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New physics behind the new muon g-2 puzzle?

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The recent measurement of the muon g-2 at Fermilab confirms the previous Brookhaven result. The leading hadronic vacuum polarization (HVP) contribution to the muon g-2 represents a crucial ingredient to establish if the Standard Model prediction differs from the experimental value. A recent lattice QCD result by the BMW collaboration shows a tension with the low-energy $e^+e^- \to$ hadrons data which are currently used to determine the HVP contribution. We refer to this tension as the new muon g-2 puzzle. In this contribution, we assess the possibility to solve this puzzle invoking new physics contributions to the $e^+e^- \to$ hadrons cross-section.

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1. Introduction

The anomalous magnetic moment of the muon, $a_\mu \equiv (g_\mu - 2)/2$, has provided a persisting hint of new physics (NP) for many years. The recent a_μ measurement by the Muon g-2 collaboration at Fermilab has confirmed the earlier result by the E821 experiment at Brookhaven, yielding the average $a_\mu^{\rm EXP} = 116592061(41)\times 10^{-11}$. The comparison of this result with the Standard Model (SM) prediction $a_\mu^{\rm SM} = 116591810(43)\times 10^{-11}$ of the Muon g-2 Theory Initiative [1] leads to an intriguing 4.2σ discrepancy [2]

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 251\,(59)\times 10^{-11}\,. \tag{1}$$

The expected forthcoming results of the Fermilab experiment plan to reach a sensitivity four-times better than the E821 one.

On the theory side, the only source of sizable uncertainties in $a_\mu^{\rm SM}$ stems from the non-perturbative contributions of the hadronic sector, which have been under close scrutiny for several years. The SM prediction $a_\mu^{\rm SM}$ in Eq. (1) has been derived by the Muon g-2 Theory Initiative [1] using $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$, the leading hadronic vacuum polarization (HVP) contribution to the muon g-2 based on low-energy $e^+e^- \to$ hadrons data. Alternatively, the HVP contribution has been computed using a first-principle lattice QCD approach [1]. Recently, the BMW lattice QCD collaboration (BMWc) computed the leading HVP contribution to the muon g-2 with sub per-cent precision, finding a value, $(a_\mu^{\rm HVP})_{\rm BMW}$, larger than $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$ [3]. If $(a_\mu^{\rm HVP})_{\rm BMW}$ is used to obtain $a_\mu^{\rm SM}$ instead of $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$, the discrepancy with the experimental result is reduced to 1.6σ only. The above results are respectively

$$(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-} = 6931\,(40)\times 10^{-11}\,, \qquad (a_\mu^{\rm HVP})_{\rm BMW} = 7075\,(55)\times 10^{-11}\,. \tag{2}$$

The present situation regarding the leading HVP contribution to the muon g-2 can be schematically represented as in Fig. 1, where $(a_\mu^{\rm HVP})_{\rm EXP}$ is the value of the HVP contribution required to exactly match $a_\mu^{\rm EXP}$ assuming no NP. The difference between the discrepancies in Fig. 1 has been referred to as the *new muon g-2 puzzle* [4].

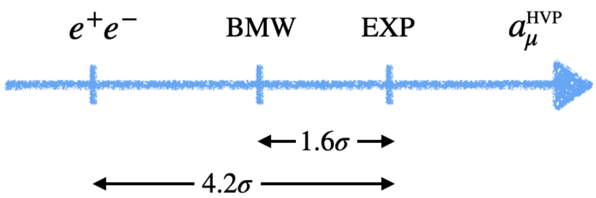

Figure 1: The new muon g-2 puzzle: 4.2σ vs. 1.6σ [4].

Assuming that both $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$ and $(a_\mu^{\rm HVP})_{\rm BMW}$ are correct, we ask whether this puzzle can be solved thanks to NP effects which would bring $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$ in agreement with $(a_\mu^{\rm HVP})_{\rm BMW}$, *without* spoiling the 1.6σ agreement of $(a_\mu^{\rm HVP})_{\rm BMW}$ with $(a_\mu^{\rm HVP})_{\rm EXP}$. Differently from what has been usually done in the literature [5], here we do not assume a direct NP contribution to Δa_μ (i.e. new states that couple directly to muons). If fact, by itself this possibility could solve the longstanding discrepancy in Eq. (1), but not the new muon g-2 puzzle. Instead, in order to solve the latter, we invoke NP that modifies the $e^+e^- \to$ hadrons cross-section $\sigma_{\rm had}$.[1] An increase of $\sigma_{\rm had}$, due to an unforeseen

