PROCEEDINGS OF SCIENCE

Impact of Dynamical Fermions on Centre Vortex Structure

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This presentation examines the centre vortex structure of Monte-Carlo generated gauge-field configurations using modern visualisation techniques. This time, the manner in which light dynamical fermion degrees of freedom impact the centre-vortex structure is explored. Focusing on the thin vortices identified by plaquettes having a non-trivial centre phase, the vortex structure is illustrated through 3D renderings of oriented plaquettes. The impact of light dynamical fermions is not subtle, changing both the density of vortices and the complexity of the vortex structures observed. The role of vortex branching points in full QCD is highlighted in the survey of results presented.

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1. Introduction

The salient emergent features of the QCD ground-state vacuum fields are: the dynamical generation of mass through chiral symmetry breaking, and the confinement of quarks. Understanding the most fundamental aspects of QCD-vacuum field structure giving rise to these phenomena is a contemporary problem of interest.

The most fundamental mechanism proposed to give rise to these phenomena are centre vortices [1–12] within the ground-state vacuum fields. Here, the eight degrees of freedom contained within an $SU(3)$ gauge-field link are replaced by one of the three cube roots of unity. It is this extreme level of simplification that makes the centre-vortex proposal so compelling as the most fundamental essence of QCD vacuum structure.

Within the pure-gauge sector of $SU(3)$-colour, there is encouraging evidence that centre vortices underpin both confinement and dynamical chiral-symmetry breaking.

- Removal of $SU(3)$ centre vortices removes confinement, while consideration of the vortices alone provides confinement [13–15].
- The planar vortex density of centre-vortex degrees of freedom scales with the lattice spacing providing an well defined continuum limit [13].
- Removal of vortices suppresses the infrared enhancement of the gluon propagator. Again the vortices alone contain the long distance structure of the gluon fields responsible for the well-known infrared enhancement [16].
- A connection between centre vortices and instantons was established through gauge-field smoothing [15]. An understanding of the phenomena linking these these degrees of freedom was illustrated in Ref. [17].
- Centre vortices have been shown to give rise to mass splitting in the low-lying hadron spectrum [18–20].
- Through studies of the nonperturbative quark propagator of the overlap-Dirac fermion operator, evidence that centre vortices underpin dynamical chiral symmetry breaking in $SU(3)$ gauge theory was reported in Ref. [19].
- The removal of centre vortex degrees of freedom from the gluon fields restores chiral symmetry [18].

But how do these results hold up in QCD where the virtual transition of gluons into dynamical quark-antiquark pairs can significantly alter the vacuum structure [21]. In this presentation we investigate how these dynamical fermions change the centre-vortex structure of the ground-state vacuum fields.

2. Centre Vortex Identification

Centre vortices are identified through a gauge fixing procedure designed to bring the lattice link variables as close as possible to the identity, up to a phase. Here, we consider the original

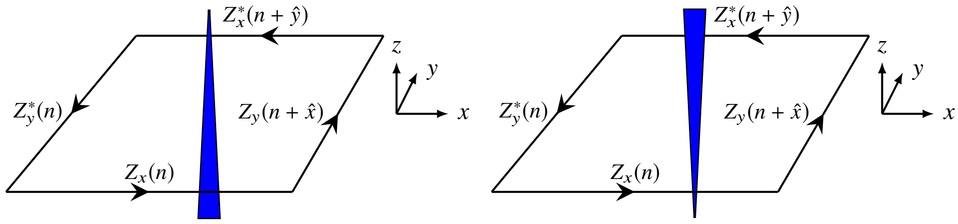

Figure 1: Rendering the centre charge of Eq. (4) associated with a plaquette in the x-y plane at lattice site n. (left) An $m = +1$ vortex with centre charge $z = \exp(2\pi i/3)$ is rendered as a jet pointing in the $+\hat{z}$ direction. (right) An $m = -1$ vortex with centre charge $z = \exp(-2\pi i/3)$ is rendered as a jet in the $-\hat{z}$ direction.

