

Direct CP violation in charmed meson decays within the standard model

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Direct CP violation in two-body $D \rightarrow PP$ and $D \rightarrow VP$ decays is studied within the framework of the topological amplitude approach for tree amplitudes and the QCD factorization approach for penguin amplitudes. It is the interference between tree and *long-distance* penguin and penguin-exchange amplitudes that pushes the CP asymmetry difference between $D^0 \rightarrow K^+K^-$ and $\pi^+\pi^-$ modes up to the per mille level. Using the same mechanism, we find that the CP asymmetry can also occur at the 10^{-3} level in many of the $D \rightarrow VP$ channels or otherwise be negligibly small. There are six golden modes which have sufficiently large branching fractions and direct CP violation at the per mille level. In particular, the direct CP asymmetry difference between $D^0 \rightarrow K^+K^{*-}$ and $\pi^+\rho^-$ is predicted to be $(-1.61 \pm 0.33) \times 10^{-3}$, which is very similar to the counterpart in the PP sector. The LHCb's observation of CP asymmetry difference can be explained within the standard model without the need of new physics. The key lies in the long-distance penguin topology (penguin and penguin-exchange) arising from final-state rescattering.

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1. Introduction

In November of 2011 LHCb announced the first evidence of CP violation in the charm sector. A nonzero value for the difference between the time-integrated CP asymmetries of the decays $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ [1]

$$\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -(0.82 \pm 0.21 \pm 0.11)\% \quad (1)$$

was reported. The significance of the measured deviation from zero is 3.5σ . This had triggered a flurry of studies exploring whether this CP violation in the charm sector was consistent with the standard model (SM) or implies new physics (NP). However, the original evidence of CP asymmetry difference was gone in 2013 and 2014 when LHCb started to use the muon tag to identify the D^0 and found a positive ΔA_{CP} [2]. In 2019, LHCb finally reported the first observation of CP asymmetry in the charm system with the result at the per mille level [3]

$$\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}. \quad (2)$$

The time-integrated asymmetry can be further decomposed into a direct CP asymmetry a_{CP}^{dir} and a mixing-induced indirect CP asymmetry characterized by the parameter ΔY_f which measures the asymmetry between $D^0 \rightarrow f$ and $\bar{D}^0 \rightarrow f$ effective decay widths; that is, $A_{CP}(f, t) \approx a_{CP}^{\text{dir}}(f) + (t/\tau(D^0))\Delta Y_f$. Based on the LHCb average of ΔY , it follows that the direct CP asymmetry difference is given by [3]

$$\Delta a_{CP}^{\text{dir}} = (-1.57 \pm 0.29) \times 10^{-3}. \quad (3)$$

In 2012 there existed two independent studies in which direct CP violation in charmed meson decays was explored based on the topological diagram approach for tree amplitudes and the QCD-inspired approach for penguin amplitudes [5–7]. Interestingly, both works predicted a ΔA_{CP} at the per mille level with a negative sign seven years before the LHCb's announcement of the first observation of CP violation in the charm sector.

2. CP violation in $D \rightarrow PP$ decays

The amplitude of the singly Cabibbo-suppressed charmed meson decay in general reads

$$A(D \rightarrow PP) = \lambda_d(\text{tree} + \text{penguin}) + \lambda_s(\text{tree}' + \text{penguin}'), \quad (4)$$

where $\lambda_p \equiv V_{cp}^* V_{up}$. Direct CP asymmetry can occur at the tree level if the tree amplitudes denoted by “tree” and “tree'” have a nontrivial strong phase difference. For example, consider the decay $D_s^+ \rightarrow K^+\eta$ with the quark-diagram amplitudes

$$A(D_s^+ \rightarrow K^+\eta) = \frac{1}{\sqrt{2}} [\lambda_d(C + P^d) + \lambda_s(A + P^s)] \cos \phi - [\lambda_d P^d + \lambda_s(T + C + A + P^s)] \sin \phi, \quad (5)$$

where T, C, A, P are color-allowed external W -emission, color-suppressed internal W -emission, W -annihilation and penguin topological amplitudes, respectively, and ϕ is the η - η' mixing angle. The tree topological amplitudes can be extracted from Cabibbo-favored $D \rightarrow PP$ decays to be (in units of 10^{-6} GeV) [4]

