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Solving the Strong CP and Dark Matter Problems with a $U(1)$ Gauged Extension of the Standard Model

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The standard model does not include a candidate for dark matter, but it can be extended to include one or more suitable particles which can serve as the Dark Matter candidate. This paper considers an extension that includes a local U(1) symmetry, an axion particle to solve the strong CP problem, and right-handed neutrinos with appropriate mass terms to address the neutrino mass and dark matter. The axion decay constant and the right-handed neutrino masses are related to the same v.e.v.s, and the model predicts a mixed Dark Matter scenario with both axion and right handed neutrino components. The model is characterized by a reduced observational signals from each DM component.

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1. Introduction

The standard model (SM) has been a successful theory in describing the nature, but it fails to address important issues such as the absence of a dark matter candidate, the generation of neutrino masses and mixings and smallness of the θ-parameter. In this paper, we propose adding a new dark sector to the standard model to provide a solution to these issues as well as the long-standing strong CP problem. We examine the θ term in the SM Lagrangian, which generates a non-vanishing electric dipole moment for the neutron, and propose a dark sector solution that breaks the Parity symmetry. The proposed model predicts a non-zero $\bar{\theta}$ value due to the extra contribution coming from gravitational effects to the axion potential, which is expected to be of the order of $O\,(10^{-10})$ or smaller.

One proposed solution to the strong CP problem is the Peccei-Quinn symmetry, a global axial symmetry that is spontaneously broken at a high scale, F_a, referred as the axion decay constant. This introduces a new pseudoscalar field, the axion, which allows for a dynamical θ that relaxes to the minimum of the potential during cosmological evolution. However, the Peccei-Quinn symmetry is not expected to be exact due to the breaking of global symmetries by gravity, and may be broken by non-renormalizable operators at the Planck scale. The axion, which can solve the dark matter problem, decay constant is typically expected to have an intermediate mass scale of $F_a \sim 10^{10-11}$ GeV, making it consistent with the mass of the RH neutrinos and allowing for the seesaw mechanism to generate sub-eV scale active neutrino masses.

We propose a new gauged $U(1)$ extension of the SM with an accidental PQ symmetry and an axion field that emerges when the additional scalar fields take vevs. This model includes an additional local gauge group $U(1)_X$ and a $\mathbb{Z}_2$ discrete symmetry. The gauge symmetry is broken at a high scale by two complex scalar v.e.v.s, resulting in one physical pseudoscalar axion field. Additionally, the model includes three RH neutrinos and two sets of Dirac fermions charged under both the color gauge group and $U(1)_X$. Anomaly constraints of $U(1)_X$ are studied in detail and the charges are fixed to ensure the operators providing RH neutrino masses appear in the higher dimensional operators suppressed by Planck scale. Moreover, the $U(1)_X$ charges are chosen to allow for the emergence of a global PQ symmetry. However, higher dimensional operators violating the global PQ symmetry at the Planck scale cause a shift in the axion minimum from $\bar{\theta} = 0$. The model can produce mixed DM density which includes a feebly interacting massive particle (FIMP) DM component and the axion field.

The proceeding is structured as follows. In Section 2, the model is presented in detail, including the anomaly constraints. Section 3 focuses on the axion field in the model and the shifting of the θ parameter from higher dimensional operators. In Section 4, we discuss the the possibility of having axion and FIMP DM in the present work. Finally, the paper concludes in Section 5.

2. The Model

The detailed description of the model can be found in Ref. [1]. In this study, a $U(1)_X$ gauge extension of the SM is introduced with additional particles including three RH neutrinos, two sets of exotic quarks, and two singlet scalars. The charges are chosen such that one RH neutrino remains a possible DM candidate, and the scalars have charges that allow for Yukawa coupling with the fermions to generate their masses at the PQ scale F_a. The charges of the SM leptons and quarks complete a vectorial representation of the $U(1)_X$ symmetry.

A $\mathbb{Z}_2$ symmetry is also included to ensure that the lightest RH neutrino N_1 remains a stable DM candidate and prevent mixing among the extra fermions with the same QCD charge. Tables (1, 2) list all the particles with their corresponding charges under the complete gauge group $SU(3)c \times SU(2)_L \times U(1)_Y \times U(1)_X \times \mathbb{Z}_2$.