[1] The possibility of reconciling the data driven and the BMWc lattice determinations of $a_\mu^{\rm HVP}$ by rescaling the KLOE

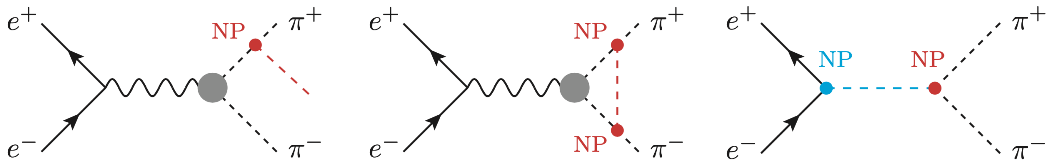

Figure 2: Examples of NP contributions to $\sigma_{\rm had}$ via FSR (first and second diagram) and via a NP tree-level mediator coupled both to hadrons and electrons (third diagram) [4].

missing contribution, has been already proposed to enhance $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$ and solve Δa_μ [7]. However, the required shift in $\sigma_{\rm had}$ is disfavoured by the electroweak fit if it occurs at $\sqrt{s} \gtrsim 1$ GeV [7]. Hence, hereafter, we will consider NP modifications of $\sigma_{\rm had}$ below the GeV scale.

2. Model-independent analysis

Let us examine the general properties of NP models aiming at solving the new muon g-2 puzzle via a modification of $\sigma_{\rm had}$. To this end, we introduce the dispersion relation

$$(a_\mu^{\rm HVP})_{e^+e^-} = \frac{\alpha}{\pi^2}\int_{m_{\pi^0}^2}^{\infty} \frac{{\rm d}s}{s}\, K(s)\, {\rm Im}\,\Pi_{\rm had}(s)\,, \tag{3}$$

where $K(s)$ is a positive-definite kernel function with $K(s) \approx m_\mu^2/3s$ for $\sqrt{s} \gg m_\mu$. This equation defines the HVP contribution to the muon g-2 in terms of the photon HVP, $\Pi_{\rm had}$, which includes possible NP effects. If the possible NP entering the photon HVP does not couple to electrons, i.e. it does not enter the hadronic cross-section at tree level, then Eq. (3) can be written as

$$(a_\mu^{\rm HVP})_{e^+e^-} = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} {\rm d}s\, K(s)\sigma_{\rm had}(s)\,, \tag{4}$$

where $\sigma_{\rm had}$ includes final-state radiation (FSR), whereas both vacuum polarization and initial-state radiation (ISR) effects are subtracted. In particular, vacuum polarization corrections can be simply accounted for by multiplying the experimental cross-section by $|\alpha/\alpha(s)|^2$, while the correction of ISR and ISR/FSR interference effects is addressed by each experimental collaboration. In this proceeding, we will focus on the region where $\sigma_{\rm had}$ is experimentally determined, i.e. $\sqrt{s} \gtrsim 0.3$ GeV, since this gives the by far dominant contribution to the dispersive integral in Eq. (4).

In Fig. 2 we show a schematic classification of how NP can enter $\sigma_{\rm had}$. The first two diagrams are representative of FSR effects, which also unavoidably affect the photon HVP at the next-to-leading order (NLO). We can safely neglect possible NP contaminations in ISR since the bounds on NP couplings to electrons are very severe. The third diagram, where NP enters the hadronic cross-section at tree level coupling both to hadrons and electrons, is due to NP that also modifies the photon HVP at NLO. Crucially, however, its dominant contribution to the muon g-2 emerges via the tree-level shift of $\sigma_{\rm had}$.

Hence, when invoking NP in $\sigma_{\rm had}$, there are two different scenarios to be considered, depending on whether NP couples only to hadrons or both to hadrons and electrons. In the following, we analyze these two possibilities and their capability to solve the new muon g-2 puzzle.

luminosity via a NP contribution to Bhabha scattering has been discussed in Ref. [6].

1. NP coupled only to hadrons. This scenario is schematically represented by the first two diagrams of Fig. 2. As remarked above, real and virtual FSR must be included in $\sigma_{\rm had}$. However, in order to establish the impact of NP in FSR (which depends on the interplay between the mass scale of NP and the experimental cuts), it would be mandatory to perform dedicated experimental analyses imposing the various selection cuts specific of each experimental setup. Since the full photon FSR effect estimated in scalar QED amounts only to 50×10^{-11} [1], and given that light NP couplings with the SM particles are tightly constrained, the NP contributions in FSR can hardly solve the new muon g-2 puzzle.

