Scales in light-nuclei production near the QCD critical point

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Based on the coalescence model, we analyse the light-nuclei production near the critical point by expanding the phase-space distribution function $f(\boldsymbol{r}, \boldsymbol{p})$ in terms of the phase-space cumulants $\sim \langle r^m p^m \rangle_c$. We show that the dominant contribution of the phase-space distribution to the yield of light nuclei is determined by the second-order phase-space cumulants. Here, we identify the fireball size, the homogeneity length, and the effective temperature, which are encoded in the second-order phase-space cumulants, as the relevant scales in explaining the yield of light nuclei. These scales are typically much larger than the correlation length of the critical fluctuations created in the rapid expansion of the heavy-ion systems, so we need to eliminate this dominant contribution of the relevant scales in order to isolate the critical contribution from the yield of light nuclei. We find that the second-order phase-space cumulants appeared in the yields of light-nuclei with different mass numbers share a similar structure. This property allows us to construct ratios of light-nuclei yields in appropriate combinations so that the effect of the relevant scales of the light-nuclei yield cancels, which isolates the critical effects.

The Tenth Annual Conference on Large Hadron Collider Physics,
16-20 May 2022
Online

*Speaker

PoS(LHCP2022)240

1. Introduction

Searching for the QCD critical point is one of the most important goals for relativistic heavy-ion collisions. Preliminary measurements of net-proton multiplicity fluctuations at the Relativistic Heavy Ion Collider (RHIC) show a non-monotonic behavior as a function of the colliding energy [1], which qualitatively agrees with the theoretical prediction [2]. Meanwhile, the realistic dynamics of the heavy-ion collision reaction is so complicated that it is non-trivial whether the observed non-monotonic behavior is unique to the critical effect. Thus, it is preferable to confirm the critical effects in multiple observables. Light-nuclei production is claimed to be related to the relative neutron density fluctuations [3] and its non-monotonic behavior [4] is the imprint of critical point. In this study, we consider the impact of the critical effect on another observable, the light-nuclei yield ratios calculated from the phase-space distribution of nucleons, $f(\boldsymbol{r}, \boldsymbol{p})$, using the coalescence model [5, 6]. In Sec. 2 we demonstrate that the size of the fireball, the homogeneity length, and the effective temperature are the relevant scales of the light-nuclei yield. The critical fluctuations induce additional correlations in the phase-space distributions and thus affect the yield of the light nuclei formed near the critical point. The critical correlation length in rapidly expanding heavy-ion systems is typically much smaller than the relevant scales of the light-nuclei production. We find that the ratios of the yield in appropriate combinations can be used to eliminate the effect of the relevant scales and isolate the critical effect from the light-nuclei yield.

2. Phase-space distribution in light-nuclei production

One of the widely used methods to calculate the production of light nuclei is the coalescence model [5–8], in which the yield is obtained as

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f_i(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A), \tag{1}$$

where the statistical factor $g_A = (2s+1)/2^A$ is given by the spin s of the light nucleus. The probability of producing light nuclei from A-nucleons at the phase-space positions $(\boldsymbol{r}_i, \boldsymbol{p}_i)$ $(i = 1, \dots, A)$ is described by the Wigner function of the spherical harmonic oscillator $W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A) = 8^{A-1} \exp[-\frac{1}{2} \sum_{i=1}^{A-1} \boldsymbol{Z}_i^2]$. The Wigner function only depends on the relative distances $\boldsymbol{Z}_i = \sqrt{\frac{i}{i+1}} (\frac{1}{i} \sum_{j=1}^{i} z_j - z_{i+1})$ $(i = 1, \dots, A-1)$ but not on the center-of-mass motion $\boldsymbol{Z}_A = A^{-1/2} \sum_{i=1}^{A} z_i$, where we redefined the phase-space variables as $z_i = \sqrt{2}(\boldsymbol{r}_i/\sigma_A, \sigma_A \boldsymbol{p}_i)$. This property stems from the fact that the nuclear interactions depend on the relative coordinates between the nucleons, namely the translational invariance of the system. The fact that the transform between the relative distances $\boldsymbol{Z}_i$ and the nucleon positions z_i is orthogonal will play an important role later.