Gauge Group	Baryon Fields			Lepton Fields						Scalar Fields
	Q^i_L	u^i_R	d^i_R	L^e_L	L^μ_L	L^τ_L	e_R	μ_R	τ_R	ϕ_h
$\mathrm{SU(2)_L}$	2	1	1	2	2	2	1	1	1	2
$\mathrm{U(1)_Y}$	1/6	2/3	−1/3	−1/2	−1/2	−1/2	−1	−1	−1	1/2
$U(1)_X$	m	m	m	n_e	n	n	n_e	n	n	0
$U(1)_{PQ}$	0	0	0	$-2q_a$	0	0	$-2q_a$	0	0	0

Table 1: SM particles and their corresponding charges under complete gauge group. All the particles are even under $\mathbb{Z}_2$ discrete gauge group. The doublets are defined as $Q^i_L = (u^i_L, d^i_L)^T$ and $L^i_L = (\nu^i_L, e^i_L)^T$, respectively.

Gauge Group	Fermions							Scalars	
	N_1	N_2	N_3	ψ_L	ψ_R	χ_L	χ_R	ϕ_1	ϕ_2
$\mathrm{SU(3)_c, SU(2)_L}$	(1, 1)	(1, 1)	(1, 1)	(3, 1)	(3, 1)	(3, 1)	(3, 1)	1	1
$U(1)_X$	n_e	n	n	α_L	α_R	β_L	β_R	$\alpha_L - \alpha_R$	$\beta_L - \beta_R$
$U(1)_{PQ}$	$-2q_a$	0	0	$-q_a$	q_a	q_a	$-q_a$	$-2q_a$	$2q_a$
$\mathbb{Z}_2$	−1	1	1	1	1	−1	−1	1	1
No. of flavors	1	1	1	N_ψ	N_ψ	N_χ	N_χ	1	1

Table 2: BSM particles and their corresponding charges under complete gauge group and global $U(1)_{PQ}$ symmetry. The $U(1)_Y$ charges are vanishing and therefore not given in the table.

The Lagrangian for the particles in Tables 1 and 2 includes the SM Lagrangian and terms for the extra particles,

$$\begin{aligned}\mathcal{L} \;=\; & \mathcal{L}_{SM} + \sum_{j=1,2,3} \frac{i}{2}\bar{N}_j\gamma^\mu D_\mu N_j + \sum_{j=1}^{N_\psi} i\bar{\psi}_j\gamma^\mu D_\mu\psi_j + \sum_{j=1}^{N_\chi} i\bar{\chi}^j\gamma^\mu D_\mu\chi^j + \sum_{j=1}^{2}(D_\mu\phi_j)^\dagger(D^\mu\phi_j) \\ & +\mathcal{L}^{Yuk}_{BSM} + \mathcal{L}^{Yuk}_{N} + \mathcal{V}(\phi_h,\phi_i)\,. \end{aligned} \tag{1}$$

The terms in the above Lagrangian consist of kinetic terms for the right-handed neutrinos, exotic fermions, and singlet scalars, as well as Yukawa couplings between the SM leptons and right-handed neutrinos. The complete Lagrangian is given by Equation 1, where D_μ represents the covariant derivative and depends on the different charge assignments of the fields. The Yukawa couplings for the right-handed neutrinos, $\mathcal{L}_N^{Yuk}$, also include operators up to dimension 5 that are suppressed by the Planck scale. These are as follows

$$\mathcal{L}_N^{Yuk} = \sum_{i=2,3}\left[y_{\mu i}\bar{L}_\mu\phi_h N_i + y_{\tau i}\bar{L}_\tau\phi_h N_i + y_{ei}\bar{L}_e\phi_h N_i\frac{\phi_1}{M_{PL}}\right] + \sum_{i,j=2,3}\frac{y_{ij}}{2}N_iN_j\frac{\phi_1\phi_2}{M_{PL}} + \frac{y_{11}}{2}N_1N_1\frac{\phi_1^\dagger\phi_2}{M_{PL}} + h.c.. \quad (2)$$

2.1 Gauge Anomaly Cancellation

In Ref. [1], we have studied the gauge anomaly cancellation in detail. The non-vanishing contributions could come from the following terms,

