

Semileptonic B decays and $|V_{xb}|$ update

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We review the status of semileptonic B decays and $|V_{xb}|$ determinations.

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1. Introduction

Semileptonic B decays are the processes of election when it comes to a precise determination of the parameters $|V_{xb}|$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. At the lowest order in the SM, they are mediated by tree level quark decays, and the presence of leptons in the final states simplifies the QCD analyses, since hadronic and leptonic currents factorize. Precision studies of semileptonic B meson decays have been made possible by the large samples of B mesons collected at the B -factories and at LHCb, and by a concomitant progress in theoretical computations, especially in lattice QCD. Since at least four decades, the parameters $|V_{xb}|$ play a central role in the analyses of the unitarity triangle and in testing the Standard Model (SM). A long-standing tension among their values, depending on whether they are extracted using exclusive or inclusive semi-leptonic B decays, has been an additional motivation for more and more refined theoretical and experimental analyses. We briefly review recent progress on the study of semileptonic B decays and on $|V_{xb}|$ determinations ¹.

2. Inclusive decays $B \rightarrow X_c \ell \nu_\ell$

In inclusive $B \rightarrow X_c \ell \nu_\ell$ decays, the final state X_c is an hadronic state originated by the charm quark. The large hierarchy between the intrinsic large 'dynamic' scale of energy release and the hadronic soft scale leads naturally to Λ_{QCD}/m_b as an expansion parameter of non-perturbative effects. The expansion for the total semileptonic width takes the form

$$\Gamma(B \rightarrow X_c \ell \nu) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[c_3 \langle O_3 \rangle + c_5 \frac{\langle O_5 \rangle}{m_b^2} + c_6 \frac{\langle O_6 \rangle}{m_b^3} + O \left(\frac{\Lambda_{QCD}^4}{m_b^4}, \frac{\Lambda_{QCD}^5}{m_b^3 m_c^2}, \dots \right) \right] \quad (1)$$

Here c_d ($d = 3, 5, 6, \dots$) are short distance coefficients, calculable in perturbation theory as a series in the strong coupling α_s , and O_d denote local operators of (scale) dimension d . The hadronic expectation values of the operators are the (normalized) forward matrix elements, encode the nonperturbative corrections and can be parameterized in terms of heavy quark expansion (HQE) parameters, whose number grows with powers of Λ_{QCD}/m_b . Similar expansions hold for sufficiently inclusive quantities as the moments of distributions of charged-lepton energy, hadronic invariant mass and hadronic energy.

At order $1/m_b^0$ in the HQE, the perturbative corrections up to order α_s^2 to the width and to the moments of the lepton energy and hadronic mass distributions are known completely [7–13]. The terms of order $\alpha_s^{n+1} \beta_0^n$, where β_0 is the first coefficient of the QCD β function, $\beta_0 = (33 - 2n_f)/3$, have also been computed following the Brodsky-Lepage-Mackenzie procedure [10, 14]. Perturbative corrections to the coefficients of the kinetic operator [15, 16] and the chromomagnetic operator [17–19] have been evaluated at order α_s . Two independent parameters, $\rho_{D,LS}^3$, are also needed to describe matrix elements of operators of dimension six, that is at order $1/m_b^3$. Their coefficients have long been known at tree level [20], and more recently α_s corrections to the coefficient of the ρ_D^3 term have been computed [21]. Starting at order Λ_{QCD}^3/m_b^3 , terms with an infrared sensitivity to the charm mass appear, at this order as a $\log m_c$ contribution [22–24].

¹For brief overviews see for example [1–6] and references therein.

Presently, the matrix elements have been identified and estimated up to the order $1/m_b^4$ and $1/m_b^5$ [25–27].

3. Inclusive $|V_{cb}|$ determination

The shapes of the kinematic distributions in the $B \rightarrow X_c \ell \nu$ decays are sensitive to the masses of the b and c quarks and the non-perturbative HQE parameters, and all these quantities are affected by the particular theoretical scheme used to define the quark masses. Non perturbative parameters can be extracted together with $|V_{cb}|$ in a global fit based on experimentally measured distributions and momenta. Global fit analyses differ by the data sets they are based onto, the theoretical scheme employed, and the order of truncation of the HQE expansion. Challenges are experimental selections applied to the data as well as to properly account for correlations.