2. NP coupled both to hadrons and electrons. If NP contributes to $\sigma_{\rm had}$ at tree level (see third diagram in Fig. 2), then only the subtracted cross-section $\sigma_{\rm had} - \Delta\sigma^{\rm NP}_{\rm had}$ should be included in Eq. (4). We note that the latter can be larger than $\sigma_{\rm had}$ if $\Delta\sigma^{\rm NP}_{\rm had} < 0$, thus requiring that the NP contribution is dominated by a *negative* interference with the SM. As $(a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-}$ has been computed using $\sigma_{\rm had}$ rather than the subtracted cross-section $\sigma_{\rm had} - \Delta\sigma^{\rm NP}_{\rm had}$, the theoretical prediction of the HVP contribution in Eq. (4) is

$$(a_\mu^{\rm HVP})_{e^+e^-} = (a_\mu^{\rm HVP})^{\rm TI}_{e^+e^-} + (a_\mu^{\rm HVP})_{\rm NP}\,, \tag{5}$$

where $(a_\mu^{\rm HVP})_{\rm NP}$ describes NP effects at LO, due to the tree-level exchange of the NP mediator (see third diagram in Fig. 2), as well as at NLO. Instead, $(a_\mu^{\rm HVP})_{\rm BMW}$ should be shifted only by NLO NP effects. Remarkably, this scenario may allow to match Eq. (5) with $(a_\mu^{\rm HVP})_{\rm EXP}$, while keeping at the same time the agreement with the BMWc lattice result.

3. Light new physics analysis

We now explore whether the second scenario envisaged above can be quantitatively realized in an explicit NP model. Motivated by the fact that the kernel function in Eq. (4) scales like $1/s$ and by the fact that modifications of $\sigma_{\rm had}$ above ~ 1 GeV are disfavoured by electroweak precision tests, we focus on the sub-GeV energy range, where the dominant contribution to $\sigma_{\rm had}$ arises from the $e^+e^- \to \pi^+\pi^-$ channel. In fact, in the SM, this channel accounts for more than 70% of the full hadronic contribution to the muon g-2. Furthermore, the requirement of having a sizeable negative interference with the SM amplitude narrows down the general class of NP models. Indeed, the interference of scalar couplings with the SM vector current is suppressed by the electron mass, while pseudoscalar and axial couplings do not interfere. Hence, we focus on the tree-level exchange of a light Z' boson with the following vector couplings to electrons and first-generation quarks

$$\mathcal{L}_{Z'} \supset (g_V^e\, \bar{e}\gamma^\mu e + g_V^q\, \bar{q}\gamma^\mu q) Z'_\mu\,, \tag{6}$$

with $q = u, d$ and $m_{Z'} \lesssim 1$ GeV.

Defining $\sigma^{\rm SM+NP}_{\pi\pi} = \sigma^{\rm SM}_{\pi\pi} + \Delta\sigma^{\rm NP}_{\pi\pi}$, the tree-level exchange of the Z' with width $\Gamma_{Z'}$ leads to

$$\frac{\sigma^{\rm SM+NP}_{\pi\pi}}{\sigma^{\rm SM}_{\pi\pi}} = \left| 1 + \frac{g_V^e (g_V^u - g_V^d)}{e^2} \frac{s}{s - m_{Z'}^2 + i m_{Z'}\Gamma_{Z'}} \right|^2, \tag{7}$$

where the pion vector form factor cancels out in the ratio.

The dispersive contribution to the muon g-2 due to SM and NP can be obtained by using $\sigma_{\rm had} - \Delta\sigma^{\rm NP}_{\rm had}$ in Eq. (4). Imposing that the current discrepancy Δa_μ is solved by NP in the hadronic cross-section, we obtain

$$\Delta a_\mu = \frac{1}{4\pi^3}\int_{s_{\rm exp}}^{\infty} {\rm d}s\, K(s)(-\Delta\sigma^{\rm NP}_{\rm had}(s)), \tag{8}$$

where the lower integration limit is $s_{\rm exp} \approx (0.3\ {\rm GeV})^2$, that is, the integral is performed in the data-driven region for the $\pi\pi$ channel. Approximating $\Delta\sigma^{\rm NP}_{\rm had} \approx \Delta\sigma^{\rm NP}_{\pi\pi}$, from Eq. (7) we find

$$\Delta\sigma^{\rm NP}_{\rm had}(s) \approx \sigma^{\rm SM}_{\pi\pi}(s) \times \frac{2\epsilon s(s-m^2_{Z'}) + \epsilon^2 s^2}{(s-m^2_{Z'})^2 + m^4_{Z'}\gamma^2}, \tag{9}$$