Monte-Carlo generated configurations and gauge transform them directly to Maximal Centre Gauge [13, 22, 23], avoiding any preconditioning [14]. In doing so, the lattice link variables $U_\mu(x)$ are brought close to the centre elements of $SU(3)$,

$$Z = \exp\left(2\pi i\, \frac{m}{3}\right) \mathbf{I}, \text{ with } m = -1, 0, 1. \tag{1}$$

This is implemented through gauge transformations Ω such that,

$$\sum_{x,\mu} \left|\text{tr}\, U_\mu^\Omega(x)\right|^2 \xrightarrow{\Omega} \max\, . \tag{2}$$

One then projects the link variables to the centre

$$U_\mu(x) \longrightarrow Z_\mu(x) \text{ where } Z_\mu(x) = \exp\left(2\pi i\, \frac{m_\mu(x)}{3}\right) \mathbf{I}, \tag{3}$$

where $m_\mu(x) = -1, 0, 1$.

The centre-line of an extended vortex in three dimensions is identified by the presence of nontrivial centre charge, z, found in the product of centre-projected links around a plaquette,

$$z = \prod_{\square} Z_\mu(x) = \exp\left(2\pi i\, \frac{m}{3}\right)\, . \tag{4}$$

A right-handed ordering of the dimensions is selected in calculating the centre charge. If $z = 1$, no vortex pierces the plaquette. If $z \neq 1$ a vortex with charge z pierces the plaquette. We refer to the centre charge of a vortex via the value of $m = \pm 1$.

3. Centre Vortex Visualisation

The centre lines of extended vortices are illustrated on the dual lattice by a jet piercing the plaquette producing a nontrivial centre charge. The orientation of the jet follows a right-handed coordinate system. For example, with reference to Eq. (4), an $m = +1$ vortex in the x-y plane is plotted in the $+\hat{z}$ direction as a blue jet. Similarly, an $m = -1$ vortex in the x-y plane is plotted in the $-\hat{z}$ direction. Figure 1 provides an illustration of this assignment.

As the centre charge transforms to its complex conjugate under permutation of the two dimensions describing the plaquette, the centre charge can be thought of as a directed flow of charge $z = \exp(2\pi i/3)$.

4. Centre Vortex Structure

The projected centre vortices (P-vortices), identified on the lattice as described above, form surfaces in four dimensional space-time, analogous to the centre line of a vortex in fluid dynamics that maps out a surface as it moves through time. As the surface cuts through the three-dimensional volume of our visualisation, a P-vortex line mapping the flow of centre charge is rendered.

Our point of focus in this presentation is to discover the impact of dynamical-fermion degrees of freedom on the vortex structure of a gauge field. Here we the PACS-CS (2 + 1)-flavour full-QCD ensembles [24], made available through the ILDG [25]. These ensembles use a $32^3 \times 64$ lattice, and employ a renormalisation-group improved Iwasaki gauge action with $\beta = 1.90$ and non-perturbatively $O(a)$-improved Wilson quarks, with $C_{\rm SW} = 1.715$. We consider their lightest u- and d-quark-mass ensemble identified by a pion mass of 156 MeV [24], and set the scale using the Sommer parameter with $r_0 = 0.4921$ fm providing a lattice spacing of $a = 0.933$ fm [24].

For comparison, we have created a matched $32^3 \times 64$ pure-gauge ensemble using the same improved Iwasaki gauge action with $\beta = 2.58$ providing a Sommer-scale spacing of $a = 0.100$ fm, facilitating comparisons with all the PACS-CS ensembles.

Figures 2 and 3 illustrate the centre-vortex structure of pure-gauge and dynamical-fermion ground-state vacuum fields respectively. Inspection of the vortices reveals the flow of centre charge, intersection points and a prevalence of branching points resembling monopole or anti-monopole contributions, where three jets emerge from or converge to a point. With the introduction of dynamical fermions the structure becomes more complicated, both in the abundance of nontrivial centre charge and in the proliferation of branching points.

Figures 2 and 3 are interactive illustrations which can be activated in Adobe Reader[1] by clicking on the image. Once activated, click and drag to rotate, Ctrl-click to translate, Shift-click or mouse wheel to zoom, and right click to access the "Views" menu. Several views have been created to facilitate and inspection of the centre-vortex structure. The presence of a percolating vortex cluster is a characteristic feature of the confining phase [26]. These illustrations are representative of the ensemble in that the vortex vacuum is typically dominated by a single large percolating cluster. Moreover, dynamical fermions significantly increase the number of vortices observed.