- from $[Gravity]^2 \times U(1)_X$ and $[SU(3)]^2 \times U(1)_X$ we obtain

$$(\alpha_L - \alpha_R) = n_\chi(\beta_R - \beta_L) \text{ where } n_\chi = \frac{N_\chi}{N_\psi}. \quad (3)$$

- the $[U(1)_X]^3$ anomaly gives

$$\alpha_L^2 + \alpha_L\alpha_R + \alpha_R^2 = \beta_L^2 + \beta_L\beta_R + \beta_R^2, \quad (4)$$

  where in the above equation we have used Eq. (3) to simplify the equation to a second order equation in the charges from the cubic equation.

- for the combinations $[SU(2)]^2 \times U(1)_X$ and $[U(1)_Y]^2 \times U(1)_X$ only the SM states contribute and they give $9m + n_e + 2n = 0$.

After redefining the above parameters as $y = \frac{\Delta\beta}{\alpha_R}$, $z = \frac{\beta_L}{\alpha_R}$ where $\Delta\beta = \beta_R - \beta_L$, we can write the above set of equations in a quadratic form as follows,

$$(n_\chi^2 - 1)y^2 + 3y(n_\chi - z) + 3(1 - z^2) = 0. \quad (5)$$

n_χ	z	y	α_L	β_L	β_R	n_e	n	m
10	1	$-\frac{3}{11}$	$-\frac{19}{11}\alpha_R$	α_R	$\frac{8}{11}\alpha_R$	$-\frac{3}{2}\alpha_R$	$\frac{27}{22}\alpha_R$	$-\frac{7}{66}\alpha_R$
10	-1	$-\frac{1}{3}$	$-\frac{7}{3}\alpha_R$	$-\alpha_R$	$-\frac{4}{3}\alpha_R$	$-\frac{11}{6}\alpha_R$	$\frac{3}{2}\alpha_R$	$-\frac{7}{54}\alpha_R$
11	1	$-\frac{1}{4}$	$-\frac{7}{4}\alpha_R$	α_R	$\frac{3}{4}\alpha_R$	$-\frac{3}{2}\alpha_R$	$\frac{5}{4}\alpha_R$	$-\frac{1}{9}\alpha_R$
11	-1	$-\frac{3}{10}$	$-\frac{23}{10}\alpha_R$	$-\alpha_R$	$-\frac{13}{10}\alpha_R$	$-\frac{9}{5}\alpha_R$	$\frac{3}{2}\alpha_R$	$-\frac{2}{15}\alpha_R$

Table 3: $U(1)_X$ charges of the exotic quarks and SM fields for different values of n_χ and z in terms of the charge α_R.

Finally, after suitably choosing the parameters value defined before, we can write down the charge assignment of the SM as well as the beyond SM particles as shown in Table 3.

3. Axion

The Lagrangian in Eq. (1) exhibits an accidental global symmetry of $U(1)$, with axial symmetry for the exotic sector, as evident from the charge assignments in Table 2. This global symmetry is referred to as the global Peccei-Quinn (PQ) symmetry. By applying a field-dependent axial transformation for the exotic quarks,

$$\psi_L \to e^{i\frac{a_1}{2v_1}},\ \psi_R \to e^{-i\frac{a_1}{2v_1}},\ \chi_L \to e^{i\frac{a_2}{2v_2}},\ \chi_R \to e^{-i\frac{a_2}{2v_2}}\ .$$

We can remove the pseudoscalar fields from the Yukawa interaction terms and obtain a coupling between the axion field and the gluons from the measure of the exotic fermions. This can be effectively termed as follows,

$$\begin{aligned}\mathcal{L}_{AGG} &= \left(\frac{N_\psi a_1}{v_1} + \frac{N_\chi a_2}{v_2}\right)\frac{g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G^{\mu\nu}} \\ &= N_\psi \frac{A}{F_a}\, \frac{g_s^2}{32\pi^2} G_{\mu\nu}\tilde{G^{\mu\nu}}\ .\end{aligned} \tag{6}$$

where $A = \frac{1}{\sqrt{n_\chi^2 v_1^2 + v_2^2}}\left[v_2 a_1 + n_\chi v_1 a_2\right]$ and $F_a = \frac{v_1 v_2}{\sqrt{n_\chi^2 v_1^2 + v_2^2}}$ for $N_\psi = 1$ are the axion field and decay constant, respectively.