A recent global analysis of the inclusive $B \rightarrow X_c \ell \nu$ has been done by HFLAV [28]. In the framework of kinetic scheme, $|V_{cb}|$ is extracted together with the b and c quark masses and 4 non-perturbative parameters (namely μ_π^2 , μ_G^2 , ρ_D^3 and ρ_{LS}^3). Details on the extraction can be found for instance in Ref. [1]. The resulting value is

$$|V_{cb}| = (42.19 \pm 0.78) \times 10^{-3} \quad (2)$$

where the quoted uncertainty includes both the experimental and the theoretical uncertainties. It is worth to mention that the latter ones are dominating. In this analysis, the excellent fit quality points toward the validity of the HQE fit, but the small χ^2 per degree of freedoms of $\chi^2/ndf = 0.32$, could be a signal of some overestimated theoretical uncertainties, or overestimated correlations between the various moments.

4. Exclusive $B \rightarrow D^{(*)}$ decays into light leptons

One can express the differential ratios for the semi-leptonic CKM favoured decays $B \rightarrow D^{(*)} \ell \nu$ in terms of the recoil parameter $\omega = p_B \cdot p_{D^{(*)}}/m_B m_{D^{(*)}}$, which corresponds to the energy transferred to the leptonic pair. For negligible lepton masses ($\ell = e, \mu$), one has

$$\begin{aligned} \frac{d\Gamma}{d\omega}(B \rightarrow D^* \ell \nu) &\propto G_F^2 (\omega^2 - 1)^{\frac{1}{2}} |V_{cb}|^2 \mathcal{F}(\omega)^2 \\ \frac{d\Gamma}{d\omega}(B \rightarrow D \ell \nu) &\propto G_F^2 (\omega^2 - 1)^{\frac{3}{2}} |V_{cb}|^2 \mathcal{G}(\omega)^2 \end{aligned} \quad (3)$$

In the heavy quark limit, both form factors are related to a single Isgur-Wise function, $\mathcal{F}(\omega) = \mathcal{G}(\omega) = \xi(\omega)$, which is normalized to unity at zero recoil, that is $\xi(\omega = 1) = 1$. There are non-perturbative corrections to this prediction, expressed at the zero-recoil point by the heavy quark symmetry under the form of powers of Λ_{QCD}/m , where $m = m_c$ and m_b . Other corrections are perturbatively calculable radiative corrections from hard gluons and photons. Latest estimates for zero recoil form factors come from lattice, and are reported in Table 1. We have also listed the form factors for $B_s \rightarrow D_s^{(*)} \ell \nu$ decays, whose lattice computations are more advantageous, because of the larger mass of the valence s quark compared to u or d quarks. Since the very recent LHCb measurement [29], these decays supply a new method for precisely determining $|V_{cb}|$.

Collaboration	Refs.	$\mathcal{F}(1)$	Refs.	$\mathcal{G}(1)$
FNAL/MILC	[30]	$0.906 \pm 0.004 \pm 0.012$	[31]	$1.054 \pm 0.004 \pm 0.008$
HPQCD	[32]	$0.895 \pm 0.010 \pm 0.024$	[33]	1.035 ± 0.040
HPQCD	[34]	0.914 ± 0.024		
		$\mathcal{F}^{B_s \rightarrow D_s^*}(1)$	$\mathcal{G}^{B_s \rightarrow D_s}(1)$	
HPQCD	[34]	$0.9020 \pm 0.0096 \pm 0.0090$	[35]	1.068 ± 0.004
Atoui et al.			[36]	1.052 ± 0.046

Table 1: Latest lattice form factor estimates at zero recoil (From Ref. [1]).

In the computation of the form factors, the advantage provided by the heavy quark symmetries has the hindrance that the differential rates in (3) vanish at zero recoil. Thus one needs to extrapolate the experimental points taken at $\omega \neq 1$ to the zero recoil point $\omega = 1$, using a parameterization of the dependence on ω of the form factors, which introduces additional uncertainties. Commonly used parameterizations are the CLN (Caprini-Lellouch-Neubert) [37], the BGL (Boyd-Grinstein-Lebed) [38] and the BCL (Bourrely-Caprini-Lellouch) [39] parameterizations. In all of them, ω is mapped onto a complex variable z via a conformal transformation; form factors are written in the form of an expansion in z , which converges rapidly in the kinematical region of heavy hadron decays.

Form factor estimates via zero recoil sum rules [40–42] give, in general, relatively higher values of $|V_{cb}|$ and a theoretical error more than twice the error in the lattice determinations. Recent progress includes form factors determinations from B meson light-cone sum rules (LCSR) beyond leading twist in the $B \rightarrow D^* \ell \nu$ channel [43].