Considering an ensemble of 200 configurations with 32 three-dimensional slices each, the average number of vortices composing the primary cluster in these $32^2 \times 64$ spatial slices is

- $3,277 \pm 156$ vortices in the Pure Gauge theory, versus
- $5,924 \pm 239$ vortices in Full QCD.

Noting that there are $32^2 \times 64 \times 3 = 196,608$ spatial plaquettes on these lattices, one sees that the presence of a vortex is still a relatively rare occurrences.

Figures 4 and 5 illustrate the secondary loop structure left behind as one removes the single large percolating structure. The introduction of dynamical fermions increases both the number of loops observed, and the complexity of their structure by a proliferation of branching points (or monopoles [27]). Figure 5 contains many views to facilitate the observation of this complexity.

[1]Open this pdf document in Adobe Reader 9 or later. Linux users can install Adobe acroread version 9.4.1, the last edition to have full 3D support. From the "Edit" menu, select "Preferences..." and ensure "3D & Multimedia" is enabled and "Enable double-sided rendering" is selected.

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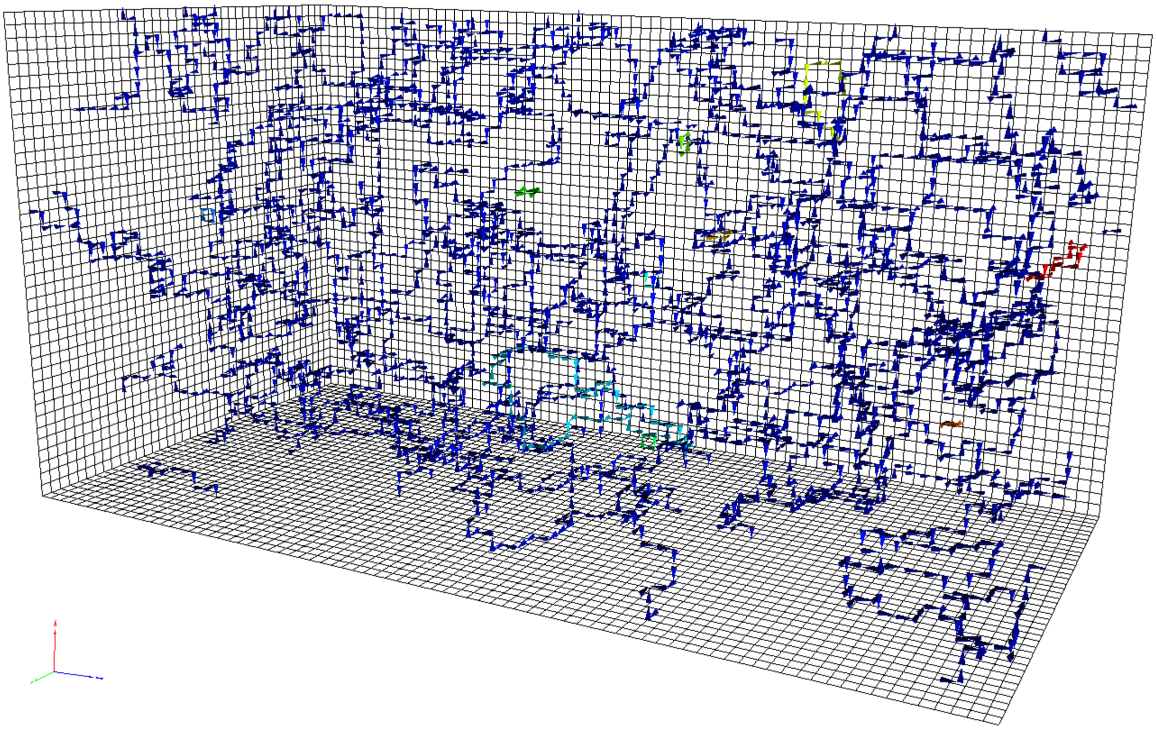

Figure 2: The centre-vortex structure of a ground-state vacuum field configuration in pure SU(3) gauge theory. (*Click to activate.*) The flow of +1 centre charge through a gauge field is illustrated by the jets. Blue jets illustrate the single percolating vortex structure, while other colours illustrate smaller structures.