$$\begin{aligned} T &= 3.113 \pm 0.011, & C &= (2.767 \pm 0.029)e^{-(151.3 \pm 0.3)^\circ}, \\ E &= (1.48 \pm 0.04)e^{i(120.9 \pm 0.4)^\circ}, & A &= (0.55 \pm 0.03)e^{i(23_{-10}^{+7})^\circ}, \end{aligned} \quad (6)$$

with E being the W -exchange amplitude. It is obvious that a large DCPV (direct CP violation) at the tree level can arise from the interference between $\lambda_d C$ and $\lambda_s T$. Our calculation yields [4]

$$a_{CP}^{\text{tree}}(D_s^+ \rightarrow K^+\eta) = (-0.75 \pm 0.01) \times 10^{-3}, \quad a_{CP}^{\text{total}}(D_s^+ \rightarrow K^+\eta) = (-0.81 \pm 0.08) \times 10^{-3}. \quad (7)$$

CP asymmetry in this mode is dominated by the tree-level one of order 10^{-3} . A great merit of the diagrammatic approach is that tree DCPV can be reliably estimated as the magnitude and phase of each tree topology can be extracted from the data.

For $D^0 \rightarrow \pi^+\pi^-$ and K^+K^- , their topological amplitudes read

$$\begin{aligned} A(D^0 \rightarrow \pi^+\pi^-) &= \lambda_d(T + E^d) + \lambda_p(P^p + PE^p + PA^p), \\ A(D^0 \rightarrow K^+K^-) &= \lambda_d(T + E^s) + \lambda_p(P^p + PE^p + PA^p), \end{aligned} \quad (8)$$

where summation over $p = d, s$ is understood, PE and PA are penguin-exchange and penguin-annihilation amplitudes, respectively, and the superscript q refers to the quark involved in the associated penguin loop. In these two decays, direct CP violation arises from the interference between tree and penguin amplitudes. The complete expression of $\Delta a_{CP}^{\text{dir}}$ is given by [5]

$$\Delta a_{CP}^{\text{dir}} = -1.30 \times 10^{-3} \left(\left| \frac{P^d + PE^d + PA^d}{T + E^s - \Delta P} \right|_{KK} \sin \delta_{KK} + \left| \frac{P^s + PE^s + PA^s}{T + E^d + \Delta P} \right|_{\pi\pi} \sin \delta_{\pi\pi} \right), \quad (9)$$

where the parameter ΔP is defined by $\Delta P \equiv (P^d + PE^d + PA^d) - (P^s + PE^s + PA^s)$.

For penguin, penguin-exchange and penguin-annihilation amplitudes, we consider two QCD-inspired approaches, namely, perturbative QCD (pQCD) and QCD factorization (QCDF). The ratio $|P/T|$ is naively expected to be of order $(\alpha_s(\mu_c)/\pi) \sim \mathcal{O}(0.1)$. It was found that

$$\begin{aligned} \left(\frac{P}{T} \right)_{\pi\pi} &\approx 0.30 e^{i110^\circ}, & \left(\frac{P}{T} \right)_{KK} &\approx 0.24 e^{i110^\circ}, & \text{pQCD + FAT,} \\ \left(\frac{P}{T} \right)_{\pi\pi} &\approx 0.23 e^{-i150^\circ}, & \left(\frac{P}{T} \right)_{KK} &\approx 0.22 e^{-i150^\circ}, & \text{QCDF + TDA,} \end{aligned} \quad (10)$$

in pQCD in conjunction with the so-called factorization-assisted topological-amplitude approach (FAT) [6] and in QCDF together with the topological diagram approach (TDA) [5, 7]. In the latter approach, we have taken into account SU(3) breaking effects by having $T_{\pi\pi} = 0.96T$, $T_{KK} = 1.27T$ and

$$\begin{aligned} \text{Solution I:} & \quad E^d = 1.10 e^{i15.1^\circ} E, & E^s &= 0.62 e^{-i19.7^\circ} E; \\ \text{Solution II:} & \quad E^d = 1.10 e^{i15.1^\circ} E, & E^s &= 1.42 e^{-i13.5^\circ} E, \end{aligned} \quad (11)$$

where E^q refers to the W -exchange amplitude associated with $c\bar{u} \rightarrow q\bar{q}$ ($q = d, s$) [4]. Based on light-cone sum rules, the ratios

$$\left| \frac{P}{T} \right|_{\pi\pi} = 0.093 \pm 0.011, \quad \left| \frac{P}{T} \right|_{KK} = 0.075 \pm 0.015, \quad \text{LCSR,} \quad (12)$$

were obtained in [8]. However, the magnitude of P/T turns out to be of order $(0.22 \sim 0.30)$ in the QCD-inspired approaches.