One of the significant consequences of the Wigner function written in the Gaussian form with respect to the relative distances is that the light nuclei constituted of different numbers of nucleons A share the same structure in the case of Gaussian phase-space distributions $f_i(\boldsymbol{r}_i, \boldsymbol{p}_i)$. To see this in a systematic manner, we expand the phase-space distribution by the phase-space cumulants [9]:

$$\frac{f(z_i)}{N_p} = \rho(z_i) = \int \frac{d^6\boldsymbol{t}_i}{(2\pi)^6} e^{-\mathrm{i}\boldsymbol{t}_i \cdot z_i} \exp\left[\sum_{\alpha \in \mathbb{N}_0^6} \frac{C_\alpha}{\alpha!} (\mathrm{i}\boldsymbol{t}_i)^\alpha \right], \tag{2}$$

where $N_p = N_n = \int d^6 z f(z)$ is the number of nucleons (where isospin asymmetry is neglected). The cumulant of the phase-space variable $C_\alpha \equiv \langle z^\alpha \rangle_c$ is defined by the multi-index order $\boldsymbol{\alpha} \in \mathbb{N}_0^6$, and $\langle \cdots \rangle = (1/N_p) \int d^6 z \cdots f(z)$ represents the average over the phase-space under a single phase-space distribution $f(\boldsymbol{r}, \boldsymbol{p})$. When the distribution is sufficiently close to the Gaussian distribution, i.e., C_α for $|\boldsymbol{\alpha}| \geq 3$ are sufficiently small, the yield of light nuclei in Eq. (1) can be diagrammatically evaluated by the perturbation to the Gaussian integration and finally gives the form:

$$N_A = g_A N_p \left[\frac{8 N_p}{\sqrt{\det(C_2 + \mathcal{I}_6)}} \right]^{A-1} \cdot [1 + O(\{C_\alpha\}_{|\alpha| \geq 3})], \tag{3}$$

Here, one can see that, at the lowest order of the perturbation determined by the second-order cumulants C_2, the phase-space distribution $f(\boldsymbol{r}, \boldsymbol{p})$ plays a similar role in light-nuclei yields of different mass numbers A. Consequently, we may construct the ratios of the light-nuclei yields, which are fixed solely by g_A (under the assumption of the common light-nuclei size $\sigma_A \equiv \sigma$):

$$R_{A,B}^{1-B,A-1} = \frac{N_p^{B-A} N_B^{A-1}}{N_A^{B-1}} = \frac{g_B^{A-1}}{g_A^{B-1}} [1 + O(\{C_\alpha\}_{|\alpha| \geq 3})], \tag{4}$$

where the dominant contribution from the second-order phase-space cumulants is canceled out. Explicitly, the canceled second-order cumulants have the form:

$$C_2 = 2 \begin{pmatrix} \frac{\langle \boldsymbol{r}\boldsymbol{r}^{\mathrm{T}} \rangle}{\sigma^2} & \langle \boldsymbol{r}\boldsymbol{p}^{\mathrm{T}} \rangle \\ \langle \boldsymbol{p}\boldsymbol{r}^{\mathrm{T}} \rangle & \sigma^2 \langle \boldsymbol{p}\boldsymbol{p}^{\mathrm{T}} \rangle \end{pmatrix}, \tag{5}$$

where $\langle \boldsymbol{a}\boldsymbol{b}^{\mathrm{T}} \rangle$ is the 3×3 matrix with the elements $\langle a_i b_j \rangle_c$ $(i, j = x, y, z)$. The diagonal elements are the variances of coordinates $\langle r_i^2 \rangle_c$ and momenta $\langle p_i^2 \rangle_c$, corresponding to the fireball size and the effective temperature of the nucleon spectrum, respectively. The non-diagonal elements are the correlation between r and p, which is related to the homogeneity length l [10]. To summarize, we can treat the fireball size, the homogeneity length, and the effective temperature as the relevant scales of the light-nuclei production.

3. Critical fluctuations in light-nuclei production

For the light nuclei created in the vicinity of the critical point, the nucleons interact with the chiral field $\sigma(\boldsymbol{r})$, and their masses are modified with a small deviation $\delta m = g_\sigma \sigma$ to the leading order. Their phase-space distribution thus contains the correction term δf [11]:

$$f = f_0 + \delta f = f_0 [1 - g_\sigma \sigma / (\gamma T)], \tag{6}$$

where $f_0 = f_\sigma|_{\sigma=0}$ denotes the background distribution without the critical contribution, and $\gamma = \sqrt{\boldsymbol{p}^2 + m^2}/m$ is the Lorentz factor. In addition to the contribution from the background f_0, the polynomials of the correction term δf also play a role in the yield of light nuclei in Eq. (1). As the first step of the study, let us borrow the forms of the static critical correlators [11]:

$$\langle \sigma(\boldsymbol{r}_1) \sigma(\boldsymbol{r}_2) \rangle_\sigma = T D(\boldsymbol{r}_1 - \boldsymbol{r}_2), \tag{7}$$

$$\langle \sigma(\boldsymbol{r}_1) \sigma(\boldsymbol{r}_2) \sigma(\boldsymbol{r}_3) \rangle_\sigma = -2T^2 \lambda_3 \int d^3 u D(\boldsymbol{r}_1 - \boldsymbol{u}) D(\boldsymbol{r}_2 - \boldsymbol{u}) D(\boldsymbol{r}_3 - \boldsymbol{u}), \tag{8}$$

where the critical propagator $D(\boldsymbol{r}_1-\boldsymbol{r}_2) \equiv \frac{1}{4\pi r}e^{-r/\xi}$ is written by $r = |\boldsymbol{r}_1-\boldsymbol{r}_2|$, the correlation length ξ, and the coupling constant λ_3 for the 3-point correlator. $\langle\cdots\rangle_\sigma$ represents the event-by-event averaging over different realizations of the sigma field.

The interaction with the sigma field induces the critical correlation, and the correlation length ξ affects the yield of light nuclei. This can be seen by using the characteristic function of the phase-space distribution with the critical contribution [12], where the final result takes the form:

$$\langle N_A\rangle_\sigma = g_A 8^{A-1} N_p^A [\det(C_2 + \mathcal{I}_6)]^{-(A-1)/2}[1 + \tilde{\Xi}(A)]. \tag{9}$$

Here we defined $\tilde{\Xi}(A) \equiv \sum_{b=2}^{A}(-1)^b C_A^b \Xi(A,b)$, where C_A^b is the binomial coefficients and $\Xi(A,b) \sim g_\sigma^b \langle\prod_{j=1}^{b}\sigma(\boldsymbol{t}_{r,j})\rangle_\sigma$ is the critical contribution which encodes the critical correlation length ξ. Although the correlation length grows up to $\xi = 1/m_\sigma$ in the static systems with m_σ being the σ mass, the correlation length is limited to the order of 2–3 fm in heavy-ion collisions due to the rapid expansion of the system [13], which is typically much smaller than the relevant scales of the light-nuclei production encoded in C_2. Considering the small value of ξ and small critical regime on the QCD phase diagram, the correlation length in the individual light-nuclei yield as shown in Eq. (9) is negligible. However, due to the similar structure related to the second-order phase-space cumulants C_2 in Eq. (9), the combination such as

$$\tilde{R}(A,B) \equiv R_{A,B}^{1-B,A-1} - g_B^{A-1} g_A^{-(B-1)} \tag{10}$$

and

$$\tilde{R}(A,B,D) \equiv R_{A,B}^{1-B,A-1} - g_B^{A-1} g_D^{-(A-1)(B-1)/(D-1)} [R_{A,D}^{1-D,A-1}]^{(B-1)/(D-1)} \tag{11}$$

greatly suppress the contribution from the background scales in C_2 which help to isolate the effects related to the correlation length. Here, the definition of $R_{A,B}^{1-B,A-1}$ in Eq. (4) is adapted to the present case as $R_{A,B}^{1-B,A-1} \equiv \langle N_B\rangle_\sigma^{A-1}\langle N_A\rangle_\sigma^{-(B-1)} N_p^{B-A}$.

4. Summary

In this study, we discussed the light-nuclei production near the QCD critical point from the viewpoint of the relevant scales of the light-nuclei production C_2 and the scale of the critical correlation length ξ. We first decomposed the phase-space distribution $f(\boldsymbol{r},\boldsymbol{p})$ in terms of various orders of phase-space cumulants C_α. Since the Wigner function in the coalescence model is approximately written in Gaussian form with respect to the relative phase-space distances of constitutive nucleons, the yield of light nuclei share a similar structure at the lowest order of the phase-space cumulants, We identified the relevant scales of the yield in the second-order cumulants: the fireball size, the homogeneity length, and the effective freeze-out temperature. The phase-space distribution of nucleons is modified by the interaction with the chiral field, which would be reflected in the yield of light nuclei. Naively, it would seem hard to separate the critical contributions from the relevant scales of the background phase-space distribution, but the structure of the lowest order of the phase-space cumulants in the yield enables us to construct combinations of light-nuclei yields to suppress the relevant scales encoded in the second-order phase-space cumulants and isolate the effects of the correlation length.

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PoS(LHCP2022)240