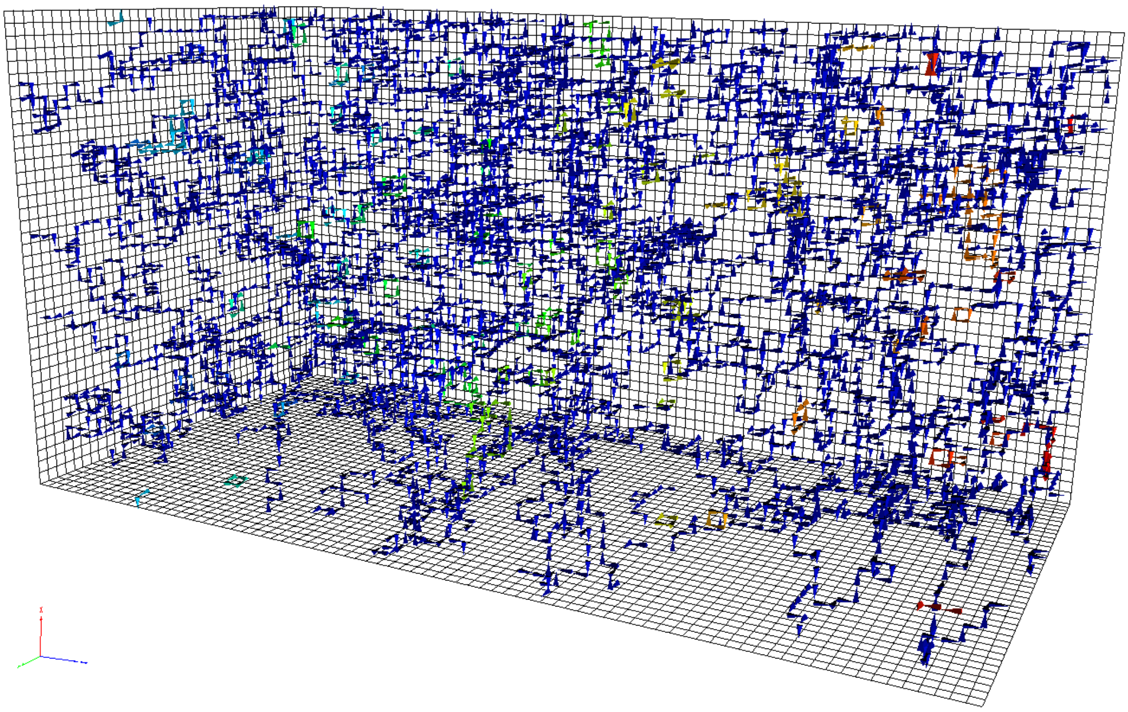

Figure 3: The centre-vortex structure of a ground-state vacuum field configuration in dynamical 2+1 flavour QCD. (*Click to activate.*) The flow of +1 centre charge through a gauge field is illustrated by the jets. Symbols are as described in Fig. 2.