3.1 Gravitational effects in the axion potential

In this study, we consider a gauged $U(1)X$ symmetry that underlies the accidental PQ symmetry $U(1)PQ$. Consequently, at the Planck scale, we expect higher-dimensional operators that conserve the gauge symmetry but violate the global $U(1)_{PQ}$ symmetry. The selection of $U(1)_X$ charges ensures that several operators are prohibited by the local gauge symmetry. Therefore, the lowest dimensional operator of this kind is of high dimension, as stated in previous works [1, 2],

$$V_{PL}(\phi_1, \phi_2) = \frac{g}{N_\psi! N_\chi!} \frac{\phi_1^{N_\psi}\phi_2^{N_\chi}}{M_{PL}^{N_\psi + N_\chi - 4}} + h.c.\,. \tag{7}$$

The coupling g is a complex number given by $g = |g|e^{i\delta}$. It is important to note that by assuming the coupling to be complex, we are not making an explicit assumption about the complex nature of the Planck scale contribution. The phase factor in the coupling can also arise from the chiral rotation of the fermions required to diagonalize the mass matrix (m_q), in which case the angle δ is approximately equal to $arg(\det[m_q])$. M_{PL} ($= 1.22 \times 10^{19}$ GeV) is the Planck mass. After taking into account the contribution from the QCD instanton effect and gravitational effect, we can write down the total axion potential as,

$$V(\bar{\theta}) = F_a^2 M_a^2 \left[\left(1 - \cos\bar{\theta}_a\right) + r_g \left(1 - \cos(p\,\bar{\theta}_a + \delta)\right)\right]\ , \tag{8}$$

where $p = -N_\psi + N_\chi = N_\psi(n_\chi - 1)$, F_a is axion decay constant, $r_g = \frac{(M_a^g)^2}{M_a^2} \ll 1$, $M_a^g = \sqrt{\frac{|g|}{N_\psi! N_\chi!}\frac{v_1^{N_\psi} v_2^{N_\chi}}{(\sqrt{2})^{N_\psi+N_\chi} M_{PL}^{N_\psi+N_\chi-4} F_a^2}}$ and $M_a = \frac{m_u m_d}{(m_u+m_d)^2}\frac{f_\pi m_\pi}{F_a}$ (m_q is q-quark mass, m_π and f_π are the pion mass and decay constant). The additional term in the axion potential will shift the axion minima from $\bar{\theta}_a = 0$. The strong CP-problem can still be resolved by the axion if the shift parameter, $\Delta\theta$, is less than or equal to the upper bound which is 10^{-10}. By minimizing the complete potential of the axion field, given by Eq.(8), we can determine the value of $\bar{\theta}_a$ (defined as $\Delta\theta$) as follows,

$$\Delta\theta = \frac{r_g |p \sin\delta|}{\left[1 + p^4 r_g^2 + 2p^2 r_g \cos\delta\right]^{1/2}} \,. \tag{9}$$

