PROCEEDINGS OF SCIENCE

Unbiased proton intermittency analysis for the detection of the QCD critical endpoint in ion collisions

F.K. Diakonos[*][†], **N.G.Antoniou**[‡], **A.S. Kapoyannis, E. Stiliaris**
Department of Physics, University of Athens, GR-15784 Athens, Greece
E-mails: fdiakono@phys.uoa.gr, akapog@phys.uoa.gr, stiliaris@phys.uoa.gr

N. Davis, V. Ozvenchuk, A. Rybicki
Institute of Nuclear Physics, Polish Academy of Sciences, Kraków, Poland
E-mails: Nikolaos.Davis@ifj.edu.pl, Vitalii.Ozvenchuk@ifj.edu.pl, Andrzej.Rybicki@ifj.edu.pl

G. Doultsinos, N. Kalntis, A. Kanargias
Department of Physics, University of Athens, GR-15784 Athens, Greece
E-mails: georgedoultsinos@gmail.com, nikos.kalntis@gmail.com, akanaryias@hotmail.com

C.N. Papanicolas
Department of Physics, University of Athens, GR-15784 Athens, Greece
The Cyprus Institute, Nicosia, Cyprus
E-mail: cnp@phys.uoa.gr, president@cyi.ac.cy

We develop an unbiased intermittency analysis scheme based on the AMIAS [1] protocol. We apply it to analyse fluctuations in transverse momentum space of protons produced in ion collisions at CERN SPS experiments NA49 and NA61/SHINE, our main goal being the unbiased determination of the intermittency index ϕ_2 [2]. We find a gradual enhancement of ϕ_2 as the production source of the considered ensemble of events changes from central to peripheral Ar+Sc collisions at $\sqrt{s_{NN}} = 16.8$GeV (NA61/SHINE) and subsequently to central Si+Si collisions at $\sqrt{s_{NN}} = 17.3$GeV (NA49). A possible explanation of these results is the gradual proximity to the QCD critical endpoint, remnant of the chiral transition, which should be located in the immediate neighbourhood of the Si+Si freeze-out state.

Corfu Summer Institute 2021 'School and Workshops on Elementary Particle Physics and Gravity'
29 August-8 September 2021
Corfu, Greece

[*]Speaker.
[†]Dedicated to the memory of Nikos Antoniou.
[‡]Deceased.

PoS(CORFU2021)013

1. Introduction

The search for the QCD critical endpoint (CEP), remnant of the chiral transition in quark matter, is the main task of the current ion collision experiments worldwide. As a unique signal for the CEP is theoretically proposed the power-law behaviour of the density-density correlation function of protons for small transverse momentum differences [2, 3]. In fact, it represents the realization of critical opalescence in particle physics and the associated power-law exponent is directly related to the isothermal critical exponent of the underlying transition. Intermittency analysis employs finite-size scaling theory to provide an experimentally accessible way to determine this power-law exponent [4]. In the context of intermittency analysis this exponent is encoded in the *intermittency index* [2, 4]. The measurement of the latter constitutes a direct path to the detection of the CEP.

In intermittency analysis one calculates the correlator ΔF_2 associated with scaled factorial moment F_2 [5] after subtraction of the background (simulated by mixed events). The calculation of F_2 is performed in the transverse momentum space of protons at midrapidity ($|y| < 0.75$). This space is divided in M^2 cells and the mean number of proton pairs per cell is estimated in an event-by-event basis. In the presence of critical fluctuations, as the number of the cells M^2 is varied, one expects the behaviour $\Delta F_2(M) \sim M^{2\phi_{2,cr}}$ with $\phi_{2,cr} = \frac{2-d_F}{2}$ and $M \gg 1$. Thus, the critical intermittency index $\phi_{2,cr}$ is directly related to the fractal dimension d_F describing the geometry of the proton clusters in transverse momentum space at the critical point. The fractal dimension d_F in turn is related to the isothermal critical exponent δ, characterizing the transition, through the relation $d_F = \frac{2}{\delta+1}$. Assuming that the chiral transition belongs to the 3D Ising universality class, as supported by strong theoretical arguments [6], then, since $\delta \approx 5$, we expect that $\phi_{2,cr} \approx \frac{5}{6}$.