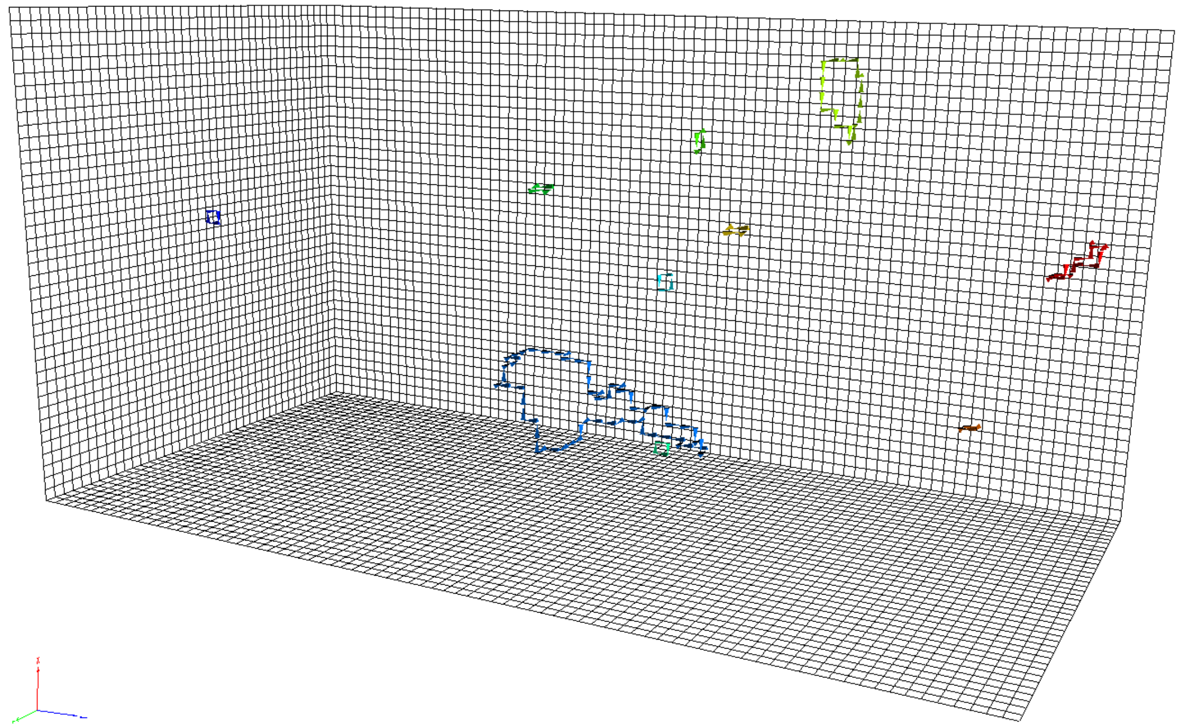

Figure 4: The centre-vortex structure of secondary loops in a ground-state vacuum field configuration of pure SU(3) gauge theory. (*Click to activate.*) The flow of +1 centre charge in the secondary loops – left behind as the single percolating structure is removed – is illustrated.