After including W -exchange, penguin-exchange and penguin-annihilation contributions, it follows that

$$\left(\frac{P^s + PE^s + PA^s}{T + E^d + \Delta P} \right)_{\pi\pi} = 0.66 e^{i134^\circ}, \quad \left(\frac{P^d + PE^d + PA^d}{T + E^s - \Delta P} \right)_{KK} = 0.45 e^{i131^\circ}, \quad (13)$$

in pQCD+FAT [6], and

$$\left(\frac{P^s + PE^s + PA^s}{T + E^d + \Delta P} \right)_{\pi\pi} = 0.32 e^{i176^\circ}, \quad \left(\frac{P^d + PE^d + PA^d}{T + E^s - \Delta P} \right)_{KK} = \begin{cases} 0.23 e^{-i164^\circ} \\ 0.23 e^{i178^\circ} \end{cases}, \quad (14)$$

in QCDF for Solutions I and II of W -exchange amplitudes E^d and E^s [4]. Comparing Eq. (13) with Eq. (10), we see that the magnitude of the ratio P/T is enhanced by a factor of 2 after including PE , PA , E and ΔP . Substituting Eq. (13) into Eq. (9) yields

$$\Delta a_{CP}^{\text{dir}} \approx -1.0 \times 10^{-3}, \quad \text{pQCD + FAT.} \quad (15)$$

It is obvious from Eq. (14) that the CP asymmetry difference is very small, of order 10^{-4} or less, in QCDF+TDA owing to the reason that the phases $\delta_{\pi\pi}$ and δ_{KK} are not far from 180° in QCDF. Even if the phases are allowed to have 25° uncertainties, the resultant $\Delta a_{CP}^{\text{dir}}$ is still too small compared to experiment. There is one crucial difference in the treatment of the penguin-exchange amplitude in pQCD and QCDF approaches. In pQCD, the factorizable penguin-exchange amplitude PE^f is proportional to $\langle PP | (\bar{u}q)_{S+P} | 0 \rangle \langle 0 | (\bar{q}c)_{S-P} | D \rangle$. It was assumed in [6] that $\langle P_1 P_2 | (\bar{q}_1 q_2) | 0 \rangle = \langle P_1 P_2 | S \rangle \langle S | \bar{q}_1 q_2 | 0 \rangle$ was dominated by the isosinglet heavier scalar resonances such as $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ with the light resonances $f_0(500)$, $f_0(980)$ and $a_0(980)$ being neglected. In QCDF, PE^f is expressed in terms of the twist-2 LCDA Φ_M and the twist-3 one Φ_m . This explains why the magnitude of the ratio P/T is enhanced by a factor of 2 after including PE , PA , E and ΔP in pQCD+FAT, but no so in QCDF+TDA.

It is known that in the diagrammatic approach, all the topological tree amplitudes except T are dominated by nonfactorizable long-distance effects. This implies that, contrary to B physics, the underlying mechanism of hadronic charm decays is dominated by nonperturbative physics. This is why even until today we still don't have a QCD-inspired theory to describe the nonleptonic decays of charmed mesons. By the same token, it is natural to expect that penguin topology P and PE also receive long-distance contributions. Indeed, we have pointed out in [7] the importance of a resonant-like final-state rescattering which has the same topology as the topological graph P and PE . In 2012 we have made the ansatz that $(P + PE)^{\text{LD}}$ is of the same order of magnitude as E and flavor independent. Very recently, this ansatz was shown to be justified in the diagrammatic approach under the SU(3) limit; that is, $(P + PE)^{\text{LD}} = E^{\text{LD}} \approx E$ [9]. Then we have

$$\begin{aligned} \left(\frac{P^s + PE^s + PA^s + (P + PE)^{\text{LD}}}{T + E^d + \Delta P} \right)_{\pi\pi} &= 0.77 e^{i114^\circ}, \\ \left(\frac{P^d + PE^d + PA^d + (P + PE)^{\text{LD}}}{T + E^s - \Delta P} \right)_{\kappa\kappa} &= \begin{cases} 0.45 e^{i137^\circ} & \text{solution I} \\ 0.45 e^{i120^\circ} & \text{solution II} \end{cases}. \end{aligned} \quad (16)$$

