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Relativistic dissipative hydrodynamics within extended relaxation time approximation

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The derivation of hydrodynamics from Boltzmann equation in the Anderson-Witting relaxation time approximation assumes the relaxation time to be independent of particle energy, and one is restricted to work in the Landau frame to ensure macroscopic conservation laws. However, the collision time scale typically depends on the microscopic interactions for any realistic system. We present a framework for consistent derivation of relativistic dissipative hydrodynamics from the Boltzmann equation with a particle energy dependent relaxation time by extending the Anderson-Witting relaxation-time approximation, and derive first-order hydrodynamic equations. We show that the obtained transport coefficients have corrections due to the energy dependence of relaxation-time compared to what one obtains from the Anderson-Witting approximation of the collision term, and discuss several interesting scaling features for the ratio of these transport coefficients.

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1. Introduction

Formulation of hydrodynamics from the relativistic Boltzmann equation in the relaxation time approximation (RTA) of the collision term [1] requires the relaxation-time to be independent of particle energy in order to ensure macroscopic conservation laws. However, the time scale of collision for any realistic system depends on the microscopic interactions [2, 3]. In this proceedings, we extend the Anderson-Witting relaxation-time approximation which allows for particle energy-dependent relaxation time, and introduce a framework for consistent derivation of relativistic hydrodynamics from the resulting Boltzmann equation [4]. Within this extended RTA framework, we derive the first-order hydrodynamic equations and demonstrate that the modified transport coefficients depends, in some cases rather dramatically, on the energy-dependence of the relaxation time. We also discuss several interesting scaling features for the ratio of these transport coefficients.

2. Extended Relaxation time approximation

We consider the following modification of RTA approximation of the collision term [3]:

$$p^\mu \partial_\mu f = -\frac{(u\cdot p)}{\tau_{\rm R}(x,p)}\left(f - f^*_{\rm eq}\right). \tag{1}$$

We shall refer r.h.s. of the above equation as extended relaxation time approximation (ERTA). Here $f^*_{\rm eq} \equiv [\exp(\beta^*(u^*\cdot p) - \alpha^*) + a]^{-1}$ is the local equilibrium distribution function, where $\beta^* \equiv 1/T^*$ is the inverse temperature, $\alpha^* \equiv \mu^*/T^*$ is the ratio of chemical potential to temperature, and $a = 1, 0, -1$ implies Fermi Dirac (FD), Maxwell Boltzmann (MB), Bose Einstein (BE) statistics respectively. The out of equilibrium distribution relaxes to this distribution with a time scale $\tau_{\rm R}(x,p)$ in the local rest frame of u^*_μ, with T^* and μ^* being the corresponding thermodynamic quantities.

2.1 Out-of-equilibrium correction to thermal distribution function

For an out of equilibrium fluid, temperature and chemical potential are auxiliary fields defined in a particular choice of hydrodynamic frame using matching conditions. Therefore, we define a local equilibrium distribution function $f_{\rm eq} \equiv [\exp(\beta(u\cdot p) - \alpha) + a]^{-1}$, with auxiliary fields T and μ, which is reached in the fluid rest frame, $u^\mu = (1,0,0,0)$. Employing Chapman-Enskog like expansion around this equilibrium distribution, we obtain the first-order gradient correction as

$$\delta f_{(1)} = \delta f_* - \frac{\tau_{\rm R}(x,p)}{(u\cdot p)}\, p^\mu \partial_\mu f_{\rm eq}. \tag{2}$$

Here, $\delta f_* \equiv f^*_{\rm eq} - f_{\rm eq}$ adds further gradient correction to $\delta f_{(1)}$ arising due to the difference between the frame variables, and vanishes for a fluid in equilibrium. To obtain δf_*, the auxiliary hydrodynamic variables u^μ, T and μ are first related to the corresponding variables u^*_μ, T^* and μ^*,

$$u^*_\mu \equiv u_\mu + \delta u_\mu, \qquad T^* \equiv T + \delta T, \qquad \mu^* \equiv \mu + \delta\mu\,. \tag{3}$$