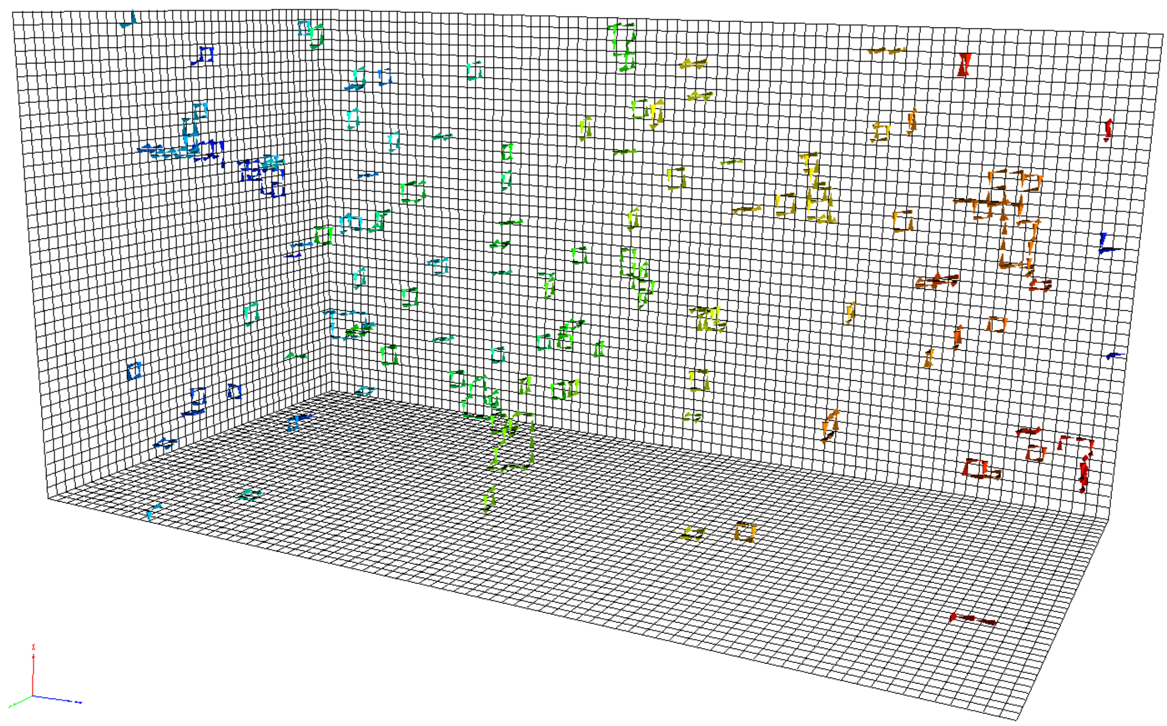

Figure 5: The centre-vortex structure of secondary loops in a ground-state vacuum field configuration of dynamical 2+1 flavour QCD. (*Click to activate.*) The flow of +1 centre charge in the secondary loops is illustrated.

5. Outlook

In summary,

- Centre-vortex structure is complex.
- We observe a proliferation of branching points in $SU(3)$ gauge theory with further enhancement as light dynamical fermion degrees of freedom are introduced in simulating QCD.
- Each ground-state configuration is dominated by a long-distance percolating centre-vortex structure.
- We have observed an approximate doubling in the size of the percolating vortex structure in in going from pure-gauge theory to full QCD.
- We have also observed an enhancement in the number of small vortex paths upon the introduction of dynamical fermions.
- Increased complexity in the vortex paths is also observed as the number of monopole-antimonopole pairs is significantly increased with the introduction of dynamical fermions.

In short, dynamical-fermion degrees of freedom radically alter the centre-vortex structure of the ground-state vacuum fields.

Having gained insight into the impact of dynamical fermions on ground-state vacuum field structure, future work will aim to quantify these effects. Here the distribution of path lengths is of particular interest as it may be possible to quantify the change in the branching probability of vortex paths in $SU(3)$ gauge theory as dynamical fermions are introduced.

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References

[1] G. 't Hooft, "On the Phase Transition Towards Permanent Quark Confinement," *Nucl. Phys. B* **138** (1978) 1–25.

[2] G. 't Hooft, "A Property of Electric and Magnetic Flux in Nonabelian Gauge Theories," *Nucl. Phys. B* **153** (1979) 141–160.

[3] L. Del Debbio, M. Faber, J. Greensite, and S. Olejnik, "Center dominance and Z(2) vortices in SU(2) lattice gauge theory," *Phys. Rev. D* **55** (1997) 2298–2306, arXiv:hep-lat/9610005.

[4] M. Faber, J. Greensite, and S. Olejnik, "Casimir scaling from center vortices: Towards an understanding of the adjoint string tension," *Phys. Rev. D* **57** (1998) 2603–2609, arXiv:hep-lat/9710039.

[5] L. Del Debbio, M. Faber, J. Giedt, J. Greensite, and S. Olejnik, "Detection of center vortices in the lattice Yang-Mills vacuum," *Phys. Rev. D* **58** (1998) 094501, arXiv:hep-lat/9801027.

[6] R. Bertle, M. Faber, J. Greensite, and S. Olejnik, "The Structure of projected center vortices in lattice gauge theory," *JHEP* **03** (1999) 019, arXiv:hep-lat/9903023.

[7] M. Faber, J. Greensite, S. Olejnik, and D. Yamada, "The Vortex finding property of maximal center (and other) gauges," *JHEP* **12** (1999) 012, arXiv:hep-lat/9910033.

[8] M. Engelhardt and H. Reinhardt, "Center projection vortices in continuum Yang-Mills theory," *Nucl. Phys. B* **567** (2000) 249, arXiv:hep-th/9907139.

[9] R. Bertle, M. Faber, J. Greensite, and S. Olejnik, "P vortices, gauge copies, and lattice size," *JHEP* **10** (2000) 007, arXiv:hep-lat/0007043.

[10] J. Greensite, "The Confinement problem in lattice gauge theory," *Prog. Part. Nucl. Phys.* **51** (2003) 1, arXiv:hep-lat/0301023.