Although the basic tool of the intermittency analysis, i.e. the (second) factorial moment, possess a strict and simple definition, its successful application to experimental data and in particular the interpretation of the corresponding outcome require some additional effort. The reasons for that have both theoretical as well as experimental origin. From the theoretical point of view there are two important questions to be answered: (i) since the considered physical system has finite size it is expected that, even if the corresponding freeze-out state lies exactly at the CEP, the power-law behaviour of the correlator will only hold between an upper and a lower scale in transverse momentum space. Which are these scales? And (ii) what happens with the power-law behaviour if the freeze-out state lies close but not exactly at the CEP? Concerning the first question one can make a rough estimation of the scales based on the typical fireball size and on the range of the hard core of the two-proton nuclear potential. Using for the former an approximate size of 7fm and for the later an approximate range of 0.3fm one finds an estimation of 0.04MeV for the lower and 600MeV for the upper scale in transverse momentum space. To answer the second question one has to utilize the Ising-QCD partition function introduced in [7] to calculate the net proton multiplicity fluctuations in the neighbourhood of the critical point. As stated in [7] one observes a gradual dilution of the power-law correlations departing from the critical point. This dilution is expressed in a twofold way: the intermittency index gets a value $\phi_2 \neq \phi_{2,cr}$ while at the same time the M^2-range for which the power-law description of these correlations is valid, shrinks continuously.

In addition to the theoretical questions of the previous paragraph there are some important experimental issues which should be taken into account when the intermittency analysis is applied to data. These can be classified in principal and methodological. In the principal issues belong the proton identification and the set of appropriate cuts to avoid artificial correlations induced by split tracks. Both are crucial for the intermittency analysis. In practice, the efficiency in proton identification is quantified by purity [8] while the cuts for the removal of split tracks are discussed in [9]. Of course the first (purity) introduces systematic errors in the analysis while the second (split track cuts) influence the statistics. The methodological issues are more involved. The first is related to the fact that for a given experimental set the values of the factorial moment $F_2(M_1)$ and $F_2(M_2)$ for two different choices of the number of cells M_1^2 and M_2^2 are strongly correlated. The second issue concerns the region of M^2-values in which the power-law fit should be performed. The choice of this region introduces a strong bias which in combination with the correlations mentioned previously and the presence of large statistical errors, particularly for $M^2 \gg 1$, makes a reliable estimation of the intermittency index ϕ_2 an extremely difficult task.

In the present work we will perform a first step towards an unbiased estimation of ϕ_2 from experimental data. To this end we will employ the AMIAS, a model independent analysis scheme [1], to calculate distributions for ϕ_2 values for different data sets containing proton transverse momenta measured in NA49 and NA61/SHINE experiments at highest SPS (CERN) energy. The choice of the analysed data sets is guided by previous published results from NA49 experiment [9] and released preliminary data from NA61/SHINE experiment [10, 11]. More specifically, in [9] it has been reported the presence of unconventional fluctuations in transverse momentum space of protons produced in central Si+Si collisions. The value $\phi_2 = 0.96^{+0.38}_{-0.25}$ found in this analysis was compatible within errors with the critical value $\phi_{2,cr} = \frac{5}{6}$ however the methodological issues mentioned above prevented a strong interpretation of the obtained result. More recently, the transverse momentum intermittency analysis of protons produced in Ar+Sc collisions at different peripheralities in NA61/SHINE experiment led to the observation of enhanced correlations in the $10-20\%$ peripherality zone which however were much weaker than in the Si+Si case. In addition, the large errors in the performed analysis did not allow to draw some conclusions concerning the presence and the interpretation of these correlations. As it will be shown in section 3 the AMIAS treatment of proton intermittency provides clearer results for the NA49 Si+Si and NA61/SHINE Ar+Sc data.

The remaining of the paper is organized as follows. In section 2 we present briefly the AMIAS scheme. In section 3 we present the results obtained by the application of AMIAS to the NA49 and NA61/SHINE data described before. We also provide an alternative parametrization of the new results allowing for a more transparent physical interpretation. Finally in section 4 we give our concluding remarks.

2. A short survey of AMIAS

The AMIAS method [1] is designed to extract physical information from experimental or simulated data with the highest possible precision and in an unbiased way. It is based on statistical concepts and is able to handle a rather large number of parameters using Monte Carlo techniques. The method requires the definition of a theory or model which links the parameters to be determined in an explicit way with the data. Although AMIAS is well suited to resolve physical parameters from data where the underlying model cannot be inverted, it can equally well address simple fit cases with noisy data.