Consequently [4],

$$\Delta a_{CP}^{\text{dir}} = \begin{cases} (-1.14 \pm 0.26) \times 10^{-3} & \text{solution I} \\ (-1.25 \pm 0.25) \times 10^{-3} & \text{solution II} \end{cases}, \quad \text{QCDF + TDA.} \quad (17)$$

Although both QCD-inspired approaches predicted a Δa_{CP} of order 10^{-3} with a negative sign, the mechanisms responsible for CP violation are quite different. $\Delta a_{CP}^{\text{dir}}$ comes from the interference between the tree $T + E$ and the *short-distance* penguin $P + PE$ amplitudes in pQCD+FAT, while it is the interference between tree and *long-distance* penguin amplitudes that pushes CP asymmetry difference up to the per mille level. As we shall see in Sec. 3 below, these two different mechanisms can be discriminated in $D \rightarrow VP$ decays.

We thus see that LHCb's observation of CP asymmetry difference can be explained within the SM without the need of NP. The key lies in the long-distance penguin topology $P + PE$ arising from final-state rescattering. A similar but different idea was proposed in Ref. [11]. When the amplitudes in Eq. (8) were re-expressed in terms of U -spin components, $\Delta U = 0$ and $\Delta U = 1$, it was argued in Ref. [11] that the ratio of $\Delta U = 0$ over $\Delta U = 1$ matrix elements required a non-perturbative enhancement of the penguin contraction of tree operators in order to accommodate the LHCb measurement of $\Delta a_{CP}^{\text{dir}}$.

3. CP violation in $D \rightarrow VP$ decays

For $D \rightarrow VP$ decays, there exist two different types of topological diagrams since the spectator quark of the charmed meson may end up in the pseudoscalar or vector meson. For topological color-allowed tree amplitude T and color-suppressed amplitude C in $D \rightarrow VP$ decays, the subscript P (V) implies that the pseudoscalar (vector) meson contains the spectator quark of the charmed meson. For the W -exchange

Table 1: Topological amplitudes obtained from Cabibbo-favored $D \rightarrow VP$ decays. The amplitude sizes are quoted in units of $10^{-6}(\epsilon \cdot p_D)$ and the strong phases in units of degrees.

| | | | | | | | | |
|-----------------|-----------------|-----------------|-----------------|-----------------|-------------------|-----------------|-----------------|----------------|
| $ T_V $ | $ T_P $ | δ_{T_P} | $ C_V $ | δ_{C_V} | $ C_P $ | δ_{C_P} | $ E_V $ | δ_{E_V} |
| 2.19 ± 0.03 | 3.56 ± 0.06 | 61 ± 5 | 1.69 ± 0.04 | 220 ± 3 | 2.02 ± 0.02 | 201 ± 1 | 0.58 ± 0.06 | 283 ± 5 |
| $ E_P $ | δ_{E_P} | $ A_V $ | δ_{A_V} | $ A_P $ | δ_{A_P} | χ^2_{\min} | fit quality | |
| 1.69 ± 0.03 | 108 ± 3 | 0.23 ± 0.02 | 77 ± 5 | 0.18 ± 0.03 | 111^{+13}_{-10} | 7.06 | 6.99% | |

amplitude E and W -annihilation A with the final state $q_1 \bar{q}_2$, the subscript P (V) denotes that the pseudoscalar (vector) meson contains the antiquark \bar{q}_2 .