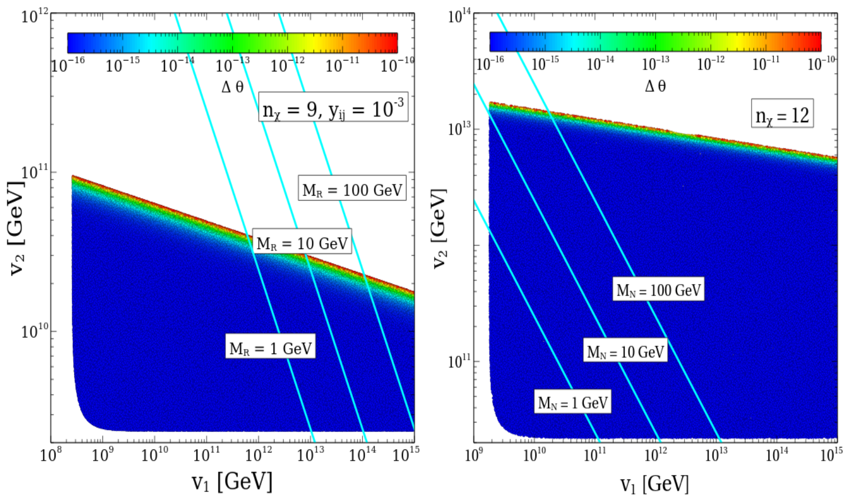

Figure 1: Scatter plot in the $v_1 - v_2$ plane. The value of $\Delta\theta$ parameter is shown by the color bar. Cyanide lines represent the different mass eigenvalues of the right handed neutrino mass matrix (as depicted in Eq. (2)) when $y_{ij} = 10^{-3}$ $(i, j = 2, 3)$. The left and right plots correspond to $N_\psi = 1$, $n_\chi = 9$ and 12, respectively.

Figure 1 displays the permissible parameter space in the v_1 versus v_2 plane for different values of n_χ (namely, $n_\chi = 9, 12$), which were numerically computed using the full expression given in Eq. (9). Increasing n_χ expands the allowable range of v_2 since a larger value of n_χ results in a more significant suppression at the Planck scale. The cyan lines represent different values of the right-handed neutrino mass for $y_{ij} = 10^{-3}$ $(i, j = 2, 3)$. In Ref. [1], we have examined these cases in greater detail, particularly those that are not significantly far from the asymptotic freedom scenario with respect to the DM density.

4. Dark Matter

In Ref. [1], we have studied DM in detail by considering axion and right handed neutrino as the dark matter candidates. In the case of axion DM, it is produced by the usual misalignment mechanism and the right handed neutrino DM is produced by the freeze-in mechanism from the decay of the bath particles. Here we discussed them very briefly and for the complete discussion on the DM, we request the interested readers to look at Ref. [1].

Axion dark matter:

In the case of axion DM, we are examining an axion that exists on a sub-eV scale mass and has the potential to become a good cold dark matter candidate. The misalignment mechanism gives rise to the axion's contribution to the density of cold dark matter, which can be expressed as [3],

$$\Omega_a h^2 \simeq 0.18\, \theta_i^2 \left(\frac{F_a}{10^{12}\,\text{GeV}} \right)^{1.19}, \tag{10}$$

where θ_i is the initial value of the misalignment angle and can take arbitrary value if the the PQ symmetry breaks before inflation. In the event that the PQ symmetry breaks after inflation, the mean value of θ_i across multiple domains is $O(1)$.

N_1 **as FIMP DM:**

As discused in Ref. [1], if $n_\chi < 12$, then axion can not give us the full amount of DM density. Fortunately, within the model, we have another promising candidate for dark matter. This is made possible by assigning $\mathbb{Z}_2$ parities to the right-handed neutrinos, such that N_1 is the lightest odd particle with respect to $\mathbb{Z}_2$ parity, and remains stable since the Higgs fields don't break this global symmetry. For N_1 to be a suitable candidate for dark matter, it must have been produced in the correct amount during the early universe. Given its extremely weak interactions, the FIMP mechanism is a natural way for this to occur, as explained in [1]. Notably, N_1 only interacts via the $U(1)_X$ gauge interaction and the non-renormalizable operators consist of extra scalars that generates its mass as shown below,

$$\mathcal{L}_{N_1} = \frac{i}{2} \bar{N}_1 \gamma^\mu \left(\partial_\mu - i\, n_e g_X Z_X \right) N_1 + y_{11} \bar{N}_1^c N_1 \frac{\phi_1^\dagger \phi_2}{M_{Pl}} + h.c. \tag{11}$$

where n_e and g_X are the charge of N_1 and gauge coupling associated with $U(1)_X$ gauge group. In our work, FIMP DM can be produced from the decay of Higgses and the $U(1)_X$ gauge boson as shown by the Eq. 11.