For a given set of parameters that defines the physical model, the principal idea behind AMIAS is that any arbitrary value assigned to the parameters constitutes a possible solution. Each of these solutions is weighted with a probability value retrieved out of the data set by a cost function. The information obtained from the given set of data is a Probability Density Function (PDF) assigned to each of the model parameters. The central values of the parameters and their uncertainties are therefore the expectation values and the standard deviations of the corresponding PDFs. The sampling method used in AMIAS allows all fit parameters to randomly vary and to yield solutions with all allowed values, including the insensitive exponential terms.

Here our aim is to use the AMIAS protocol for the detection of power-law behaviour in the proton correlator $\Delta F_2(M)$ in transverse momentum space for $M \gg 1$. Using this correlator as input we will employ AMIAS to determine the associated intermittency index ϕ_2. The power of AMIAS is that all quantities related to the theoretical model which is checked, can be incorporated in the AMIAS protocol as parameters for which it will be searched. To provide an example, even the range of M^2 values, within which the power-law is assumed to be valid, can be considered as parameter of the model.

Before applying AMIAS to detect a power-law behaviour in the correlators $\Delta F_2(M)$ calculated from NA49 and NA61/SHINE data, we checked its ability to detect a power-law behaviour in a simulated data set containing 99.3% protons with randomly chosen, uncorrelated transverse momenta and 0.7% protons with Lévy distributed transverse momentum differences. The power-law exponent in the correlator of the Lévy component was chosen to coincide with the critical intermittency index $\phi_{2,cr} = \frac{5}{6}$ associated with the QCD CEP. The result of this analysis is shown in Fig. 1. It is remarkable that the AMIAS method allows the determination of ϕ_2 with an accuracy

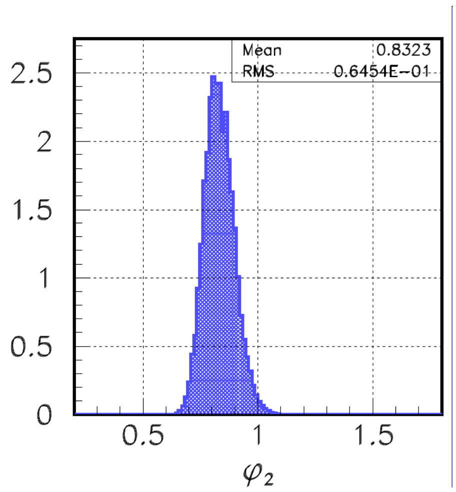

Figure 1: The non-normalized distribution of ϕ_2 values obtained by the AMIAS scheme applied to simulated data with 99.3% noise. The correlator of the very weak Lévy component in the absence of noise should lead to $\phi_2 = \phi_{2,cr} = \frac{5}{6}$.

better than 8% despite the huge noise. More explicitly, the AMIAS result is $\langle \phi_2 \rangle = 0.83 \pm 0.06$ which should be compared with $\phi_{2,cr} = \frac{5}{6}$. This is a clear indication of the ability of AMIAS to overcome the methodological difficulties accompanying the intermittency analysis, as discussed in the previous section.

3. Application of AMIAS to the intermittency analysis of SPS data

In this section we first employ AMIAS to calculate the intermittency index ϕ_2 for the proton transverse momentum correlator $\Delta F_2(M)$ measured in Si+Si central collisions at 158A GeV/c beam momentum attempting to improve the result of the NA49 experiment published in [9]. Next we estimate the intermittency index ϕ_2 for the NA61/SHINE preliminary measurements of the proton transverse momentum correlator ΔF_2 in Ar+Sc collisions at 150A GeV/c beam momentum for different peripheralities in the range $0-20\%$ [11]. We concentrate on Ar+Sc collisions since the associated freeze-out states are expected to lie relatively close to the freeze-out state produced in central Si+Si collisions in the NA49 experiment where intermittent fluctuations with critical characteristics have been observed [9]. Furthermore, we consider various peripheralitites since there is experimental evidence as well as theoretical understanding [12] that changes in the peripherality influence the freeze-out conditions in a prescribed mild manner.

In fact it is expected that more peripheral Ar+Sc collisions can approach the freeze-out conditions of central Si+Si collisions at similar energy. Unfortunately, experimental limitations restrict the range of peripherality variation in the interval $0-20\%$. In the present analysis we have used the proton transverse momentum correlator $\Delta F_2(M)$ calculated in the peripherality zones $0-5\%$, $5-10\%$, $10-15\%$, $15-20\%$, $0-10\%$, $10-20\%$ and $0-20\%$. For each such a dataset we have determined the distribution of ϕ_2 and in addition we have calculated the ϕ_2-distribution for the correlator measured in central Si+Si collisions at the same energy. The obtained results are summarized in Table I. In the same table we include some information concerning the freeze-out conditions of each formed final state. We parametrize this information through the number of wounded nucleons N_w which can be estimated by means of a geometrical Glauber simulation [13].