where we introduced the effective coupling $\epsilon \equiv g^e_V(g^u_V - g^d_V)/e^2$ and the adimensional width parameter $\gamma \equiv \Gamma_{Z'}/m_{Z'}$. If both the $Z' \to ee$ and $Z' \to \pi^+\pi^-$ channels are kinematically open, the associated decay widths (normalized to $m_{Z'}$) read, respectively

$$\gamma_{ee} \approx \frac{(g^e_V)^2}{12\pi} = 2.7\times 10^{-10}\left(\frac{g^e_V}{10^{-4}}\right)^2, \qquad \gamma_{\pi\pi} = \frac{(g^u_V - g^d_V)^2}{48\pi}|F^V_\pi(m^2_{Z'})|^2\left(1-\frac{4m^2_\pi}{m^2_{Z'}}\right)^{3/2}, \tag{10}$$

where $|F^V_\pi(m^2_{Z'})|^2$ (normalized to $F^V_\pi(0) = 1$) can be enhanced up to a factor of 45 by the ρ resonance [8]. In the following, we are going to inspect whether the region of the parameter space of the

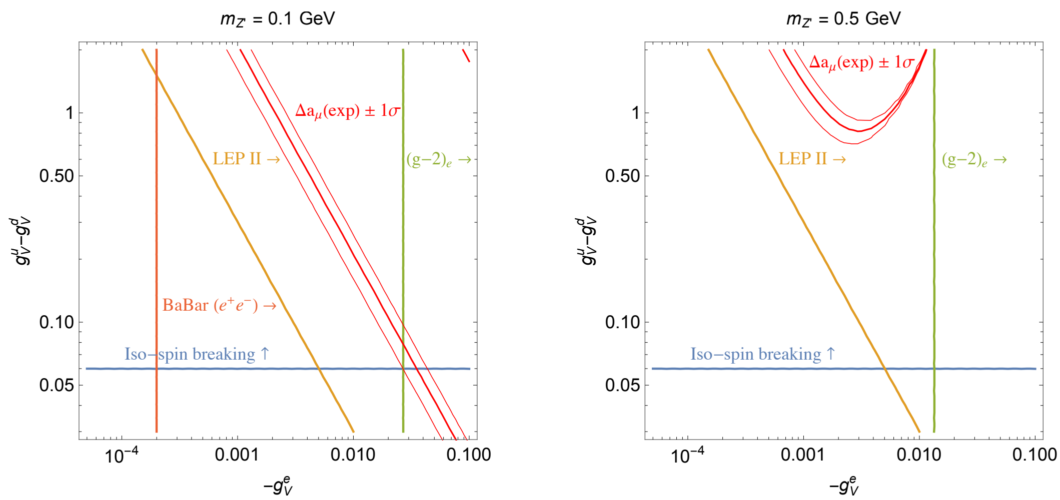

Figure 3: Z' contribution to Δa_μ via a modification of $\sigma_{\rm had}$ vs. Z' constraints [4].

Z' model needed to explain Δa_μ is allowed by experimental constraints. These can be divided for convenience in three classes: 1. semi-leptonic processes; 2. purely leptonic processes and 3. purely hadronic, iso-spin violating observables. The interplay of the above constraints in the plane $-g^e_V$ vs. $g^u_V - g^d_V$ is displayed in Fig. 3 for two representative scenarios where $m_{Z'} = 0.1$ and 0.5 GeV. The directions of the arrows indicate the excluded regions by the different experimental bounds. Instead, the red band is the region favoured by the explanation of the muon g-2 discrepancy. From Fig. 3 it is clear that, irrespectively of the Z' mass, there are always at least two independent bounds preventing to solve the new muon g-2 puzzle.

4. Conclusions

The recent lattice QCD result by the BMW collaboration shows a tension with the low-energy $e^+e^- \to$ hadrons data currently used to determine the HVP contribution to the muon g-2. A possible way to restore full consistency into the picture is to postulate a *negative* shift in $\sigma_{\rm had}$ due to NP. We showed that this scenario requires the presence of a light NP mediator that modifies the experimental cross-section $\sigma_{\rm had}$. However, this non-trivial setup, where NP hides in $e^+e^- \to$ hadrons data, is excluded by a number of experimental constraints. Alternative confirmations of the e^+e^- determinations of the HVP contribution to the muon g-2, based on either additional lattice QCD calculations or direct experimental measurements, as proposed by the MUonE experiment [9] will hence be crucial to shed light on this intriguing puzzle.

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