The equilibrium distribution $f^*_{\rm eq}$ is then Taylor expanded about u^μ, T and μ to obtain

$$\delta f_* = \left[-\frac{p_\mu \delta u^\mu}{T} + \frac{(u\cdot p - \mu)\delta T}{T^2} + \frac{\delta\mu}{T}\right] f_{\rm eq}\tilde{f}_{\rm eq} + O\left(\delta^2\right), \tag{4}$$

where $\tilde{f}_{\rm eq} \equiv 1 - a f_{\rm eq}$. Using above expression in Eq. (2), $\delta f_{(1)}$ simplifies to

$$\delta f_{(1)} = \Bigg[\tau_{\rm R}(x,p)\left\{\left(\beta(u\cdot p)\,(\chi_b - 1/3) + \frac{\beta m^2}{3(u\cdot p)} - \chi_a\right)\theta + \frac{\beta}{u\cdot p}\,p^\mu p^\nu \sigma_{\mu\nu} + \left(\frac{n}{\mathcal{E}+\mathcal{P}} - \frac{1}{u\cdot p}\right)p^\mu \nabla_\mu \alpha\right\} - \beta\, p\cdot\delta u + \beta^2\,(u\cdot p - \mu)\,\delta T + \beta\,\delta\mu\Bigg] f_{\rm eq}\,\tilde{f}_{\rm eq}\,. \tag{5}$$

Here $\mathcal{E}$, $\mathcal{P}$, and n represents the energy density, equilibrium pressure, and the net number density. The projection operator $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is orthogonal to the hydrodynamic four-velocity u^μ, $\nabla^\alpha = \Delta^{\mu\alpha}\partial_\mu$ represents the space-like derivatives, $\theta \equiv \partial_\mu u^\mu$ is the expansion scalar and $\sigma^{\mu\nu} \equiv \frac{1}{2}(\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{1}{3}\theta\Delta^{\mu\nu}$ is the velocity stress tensor. We work with the Minkowskian metric tensor $g^{\mu\nu} \equiv {\rm diag}(+,-,-,-)$. The dimensionless quantities χ_a and χ_b in Eq. (5) are defined as

$$\chi_a \equiv \frac{(\mathcal{E}+\mathcal{P})J^-_{2,0} - nJ^+_{3,0}}{J^+_{3,0}J^+_{1,0} - J^-_{2,0}J^-_{2,0}}, \qquad \chi_b \equiv \frac{(\mathcal{E}+\mathcal{P})J^+_{1,0} - nJ^-_{2,0}}{\beta(J^+_{3,0}J^+_{1,0} - J^-_{2,0}J^-_{2,0})}. \tag{6}$$

The integrals $J^\pm_{n,q}$ appearing in the above expressions, and $K^\pm_{n,q}$, $y^\pm_{n,q}$ appearing in the expression of coefficients (9) below, can be found in Ref. [4].

2.2 Hydrodynamic frame and matching conditions

We determine the quantities δT, δu^μ and $\delta\mu$ appearing in Eq. (5) by imposing the Landau frame conditions: $u_\nu T^{\mu\nu} = \mathcal{E}u^\mu$ and the matching conditions: $\mathcal{E} = \mathcal{E}_{\rm eq}$ and $n = n_{\rm eq}$ at first-order in gradients, i.e., with $f = f_1 \equiv f_{\rm eq} + \delta f_{(1)}$,

$$\delta u^\mu = C_1\frac{(\nabla^\mu\alpha)}{T}\,, \qquad \delta T = C_2\,\theta\,, \qquad \delta\mu = C_3\,\theta\,. \tag{7}$$

The expressions of dimensionless variables C_1, C_2 and C_3 can be found in Ref. [4]. The coefficients C_1, C_2 and C_3 vanishes when the relaxation time is particle energy independent, which in turn leads to vanishing of δu^μ, δT, and $\delta\mu$, and the first order viscous correction $\delta f_{(1)}$ reduces to what is obtained from RTA approximation.

3. First order transport coefficients

The first-order correction to the equilibrium distribution function is completely determined by Eq. (5) together with Eq. (7). Using this, the relativistic Navier-Stokes expression for dissipative quantities is obtained to be

$$\pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu}, \qquad \Pi = -\zeta\,\theta, \qquad n^\mu = \kappa_n \nabla^\mu\alpha. \tag{8}$$

where the transport coefficients are given by,

$$\eta = \frac{K^+_{3,2}}{T}\,, \quad \zeta = -C_2\left(\frac{\mathcal{E}+\mathcal{P}-\mu\,n}{T}\right) - C_3\,n + y^+_{3,1}\,, \quad \kappa_n = C_1\,\frac{n}{T} + \left(\frac{n}{\mathcal{E}+\mathcal{P}}\right)K^-_{2,1} - K^+_{1,1}\,. \tag{9}$$

We now discuss the behavior of the transport coefficients by considering a power-law parametrization for the energy dependence of the relaxation time [2, 5, 6], i.e., $\tau_{\rm R}(x,p) = \tau_{\rm eq}(x)\,\left(\frac{u\cdot p}{T}\right)^\ell$. Here $\tau_{\rm eq}(x)$ only depends on the space-time coordinates and ℓ is a constant.