By performing a χ^2 fit to the Cabibbo-favored (CF) $D \rightarrow VP$ decays, we have extracted the magnitudes and strong phases of the topological amplitudes T_V, C_V, E_V, A_V and T_P, C_P, E_P, A_P from the measured partial widths and found five solutions with local χ^2 minima restricted to $\chi^2_{\min} < 10$. [10]. Although all five solutions generally fit the CF modes well, they led to very different predictions for some of the singly Cabibbo-suppressed (SCS) decays. Especially, the $D^0 \rightarrow \pi^0 \omega$, $D^+ \rightarrow \pi^+ \rho^0$ and $D^+ \rightarrow \pi^+ \omega$ decays were very useful in discriminating among different solutions. We found that one of the solutions which we called (S3') gives a best accommodation of SCS data, while other solutions were ruled out. The magnitudes and relative phases of various topological amplitudes are listed in Table 1. The topological amplitudes of all these solutions respect the hierarchy pattern:

$$|T_P| > |T_V| \gtrsim |C_P| > |C_V| \gtrsim |E_P| > |E_V| > |A_{P,V}|. \quad (18)$$

As in the case of $D \rightarrow PP$ decays, we make a similar ansatz for the long-distance contributions to $P + PE_{V,P}$, namely,

$$(P + PE_V)^{\text{LD}} \approx E_P, \quad (P + PE_P)^{\text{LD}} \approx E_V. \quad (19)$$

The calculated CP asymmetries of the SCS $D \rightarrow VP$ decays are exhibited in Table 2. From Table 2 we identify several golden modes which have large branching fractions and sizeable CP asymmetries at the order of 10^{-3} :

$$D^0 \rightarrow \pi^+ \rho^-, K^+ K^{*-}, \quad D^+ \rightarrow \eta \rho^+, K^+ \bar{K}^{*0}, \quad D_s^+ \rightarrow \pi^+ K^{*0}, \pi^0 K^{*+}. \quad (20)$$

It is interesting to notice that the CP asymmetry difference defined by

$$\Delta a_{CP}^{VP} \equiv a_{CP}(K^+ K^{*-}) - a_{CP}(\pi^+ \rho^-), \quad (21)$$

in analogy to ΔA_{CP} defined in Eq. (2) for the corresponding PP final states, is predicted to be $(-1.61 \pm 0.33) \times 10^{-3}$, which is very similar to the observed CP asymmetry difference between $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$. This is an attractive and measurable observable in the near future. It is thus desirable to first search for CP violation in the aforementioned golden modes.

From Table 2 it is evident that the predicted CP asymmetries given in Ref. [12] based on the pQCD+FAT approach are generally smaller than ours by one to two orders of magnitude. In this approach the factorizable penguin-exchange amplitude $PE_{V(P)}^f$ is proportional to $\langle VP(PV) | (\bar{u}q)_{S+P} | 0 \rangle \langle 0 | (\bar{q}c)_{S-P} | D \rangle$. It was evaluated in the pole model by assuming its dominance by resonant pseudoscalars. It was shown in Ref. [12] that there was a numerical coincidence that the short-distance P_V and PE_V^f canceled each other. As a consequence, CP asymmetries in $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ decays are very small of order 10^{-5} . They are also very small in QCDF+TDA for a reason quite different from FAT: various phase angles become smaller or even close to zero after including the contributions from PE, PA and W -exchange to the ratio of P/T . It is the long-distance penguin topology that explains why our predictions of CP asymmetries in $D^0 \rightarrow \pi^\pm \rho^\mp$ and $D^0 \rightarrow K^\pm K^{*\mp}$ are much bigger than those in pQCD+FAT.

Table 2: Direct CP asymmetries of singly Cabibbo-suppressed $D \rightarrow VP$ decays (in units of 10^{-3}), where $a_{\text{dir}}^{(\text{tree})}$ denotes CP asymmetry arising from purely tree amplitudes. The superscript (t+p) denotes tree plus QCD-penguin amplitudes, (t+pe+pa+s) for tree plus PE, PA and S amplitudes, (t+pe^{LD}) for tree plus long-distance PE amplitude induced from final-state rescattering and “tot” for the total amplitude. The predictions from [12] in the FAT approach with the $\rho - \omega$ mixing are listed in the last column for comparison.