Reaction	centrality (%)	N_w	$\langle\phi_2\rangle$ $(\delta\phi_2)$
Ar+Sc	0-10	62(0.6)	-0.77(49)
Ar+Sc	10-20	45.9(0.5)	0.39(08)
Ar+Sc	0-5	66.6(0.9)	-0.51(34)
Ar+Sc	5-10	57.3(0.4)	–
Ar+Sc	10-15	49.4(0.4)	0.38(10)
Ar+Sc	15-20	42.4(0.5)	0.41(12)
Ar+Sc	0-20	54(0.6)	0.16(06)
Si+Si	0-12	37(3)	0.92(18)

Table 1: AMIAS $\langle\phi_2\rangle$ and corresponding error $\delta\phi_2$ results vs the estimated mean number of wounded nucleons N_w for central Si+Si and different Ar+Sc peripherality ranges.

It is interesting to plot the intermittency index ϕ_2, calculated with AMIAS, as a function of the number of wounded nucleons, similarly to the plot presented in [14] using ϕ_2-values obtained through ordinary fitting procedure. In Fig. 2 we show such a plot including only systems with

$N_w < 55$. Since N_w is a quantity related to the effective size of the produced fireball, it is expected to depend smoothly on the freeze-out temperature T [15]. We clearly observe the tendency of the most peripheral Ar+Sc freeze-out states to approach the Si+Si freeze-out state since N_w is decreasing with increasing peripherality. It is remarkable that, at the same time, the intermittency index ϕ_2 increases too, indicating the proximity to the critical region. Clearly the ϕ_2 value for the freeze-out state associated with the most peripheral Ar+Sc collisions is still far from the critical value (0.41 compared to 0.83) indicating that this system has not entered yet into the critical region [7]. The overall picture provides us with a rough estimation of the size of the critical region quantified by the differences $|\phi_2 - \frac{5}{6}|$ and $|N_w - N_{w,Si}|$.

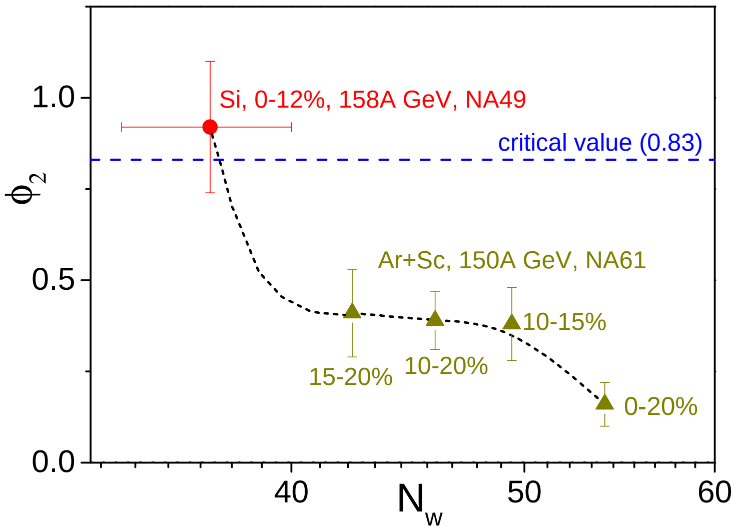

Figure 2: The intermittency index ϕ_2 versus the number of wounded nucleons N_w for the systems: Si+Si at 158A GeV/c and peripherality $0-12\%$ (NA49 experiment, red circle), Ar+Sc at 150A GeV/c and peripheralities $0-20\%$, $10-20\%$, $10-15\%$ and $15-20\%$ (NA61/SHINE experiment, light green triangles). The dashed blue line indicates the critical ϕ_2 value $\phi_{2,cr} = \frac{5}{6}$.