5. Exclusive $|V_{cb}|$ determination

In 2017, for the first time, the unfolded fully-differential decay rate and associated covariance matrix have been published, by the Belle collaboration [44], prompting independent determinations of $|V_{cb}|$ [32, 45–49]. Based on the Belle measurement [44], the accuracy of the CLN parameterization was questioned in both $B \rightarrow D \ell \nu$ [50] and $B \rightarrow D^* \ell \nu$ [45, 46] channels, in favour of the BGL one [45–47]. Higher central values (closer to the inclusive values) were found in the latter approach, and the possibility to have solved the long standing inclusive/exclusive tension was aired. However, one year later, more data were provided by Belle [51] and in 2019 by BaBar [52]. Both these analyses showed no sign of discrepancy on $|V_{cb}|$ between the BGL and CLN parameterizations, within the uncertainties. Belle also in this case released the data in a format that allows them to be fitted by outside groups, prompting a new analysis by some among the authors of the 2017 fits [53], which gave consistent results with both CLN and BGL parameterizations, in different configurations. The situation described above is summarized in Fig. 1². The exclusive values are compared with the inclusive HFLAV average of Eq. (2). A recent estimate from QCD LCSR [54] is reported as well, together with the recent $|V_{cb}|$ determination by LHCb, the first one at a hadron collider and the first one to use B_s^0 decays [29].

²For details refer to Ref. [1].

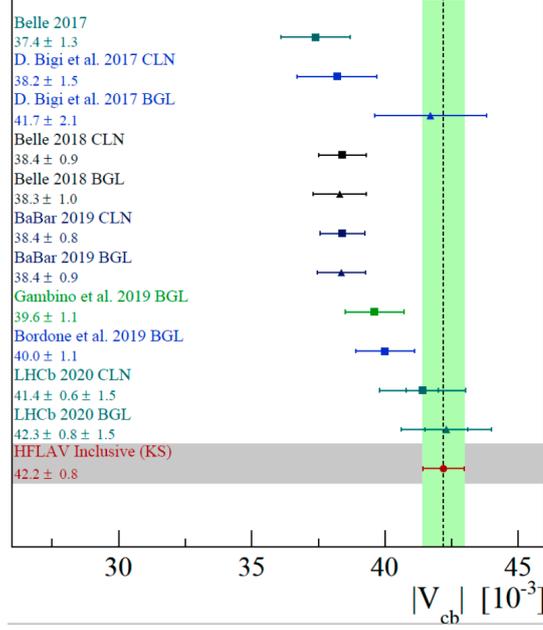


Figure 1: Recent $|V_{cb}|$ determinations (see text).

The ultimate way out of parametrization dependence is to compute the form factors at nonzero recoil values. In the case of $B \rightarrow D\ell\nu$ decays, the form factors in the unquenched lattice-QCD approximation have been made available for a range of non-zero recoil momenta since 2015 by the FNAL/MILC [31] and the HPQCD Collaboration [33]. Both estimates are in good agreement. Recently, the HPQCD Collaboration has presented a lattice QCD determination of the $B_s \rightarrow D_s\ell\nu_\ell$ scalar and vector form factors over the full physical range of momentum transfer [55].

The LHCb collaboration has measured the ratio of the branching fractions $\Lambda_b^0 \rightarrow p\mu^-\bar{\nu}_\mu$ and $\Lambda_b^0 \rightarrow \Lambda_c^+\mu^-\bar{\nu}_\mu$ [56], from which they have determined the first direct measurement of the ratio $|V_{ub}|/|V_{cb}|$. Even if this is not a direct measurement of $|V_{cb}|$, by taking $|V_{ub}|$ from external inputs it is possible to determine $|V_{cb}|$. The LHCb analysis, besides being the first one to use B -baryon decays, has paved the way to extract $|V_{ub}|/|V_{cb}|$ from the ratio $\mathcal{B}(B_s^0 \rightarrow K^-\mu^+\nu_\mu)/\mathcal{B}(B_s^0 \rightarrow D_s^-\mu^+\nu_\mu)$ [57].

