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Mathematical expressions for quantum fluctuations of energy for different energy-momentum tensors

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Expressions for the quantum fluctuations of energy density have been derived for the subsystems consisting of hot relativistic gas of particles with spin-$\frac{1}{2}$ and mass m. Our expressions for the fluctuation depend on the form of energy-momentum tensor which in turn depends on the choice of pseudo-gauge. These results suggest that quantum fluctuations of energy should be considered seriously in the case of very small thermodynamic systems.

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1. Introduction

Quantum and statistical fluctuations intrinsic to any many-body system [1] have an important role as they contain crucial information about the possible phase transitions [1–15], dissipative phenomena [16–34], and the formation of the large scale structures in the universe [35–43]. Following the footsteps of our previous works [44–49], we analyze and study the pseudo-gauge dependence on the quantum fluctuations of energy, which means, using the pseudo-gauge transformation we can choose different form of the energy-momentum tensor for the description of the system. Any energy-momentum tensor $\hat{T}^{\mu\nu}$ satisfying the conservation equation $\partial_\mu \hat{T}^{\mu\nu} = 0$ can be used to construct a new conserved energy-momentum tensor as [50–52]

$$\hat{T}'^{\mu\nu} = \hat{T}^{\mu\nu} + \partial_\lambda \hat{A}^{\nu\mu\lambda} \quad \text{with} \quad \hat{A}^{\nu\mu\lambda} = -\hat{A}^{\nu\lambda\mu}. \tag{1}$$

For the current analysis, we consider a system of spin-$\frac{1}{2}$ particles and study the effects of different pseudo-gauges. We choose three different forms of energy-momentum tensor, such as canonical form from the Noether theorem [53–55], the de Groot-van Leeuwen-van Weert (GLW) form [56], and the Hilgevoord-Wouthuysen (HW) form [57, 58]. These forms are currently being discussed widely in the context of heavy-ion collisions for the study of spin polarization [52, 59], as the pseudo-gauge choices can also be applied for the spin tensor $\hat{S}^{\lambda,\mu\nu}$ which is a part of the total angular momentum tensor $\hat{J}^{\lambda,\mu\nu} = \hat{L}^{\lambda,\mu\nu} + \hat{S}^{\lambda,\mu\nu}$ [51, 52, 60–64]. We calculate the quantum fluctuations of the $\hat{T}^{00}$ component of the energy-momentum tensor and find that even though $\hat{T}^{00}$ depends on pseudo-gauge, its thermal average value is independent of it. In addition, we see that for small size subsystems the fluctuations are pseudo-gauge dependent but become independent if the size of the system is large. This analysis might be useful for the understanding of the concept of energy-density in the context of relativistic heavy-ion collisions [65–76]. Our conclusion is that the quantum fluctuations of energy-density have no physical significance in small size subsystems and a specific pseudo-gauge must be chosen in order to describe the system.

2. Basic definitions

As in our previous studies [44–46], here we assume a subsystem S_a inside a larger thermodynamic system S_V consisting of spin-$\frac{1}{2}$ particles having mass m with no conserved charges. The volume V of the system S_V is large enough to perform integrals over particle momentum. [1] We describe our system by a spin-$\frac{1}{2}$ field in thermal equilibrium where the field operator is [77]

$$\psi(t,\boldsymbol{x}) = \sum_r \int \frac{d^3k}{(2\pi)^3\sqrt{2\omega_{\boldsymbol{k}}}} \Big(U_r(\boldsymbol{k}) a_r(\boldsymbol{k}) e^{-ik\cdot x} + V_r(\boldsymbol{k}) b_r^\dagger(\boldsymbol{k}) e^{ik\cdot x} \Big), \tag{2}$$

with $a_r(\boldsymbol{k})$ and $b_r^\dagger(\boldsymbol{k})$ being the annihilation and creation operators for particles and antiparticles, respectively, satisfying the anti-commutation relations, $\{a_r(\boldsymbol{k}), a_s^\dagger(\boldsymbol{k}')\} = (2\pi)^3 \delta_{rs} \delta^{(3)}(\boldsymbol{k} - \boldsymbol{k}')$ and $\{b_r(\boldsymbol{k}), b_s^\dagger(\boldsymbol{k}')\} = (2\pi)^3 \delta_{rs} \delta^{(3)}(\boldsymbol{k} - \boldsymbol{k}')$. The index r is the polarization degree of freedom, and $U_r(\boldsymbol{k})$ and $V_r(\boldsymbol{k})$ are the Dirac spinors where $\omega_{\boldsymbol{k}} = \sqrt{\boldsymbol{k}^2 + m^2}$ is the energy of a particle.