| Mode | $a_{\text{dir}}^{(\text{tree})}$ | $a_{\text{dir}}^{(\text{t+p})}$ | $a_{\text{dir}}^{(\text{t+pe+pa+s})}$ | $a_{\text{dir}}^{(\text{t+pe}^{\text{LD}})}$ | $a_{\text{dir}}^{(\text{tot})}$ | $a_{\text{dir}}^{(\text{tot})}$ [12] |
|------------------------------------|----------------------------------|---------------------------------|---------------------------------------|--|---------------------------------|--------------------------------------|
| $D^0 \rightarrow \pi^+ \rho^-$ | 0 | -0.00 ± 0.00 | -0.011 ± 0.000 | 0.77 ± 0.22 | 0.76 ± 0.22 | -0.03 |
| $D^0 \rightarrow \pi^- \rho^+$ | 0 | 0.01 ± 0.00 | 0.008 ± 0.001 | -0.13 ± 0.08 | -0.11 ± 0.08 | -0.01 |
| $D^0 \rightarrow \pi^0 \rho^0$ | 0 | -0.01 ± 0.00 | -0.004 ± 0.000 | 0.28 ± 0.16 | 0.27 ± 0.16 | -0.03 |
| $D^0 \rightarrow K^+ K^{*-}$ | 0 | -0.01 ± 0.01 | 0.011 ± 0.000 | -0.85 ± 0.24 | -0.85 ± 0.24 | -0.01 |
| $D^0 \rightarrow K^- K^{*+}$ | 0 | -0.03 ± 0.00 | -0.009 ± 0.000 | 0.08 ± 0.09 | 0.04 ± 0.09 | 0 |
| $D^0 \rightarrow K^0 \bar{K}^{*0}$ | -0.03 ± 0.02 | -0.03 ± 0.02 | -0.03 ± 0.02 | -0.03 ± 0.02 | -0.03 ± 0.02 | -0.7 |
| $D^0 \rightarrow \bar{K}^0 K^{*0}$ | 1.07 ± 0.12 | 1.07 ± 0.12 | 1.07 ± 0.12 | 1.07 ± 0.12 | 1.07 ± 0.12 | -0.7 |
| $D^+ \rightarrow \pi^+ \rho^0$ | 0 | 0.33 ± 0.02 | 0.10 ± 0.01 | 0.83 ± 1.36 | 1.26 ± 1.34 | 0.5 |
| $D^+ \rightarrow \pi^0 \rho^+$ | 0 | 0.10 ± 0.01 | 0.04 ± 0.00 | -0.58 ± 0.52 | -0.44 ± 0.52 | 0.2 |
| $D^+ \rightarrow \eta \rho^+$ | -1.85 ± 0.51 | -1.97 ± 0.54 | -1.93 ± 0.55 | -2.31 ± 0.92 | -2.50 ± 0.98 | -0.6 |
| $D^+ \rightarrow \eta' \rho^+$ | 0.23 ± 0.05 | 0.20 ± 0.05 | 0.21 ± 0.05 | 0.39 ± 0.16 | 0.34 ± 0.15 | 0.5 |
| $D^+ \rightarrow K^+ \bar{K}^{*0}$ | -0.11 ± 0.01 | -0.14 ± 0.01 | -0.11 ± 0.01 | -0.77 ± 0.24 | -0.80 ± 0.24 | 0.2 |
| $D^+ \rightarrow \bar{K}^0 K^{*+}$ | -0.04 ± 0.01 | -0.05 ± 0.01 | -0.05 ± 0.01 | -0.06 ± 0.06 | 0.04 ± 0.07 | 0.04 |
| $D_s^+ \rightarrow \pi^+ K^{*0}$ | 0.18 ± 0.02 | 0.24 ± 0.02 | 0.19 ± 0.02 | 1.25 ± 0.41 | 1.32 ± 0.41 | -0.1 |
| $D_s^+ \rightarrow \pi^0 K^{*+}$ | 0.13 ± 0.02 | 0.12 ± 0.03 | 0.11 ± 0.03 | 1.35 ± 0.40 | 1.31 ± 0.40 | -0.2 |
| $D_s^+ \rightarrow K^+ \rho^0$ | 0.14 ± 0.03 | 0.11 ± 0.02 | 0.15 ± 0.03 | -0.26 ± 0.12 | -0.29 ± 0.12 | 0.3 |
| $D_s^+ \rightarrow K^0 \rho^+$ | 0.06 ± 0.02 | 0.08 ± 0.02 | 0.08 ± 0.02 | -0.10 ± 0.10 | -0.07 ± 0.10 | 0.3 |

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