limit. One of the biggest advantages of this equivalence is that one can use CGC techniques to study the gluon TMDs such as using MV model for calculating different TMDs and studying their rapidity evolution by using JIMWLK equation.

One of the most challenging question that needs to be answered related with the equivalence of these two frameworks is whether it holds beyond the leading order. The equivalence between the CGC and the TMD framework have been proven for dijet production beyond the correlation limit and for three jet production in the correlation limit. Both of these studies are tree level leading order calculations. The question whether this equivalence can be shown at next to leading order is still waiting for an answer.

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