6. Inclusive $|V_{ub}|$ determination

In order to extract $|V_{ub}|$ from semileptonic $B \rightarrow X_u\ell\nu$ decays one has to reduce the $b \rightarrow c$ semileptonic background through experimental cuts. Such cuts enhance the relevance of the so-called threshold region in the phase space, jeopardizing the use of HQE. In order to face this problem, that is absent in the inclusive determination of $|V_{cb}|$, different theoretical schemes have been devised, which are tailored to analyze data in the threshold region, but differ in their treatment of perturbative corrections and the parametrization of non-perturbative effects. In Table 2 we present the results of four theoretical different approaches, that is ADFR [58–60], BLNP [61–63], DGE [64] and GGOU [65], as analyzed by BaBar [66, 67], Belle [68, 69] and HFLAV [70] collaborations. The most recent (2016) averaged HFLAV determinations [70] are the ones used in the latest (2019) update of PDG [71]. The results are consistent within the uncertainties. The most recent estimates (2020, still

Inclusive decays	$ V_{ub} \times 10^3$			
	ADFR	BNLP	DGE	GGOU
Belle 2020 [69]	$4.01^{+0.08,+0.15,+0.18}_{-0.08,-0.16,-0.18}$	$4.01^{+0.08,+0.15,+0.18}_{-0.08,-0.16,-0.19}$	$4.12^{+0.08,+0.16,+0.11}_{-0.09,-0.16,-0.12}$	$4.11^{+0.08,+0.16,+0.08}_{-0.09,-0.16,-0.09}$
HFLAV 2016 [70]	$4.08 \pm 0.13^{+0.18}_{-0.12}$	$4.44 \pm 0.15^{+0.21}_{-0.22}$	$4.52 \pm 0.16^{+0.15}_{-0.16}$	$4.52 \pm 0.15^{+0.11}_{-0.14}$
BaBar 2011 [66]	$4.29 \pm 0.24^{+0.18}_{-0.19}$	$4.28 \pm 0.24^{+0.18}_{-0.20}$	$4.40 \pm 0.24^{+0.12}_{-0.13}$	$4.35 \pm 0.24^{+0.09}_{-0.10}$
Belle 2009 [68]	$4.48 \pm 0.30^{+0.19}_{-0.19}$	$4.47 \pm 0.27^{+0.19}_{-0.21}$	$4.60 \pm 0.27^{+0.11}_{-0.13}$	$4.54 \pm 0.27^{+0.10}_{-0.11}$

Table 2: Status of inclusive $|V_{ub}|$ determinations.

preliminary), provided by Belle [69], are also included in Table 2: the uncertainties are statistical, systematic and from the theory calculation, respectively. Their arithmetic average of the results obtained from the four different QCD predictions gives [69] $|V_{ub}| = (4.06 \pm 0.09 \pm 0.16 \pm 0.15) \times 10^{-3}$. This value is smaller than the previous inclusive measurements, reducing the discrepancy with the exclusive measurement of about 2-3 to 1.4 standard deviations.

7. Exclusive $|V_{ub}|$ determination

The CKM-suppressed decay $B \rightarrow \pi \ell \nu$ with light final leptons is the typical exclusive channel used to extract $|V_{ub}|$. It is well-controlled experimentally and several measurements have been performed by both BaBar and Belle collaborations [72–78]. Since the u -quark is not heavy, heavy quark symmetries are not as binding as in $b \rightarrow c$ decays. Lattice determinations of the form factors in this channel have been obtained by the HPQCD [79, 80], the Fermilab/MILC [81, 82] and the RBC/UKQCD [83] collaborations. The HFLAV $|V_{ub}|$ determination comes from a combined fit of a $B \rightarrow \pi$ form factor parameterization to theory predictions and the average q^2 spectrum in data. The theory input included in the fit are the results from the FLAG lattice average [84] and the LCSR result at $q^2 = 0 \text{ GeV}^2$ [85]. The results give [28] $|V_{ub}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3}$, where the first error comes from the experiment and the second one from the theory.

Decay modes with charmless meson states heavier than the charged pion have been studied to a lesser extent; in general they present more challenges in the experimental reconstruction due to possibly higher backgrounds, more complex form-factor dependencies and wider decay widths. No lattice unquenched QCD calculation of their form factors is available yet, exception made for preliminary results from the SPQcdR Collaboration [86]. The LCSR computation of form factors for $B \rightarrow \rho/\omega \ell \bar{\nu}_\ell$ decays has been exploited to estimate $|V_{ub}|$ [87, 88]. The $B \rightarrow \eta'$ form factors have also been computed in the LCSR framework [94]. Measurements of the branching fractions for $B^+ \rightarrow \eta/\eta' \ell \nu_\ell$, where ℓ stands for either an electron or a muon, have been reported by CLEO [89, 90], BaBar [74, 77, 91, 92] and Belle [93],