[1]Metric $g_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$ is used. Three-vectors are shown in bold font and a dot is used to denote the scalar product of both four- and three-vectors, i.e., $a^\mu b_\mu = a \cdot b = a^0 b^0 - \boldsymbol{a} \cdot \boldsymbol{b}$.

The following expectation values are required to calculate thermal averages [78–80] of the energy-density operator $\hat{T}_a^{00}$

$$\langle a_r^\dagger(\boldsymbol{k}) a_s(\boldsymbol{k}')\rangle = (2\pi)^3 \delta_{rs}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}') f(\omega_{\boldsymbol{k}}), \tag{3}$$

$$\langle a_r^\dagger(\boldsymbol{k}) a_s^\dagger(\boldsymbol{k}') a_{r'}(\boldsymbol{p}) a_{s'}(\boldsymbol{p}')\rangle = (2\pi)^6 \Big(\delta_{rs'}\delta_{r's}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p}')\,\delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}) - \delta_{rr'}\delta_{ss'}\delta^{(3)}(\boldsymbol{k}-\boldsymbol{p})\,\delta^{(3)}(\boldsymbol{k}'-\boldsymbol{p}')\Big) f(\omega_{\boldsymbol{k}}) f(\omega_{\boldsymbol{k}'}). \tag{4}$$

where $f(\omega_{\boldsymbol{k}})$ is the Fermi–Dirac distribution function for particles. $\hat{T}_a^{00}$ is an energy-density operator expressed below of the subsystem S_a which is placed at the origin of coordinate system [50], and we use Gaussian profile to define our subsystem S_a in order to have no sharp-boundary effects

$$\hat{T}_a^{00} = \frac{1}{(a\sqrt{\pi})^3}\int d^3\boldsymbol{x}\; \hat{T}^{00}(x)\; \exp\left(-\frac{\boldsymbol{x}^2}{a^2}\right). \tag{5}$$

Then, we calculate the variance (σ^2) and the normalized standard deviation (σ_n) using the expressions below in order to find the fluctuation of the energy density of the subsystem S_a,

$$\sigma^2(a,m,T) = \langle :\hat{T}_a^{00} :: \hat{T}_a^{00}:\rangle - \langle :\hat{T}_a^{00}:\rangle^2, \quad \sigma_n(a,m,T) = \frac{\left(\langle :\hat{T}_a^{00} :: \hat{T}_a^{00}:\rangle - \langle :\hat{T}_a^{00}:\rangle^2\right)^{1/2}}{\langle :\hat{T}_a^{00}:\rangle}. \tag{6}$$

where $\langle :\hat{T}_a^{00}:\rangle$ is the thermal expectation value of $\hat{T}_a^{00}$ after doing normal ordering.

3. Energy density fluctuation in different pseudo-gauges

3.1 Canonical framework

The canonical form of energy-momentum tensor is given as [77]

$$\hat{T}^{\mu\nu}_{\rm Can} = \frac{i}{2}\bar{\psi}\gamma^\mu \overleftrightarrow{\partial}^\nu \psi. \tag{7}$$

where the thermal expectation value of $\hat{T}^{00}_{{\rm Can},a}$ for the subsystem S_a is calculated as

$$\langle :\hat{T}^{00}_{{\rm Can},a}:\rangle = 4\int \frac{d^3k}{(2\pi)^3}\,\omega_{\boldsymbol{k}}\; f(\omega_{\boldsymbol{k}}) \equiv \varepsilon_{\rm Can}(T). \tag{8}$$

with the factor 4 representing the spin degeneracy ($g_s = (2s+1)$). The canonical energy-density $\varepsilon_{\rm Can}(T)$, Eq. (8), is independent of both time and the system size a indicating the system's spatial uniformity. Then we calculate the energy-density fluctuation for $\hat{T}^{\mu\nu}_{\rm Can}$ as

$$\sigma^2_{\rm Can}(a,m,T) = 2\int dK\, dK' f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'})) \times \Big[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}'})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' + m^2) e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}'})^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' - m^2) e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\Big], \tag{9}$$

where $dK \equiv d^3k/((2\pi)^3 2\omega_{\boldsymbol{k}})$. In Eq. 9, we neglect a temperature-independent term to remove all vacuum divergences [44].