[11] M. Engelhardt, M. Quandt, and H. Reinhardt, "Center vortex model for the infrared sector of SU(3) Yang-Mills theory: Confinement and deconfinement," *Nucl. Phys. B* **685** (2004) 227–248, arXiv:hep-lat/0311029.

[12] J. Greensite, "Confinement from Center Vortices: A review of old and new results," *EPJ Web Conf.* **137** (2017) 01009, arXiv:1610.06221 [hep-lat].

[13] K. Langfeld, "Vortex structures in pure SU(3) lattice gauge theory," *Phys. Rev.* **D69** (2004) 014503, arXiv:hep-lat/0307030.

[14] A. O'Cais, W. Kamleh, K. Langfeld, B. Lasscock, D. Leinweber, *et al.*, "Preconditioning Maximal Center Gauge with Stout Link Smearing in SU(3)," *Phys.Rev.* **D82** (2010) 114512, arXiv:0807.0264 [hep-lat].

[15] D. Trewartha, W. Kamleh, and D. Leinweber, "Connection between center vortices and instantons through gauge-field smoothing," *Phys. Rev.* **D92** no. 7, (2015) 074507, arXiv:1509.05518 [hep-lat].

[16] J. C. Biddle, W. Kamleh, and D. B. Leinweber, "Gluon propagator on a center-vortex background," *Phys. Rev.* **D98** no. 9, (2018) 094504, arXiv:1806.04305 [hep-lat].

[17] J. C. Biddle, W. Kamleh, and D. B. Leinweber, "Visualization of center vortex structure," *Phys. Rev. D* **102** no. 3, (2020) 034504, arXiv:1912.09531 [hep-lat].

[18] D. Trewartha, W. Kamleh, and D. Leinweber, "Centre vortex removal restores chiral symmetry," *J. Phys.* **G44** no. 12, (2017) 125002, arXiv:1708.06789 [hep-lat].

[19] D. Trewartha, W. Kamleh, and D. Leinweber, "Evidence that centre vortices underpin dynamical chiral symmetry breaking in SU(3) gauge theory," *Phys. Lett.* **B747** (2015) 373–377, arXiv:1502.06753 [hep-lat].

[20] E.-A. O'Malley, W. Kamleh, D. Leinweber, and P. Moran, "SU(3) centre vortices underpin confinement and dynamical chiral symmetry breaking," *Phys. Rev. D* **86** (2012) 054503, arXiv:1112.2490 [hep-lat].

[21] P. J. Moran and D. B. Leinweber, "Impact of Dynamical Fermions on QCD Vacuum Structure," *Phys. Rev. D* **78** (2008) 054506, arXiv:0801.2016 [hep-lat].

[22] L. Del Debbio, M. Faber, J. Greensite, and S. Olejnik, "Center dominance and Z(2) vortices in SU(2) lattice gauge theory," *Phys. Rev.* **D55** (1997) 2298–2306, hep-lat/9610005.

[23] K. Langfeld, H. Reinhardt, and O. Tennert, "Confinement and scaling of the vortex vacuum of SU(2) lattice gauge theory," *Phys. Lett.* **B419** (1998) 317–321, arXiv:hep-lat/9710068.

[24] **PACS-CS** Collaboration, S. Aoki *et al.*, "2+1 Flavor Lattice QCD toward the Physical Point," *Phys. Rev. D* **79** (2009) 034503, arXiv:0807.1661 [hep-lat].

[25] M. G. Beckett, B. Joo, C. M. Maynard, D. Pleiter, O. Tatebe, and T. Yoshie, "Building the International Lattice Data Grid," *Comput. Phys. Commun.* **182** (2011) 1208–1214, arXiv:0910.1692 [hep-lat].

[26] M. Engelhardt, K. Langfeld, H. Reinhardt, and O. Tennert, "Deconfinement in SU(2) Yang-Mills theory as a center vortex percolation transition," *Phys. Rev. D* **61** (2000) 054504, arXiv:hep-lat/9904004.

[27] F. Spengler, M. Quandt, and H. Reinhardt, "Branching of Center Vortices in SU(3) Lattice Gauge Theory," *Phys. Rev.* **D98** no. 9, (2018) 094508, arXiv:1810.04072 [hep-th].

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