4. Concluding remarks

We have presented an improved intermittency analysis scheme employing the AMIAS protocol for the unbiased determination of the intermittency index ϕ_2 in proton transverse momentum space for data sets produced in Ar+Sc (NA61/SHINE experiment) and Si+Si (NA49 experiment) collisions at highest SPS (CERN) energy. The Ar+Sc system has been analysed for different peripheralities. The obtained results indicate a slight increase of ϕ_2 as the peripherality increases. In addition, the AMIAS analysis for the Si+Si system leads to a ϕ_2 value compatible with the NA49 ϕ_2 measurement [9] supporting further the scenario that the CEP is in the vicinity of the Si+Si freeze-out state. An interesting parametrization of the different freeze-out states, which has been analysed with AMIAS, is provided by the number of wounded nucleons N_w. It allows for an alternative representation of the intermittency analysis results in the (N_w, ϕ_2) plane. The freeze-out states of Ar+Sc with decreasing N_w (increasing peripherality) form a smooth path in the (N_w, ϕ_2) plane indicating the proximity to the critical endpoint located approximately at $(38, 0.8\bar{3})$. Apparently, the Si+Si freeze-out state is very close to this location. In the performed AMIAS analysis the parameter space consisted of two parameters: the power-law exponent ϕ_2 and the associated amplitude $\Delta F_2(1)$. The success of the performed analysis suggests that it would be interesting to enlarge the parametric space in the AMIAS search, adding as parameters the upper and lower scale of the validity of the power-law. Furthermore, one could also circumvent the calculation of the correlator and apply the AMIAS protocol directly to the factorial moment $F_2(M)$. To achieve this one should add to the parameter space also the noise level as an additional parameter. In such a case no subtraction of the background, simulated by mixed events, would be needed. Thus, the AMIAS based intermittency analysis provides a general platform for an improved and unbiased search for traces of critical fluctuations in ion collision data.

References

[1] E. Stiliaris, C. N. Papanicolas, AIP Conf. Proc. **904**, 257 (2007); C. N. Papanicolas, E. Stiliaris, arXiv:1205.6505 (2012).

[2] N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis and K. S. Kousouris, Phys. Rev. Lett. **97**, 032002 (2006).

[3] Y. Hatta and M. A. Stephanov, Phys. Rev. Lett. **91**, 102003 (2003).

[4] N. G. Antoniou, F. K. Diakonos, X. N. Maintas and C. E. Tsagkarakis, Phys. Rev. D **97**, 034015 (2018).

[5] A. Bialas and R. Peshanski, Nucl. Phys. B **273**, 703 (1986); A. Bialas and R. Peshanski, Nucl. Phys. B **308**, 857 (1988).

[6] S. Gavin, A. Gocksch and R. D. Pisarski, Phys. Rev. D **49**, 3079 (1994); M. Stephanov, K. Rajagopal and E. Shuryak, Phys. Rev. Lett. **81**, 4816 (1998); M. A. Halasz, A. D. Jackson, R. E. Shrock, M. A. Stephanov and J. J. M. Verbaarschot, Phys. Rev. D **58**, 096007 (1998); J. Berges and K. Rajagopal, Nucl. Phys. B **538**, 215 (1999); F. Karsch, E. Laermann and Ch. Schmidt, Phys. Lett. B **520**, 41 (2001).

[7] N. G. Antoniou and F. K. Diakonos, J. Phys. G: Nucl. Part. Phys. **46**, 035101 (2019).

[8] N. Davis, *"Searching for the chiral critical point of quark matter in relativistic ion collisions"* (PhD thesis), http://hdl.handle.net/10442/hedi/36113.

[9] T. Anticic *et al.*, Eur. Phys. J. C **75**, 587 (2015).

[10] M. Mackowiak-Pawlowska [NA61/SHINE Collaboration], Nucl. Phys. A **1005**, 121753 (2021), arXiv:2002.04847 [nucl-ex].

[11] N. Davis [NA61/SHINE Collaboration], Acta Phys. Polon. Supp. **13**, 637 (2020), arXiv:2002.06636 [nucl-ex].

[12] F. Becattini, J. Manninen, M. Gazdzicki, Phys. Rev. C **73**, 044905 (2006); F. Becattini, M. Bleicher, E. Grossi, J. Steinheimer and R. Stock, Phys. Rev. C **90**, 054907 (2014).

PoS(CORFU2021)013

[13] R. J. Glauber, Nucl. Phys. A **774**, 3 (2006).

[14] S. Pulawski [NA61/SHINE Collaboration], *"Status and plans of the NA61 Experiment,"* News from the Experiments at CERN (131st Meeting of the SPSC), 16-17 October 2018, https://indico.cern.ch/event/758114.

[15] T. Anticic *et al.*, Phys. Rev. C **70**, 034902 (2004).