8. The $|V_{cb}|$ and $|V_{ub}|$ puzzles

The discrepancy between inclusive and exclusive values of $|V_{cb}|$ ($|V_{ub}|$) is generally referred to as the $|V_{cb}|$ ($|V_{ub}|$) puzzle. In Fig. 2 HFLAV [28] exclusive and inclusive determinations of $|V_{cb}|$ are summarized and compared with the analogous determinations of $|V_{ub}|$. The magenta and green vertical bands represent the two different exclusive determinations of $|V_{cb}|$: both show a

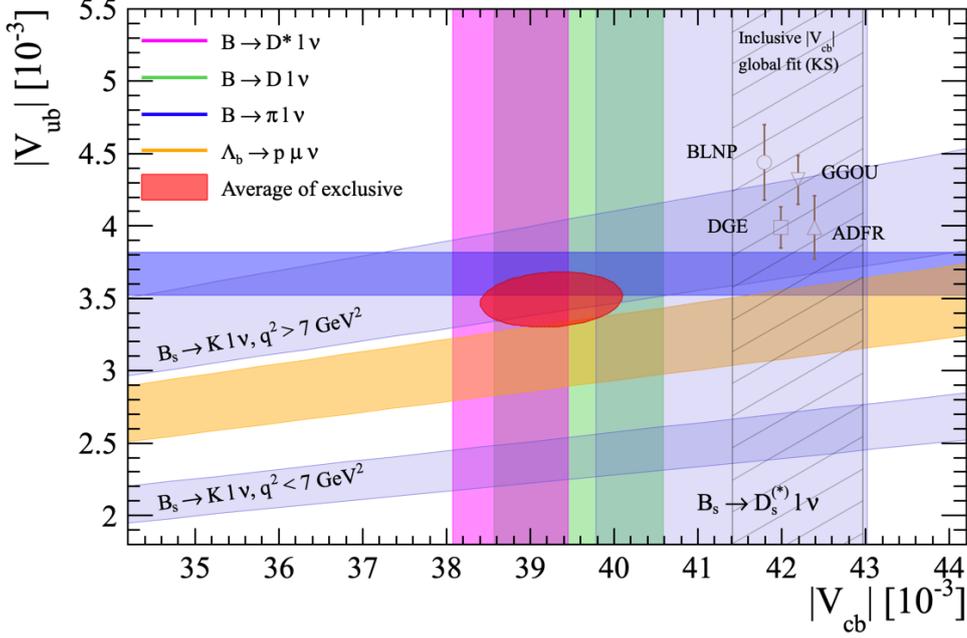


Figure 2: $V_{ub} - V_{cb}$ HFLAV averages.

discrepancy with the striped vertical band, which is relative to the $|V_{cb}|$ inclusive determination in the kinetic scheme. The bands relative to the exclusive $B \rightarrow D$ and $B \rightarrow D^*$ decays are the HFLAV averages done with the CLN parameterizations. Also considering the slightly larger uncertainty associated with the BGL fit, the discrepancy with the inclusive determination remains significant, amounting to about 3σ . The grey vertical band corresponds to the LHCb result with $B_s \rightarrow D_s^{(*)} \mu \nu \mu$ decays: although affected by large uncertainties, it is compatible with both inclusive and exclusive determinations of $|V_{cb}|$. The oblique bands represent the constraints on the $|V_{ub}|/|V_{cb}|$ ratio determined by LHCb in baryon decays [56] and more recently in $B_s^0 \rightarrow K^- \mu^+ \nu \mu$ decays [57].

In Fig. 2, the blu horizontal band gives the exclusive $|V_{ub}|$ determination from semileptonic $B \rightarrow \pi$ decay. The values for inclusive $|V_{ub}|$ with different QCD calculations are given by the four points with vertical error bars. The shift along the x-axis of these four points is just arbitrary and has no meaning. By considering their arithmetic average, and the latest Belle results [69], the observed $|V_{ub}|$ discrepancy reduces from 2-3 to 1.4 standard deviations, as discussed in Sect.6.

It is also possible to determine $|V_{cb}|$ and $|V_{ub}|$ indirectly, using the CKM unitarity relations together with CP violation and flavour data, excluding direct information on decays, as done by the CKMfitter [95] and by the UTfit Collaborations [96].

Future prospects seems to be promising. Belle II is expected to reduce the experimental uncertainty on inclusive and exclusive $|V_{cb}|$ to 1% and 1.5%, and on inclusive and exclusive $|V_{ub}|$ to 3% and 2%, respectively [97]. A synergy between theoretical advances, Belle II and the upgraded LHCb may well say the final word on the present puzzles.

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