3.2 de Groot-van Leeuwen-van Weert framework

The de Groot-van Leeuwen-van Weert form of energy-momentum tensor is given as [56]

$$\hat{T}^{\mu\nu}_{\rm GLW} = \frac{1}{4m}\Big[-\bar{\psi}(\partial^\mu\partial^\nu\psi) + (\partial^\mu\bar{\psi})(\partial^\nu\psi) + (\partial^\nu\bar{\psi})(\partial^\mu\psi) - (\partial^\mu\partial^\nu\bar{\psi})\psi \Big]. \tag{10}$$

In this case we obtain expressions for the thermal average and fluctuation as

$$\langle :\hat{T}^{00}_{{\rm GLW},a}: \rangle = 4\int \frac{d^3k}{(2\pi)^3}\,\omega_{\boldsymbol{k}}\, f(\omega_{\boldsymbol{k}}) \equiv \varepsilon_{\rm GLW}(T) \tag{11}$$

$$\begin{aligned}\sigma^2_{\rm GLW}(a,m,T) = \frac{1}{2m^2}\int dK dK' f(\omega_{\boldsymbol{k}})(1-f(\omega_{\boldsymbol{k}'}))\Big[(\omega_{\boldsymbol{k}}+\omega_{\boldsymbol{k}'})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} - \boldsymbol{k}\cdot\boldsymbol{k}' + m^2\right)\\ \times e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}-\omega_{\boldsymbol{k}'})^4\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} - \boldsymbol{k}\cdot\boldsymbol{k}' - m^2\right)e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\Big]\end{aligned} \tag{12}$$

respectively. We again discard a divergent term which is temperature-independent. We note that, even though the thermal averages for the canonical and GLW energy-momentum tensors are the same, $\langle :\hat{T}^{00}_{{\rm Can},a}: \rangle = \langle :\hat{T}^{00}_{{\rm GLW},a}: \rangle$, their fluctuations are not, $\sigma^2_{\rm Can}(a,m,T) \neq \sigma^2_{\rm GLW}(a,m,T)$.

3.3 Hilgevoord-Wouthuysen framework

The Hilgevoord-Wouthuysen form of energy-momentum tensor is defined as below [57, 58]

$$\hat{T}^{\mu\nu}_{\rm HW} = \hat{T}^{\mu\nu}_{\rm Can} + \frac{i}{2m}\left(\partial^\nu\bar{\psi}\sigma^{\mu\beta}\partial_\beta\psi + \partial_\alpha\bar{\psi}\sigma^{\alpha\mu}\partial^\nu\psi\right) - \frac{i}{4m}g^{\mu\nu}\partial_\lambda\left(\bar{\psi}\sigma^{\lambda\alpha}\overleftrightarrow{\partial}_\alpha\psi\right), \tag{13}$$

with $\sigma_{\mu\nu} \equiv (i/2)\left[\gamma_\mu,\gamma_\nu\right]$. Here the thermal average and fluctuation are calculated respectively as

$$\langle :\hat{T}^{00}_{{\rm HW},a}: \rangle = 4\int \frac{d^3k}{(2\pi)^3}\,\omega_{\boldsymbol{k}}\, f(\omega_{\boldsymbol{k}}) \equiv \varepsilon_{\rm HW}(T) \tag{14}$$

$$\begin{aligned}\sigma^2_{\rm HW}(a,m,T) = \frac{2}{m^2}\int dK dK' f(\omega_{\boldsymbol{k}})\Big[\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' + m^2\right)^2\left(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} - \boldsymbol{k}\cdot\boldsymbol{k}' + m^2\right)\\ \times e^{-\frac{a^2}{2}(\boldsymbol{k}-\boldsymbol{k}')^2} - (\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} + \boldsymbol{k}\cdot\boldsymbol{k}' - m^2)^2(\omega_{\boldsymbol{k}}\omega_{\boldsymbol{k}'} - \boldsymbol{k}\cdot\boldsymbol{k}' - m^2)\times e^{-\frac{a^2}{2}(\boldsymbol{k}+\boldsymbol{k}')^2}\Big](1-f(\omega_{\boldsymbol{k}'})).\end{aligned} \tag{15}$$

It can be seen easily from Eqs. (8), (11), and (14) that $\varepsilon_{\rm Can}(T) = \varepsilon_{\rm GLW}(T) = \varepsilon_{\rm HW}(T)$, while the fluctuations of $:\hat{T}^{00}_a:$ are different for different choices of pseudo-gauge. Eqs. (9), (12), and (15) are used to calculate the fluctuations of the energy-density of the subsystem S_a of the larger system S_V. Both the energy density (ε) and fluctuation (σ) can be extended to incorporate other degeneracy factors such as isospin or color charge degrees of freedom.

4. Summary

We have calculated the mathematical expressions for quantum energy-density fluctuations for the subsystems of hot relativistic gas of spin-$\frac{1}{2}$ particles. Our results show that even though the energy-density for all choices of pseudo-gauge are the same, still their fluctuations depend on the forms of pseudo-gauge, which means that the quantum fluctuations are pseudo-gauge dependent [81–83] and should be kept in mind during the experimental measurements.

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