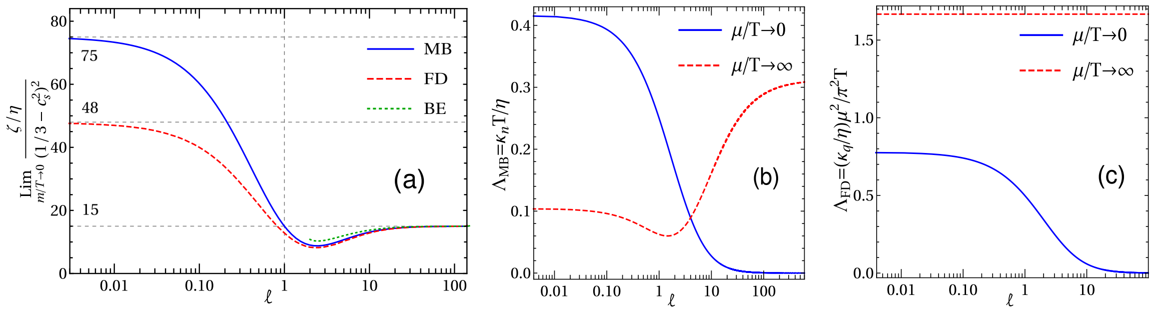

Figure 1: Scaling behavior of ratio of transport coefficients with ℓ for different equilibrium statistics. Figures adapted from [4].

Dissipation in a nonconformal plasma ($m \neq 0$) in absence of chemical potential ($\mu = 0$) is due to the shear stress as well as the bulk stress. Therefore, it is instructive to study the behavior of the ratio of bulk to shear viscous coefficients, ζ/η, with the conformality measure, $1/3 - c_s^2$. In small m/T limit, the quantity ζ/η has the leading behavior,

$$\frac{\zeta}{\eta} = \Gamma\left(\frac{1}{3} - c_s^2\right)^2 \tag{10}$$

for both MB and FD statistics. The scaling relation also holds for BE statistics for $\ell \gtrsim 2$. However, for $\ell\,(< 2)$, the scaling is more subtle as the soft momenta governs the behavior in small mass limit [7]. The behavior of Γ as a function of ℓ in this ultrarelativistic limit is shown in Fig. 1(a). We observe a non-monotonic behavior of Γ as a function of ℓ with a minimum at $\ell \approx 2.5$. The strong dependence of the relaxation rate on the particle energy $\left(\tau_{\rm R} \propto (u \cdot p)^{\ell}\right)$ results in $\Gamma \longrightarrow 15$ for large ℓ (irrespective of equilibrium statistics), as can be seen in Fig. 1(a).

For a conformal system ($m = 0$) with conserved charges ($\mu \neq 0$), the dissipation is due shear stress and charge conduction. In Fig. 1(b), we show the scaling behavior of the ratio of the coefficient of charge conductivity κ_n to the coefficient of shear viscosity η for MB statistics as a function of ℓ. The scaling of the ratio of coefficient of thermal conductivity $\kappa_q = \kappa_n \left(\frac{\mathcal{E}+\mathcal{P}}{nT}\right)^2$ to η with ℓ for FD statistics is shown in Fig. 1(c). The ratios in both small and large μ/T limits scale as

$$\frac{\kappa_n}{\eta} = \Lambda_{\rm MB}\,\frac{1}{T}, \qquad \frac{\kappa_q}{\eta} = \Lambda_{\rm FD}\,\frac{\pi^2 T}{\mu^2}. \tag{11}$$

From Figs. 1(b) and 1(c), it can be seen that these ratios tend to constant values in both small and large μ/T limit. In Fig. 1(b), it is interesting to note the monotonous decrease of $\Lambda_{\rm MB}$ with ℓ in the $\mu/T \longrightarrow 0$ limit (solid blue curve), starting from $\Lambda_{\rm MB} = 5/12$ for $\ell = 0$ and approaching zero as $\ell \longrightarrow \infty$. Similar behavior is observed in Fig. 1(c) for FD statistics (solid blue curve). The constant red dashed line in Fig. 1(c) corresponds to $\Lambda_{\rm FD} = 5/3$ at large μ/T, indicating that the ratio becomes ℓ-independent in this limit. In contrast, a non-trivial behavior of $\Lambda_{\rm MB}$ in the large μ/T limit is observed in Fig. 1(b) (red dashed curve).

The present formulation can be extended to derive higher-order hydrodynamic equations. Further, the framework provides freedom to specify the nature of interactions through the energy dependence of relaxation time (which can be any general function of particle energy/momentum), and can therefore be used to study the bulk properties of the evolving nuclear matter formed in heavy-ion collisions. We leave these interesting studies for future work.

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