In Fig. (2), we present scatter plots in the $v_1 - M_{N_1}$ (LP) and $M_{Z_X} - g_X$ (RP) planes, while satisfying the constraint on the $\Delta\theta$ parameter and the limit on the DM relic density. The cyan points correspond to the contribution from scalar decays, the green points denote the contribution from Z_X decay, and the red points indicate the total contribution from both decays as well as the axion contribution. The contribution from scalar decays to DM is proportional to $\Omega_{N_1}^{FIMP} h^2 \propto \frac{M^3 N_1}{v_1^2}$, which explains the linear relationship among the cyan points until $v_1 < 5 \times 10^{11}$, consistent with the analytical formula shown in Ref. [1]. Beyond this value, the relic density of N_1 is controlled by the other v.e.v. v_2, which is limited by the constraint on the θ parameter. Consequently, the dependence on v_1 weakens and the correlation between mass and v.e.v. is disappeared. We observe that no allowed points exist for $M_{N_1} > 100$ GeV, as the couplings $g_{H_1 N_1 N_1} = \frac{M_{N_1}}{v_1}$ and $g_{H_2 N_1 N_1} = \frac{M_{N_1}}{v_2}$ become too large, resulting in a high energy density of N_1 for the chosen parameters. The contribution from Z_X decay is proportional to the ratio of the dark matter mass and the Z_X mass, resembling the SuperWIMP scenario, leading to a sharp correlation among the green points. Finally, the red points correspond to the total contribution, which is the sum of the scalar and gauge boson contributions. In the RP, the colors represent same kind of production and exhibits the linear relation between M_{Z_X} and g_X^{eff} which is consistent with the analytical formula of M_{Z_X} in terms of g_X and v.e.v.s $v_{1,2}$.

Finally, in Ref. [1], we have studied in detail the axion and FIMP dark matter and tried to constraint the different parameters after applying DM relic density bound. For more discussion on DM, we refer the readers to the DM part of Ref. [1].

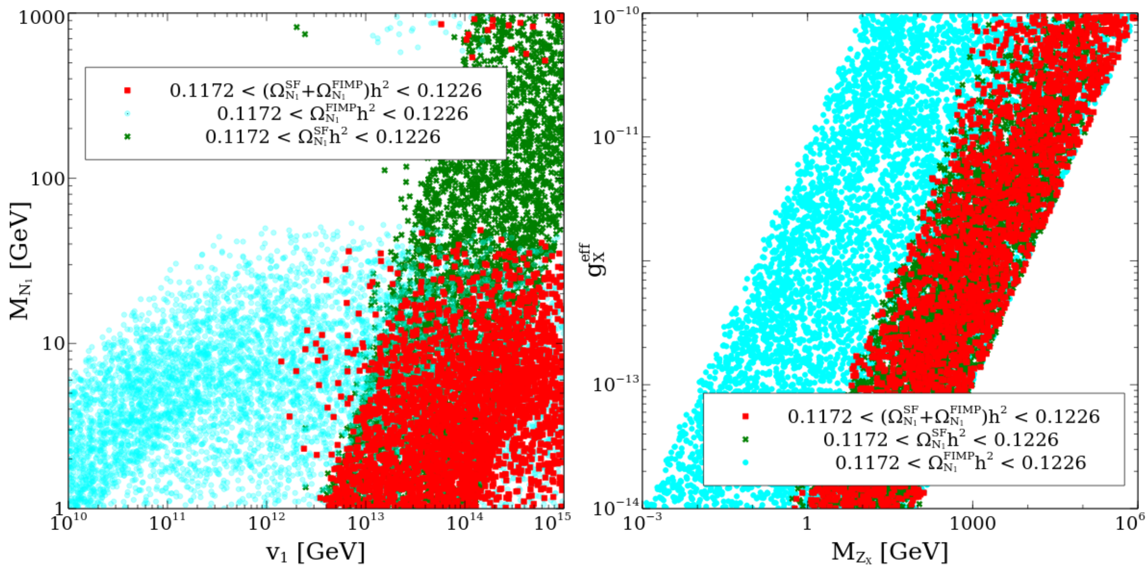

Figure 2: Scatter plots in the $v_1 - M_{N_1}$ (LP) and $M_{Z_X} - g_X$ (RP) planes for $n_\chi = 10$ after satisfying the DM relic density bound and considering production from the decays of different parent particles. Cyan points correspond to contribution from the scalars decays, green points correspond to contribution from Z_X decay and the red points are the total contribution. The model parameters are fixed at $\theta'_{12} = 10^{-3}$ and $M_{h_{1,2}} = 100$ TeV. Both the plots have been adopted from Ref. [1].

5. Conclusion

We propose a new $U(1)_X$ extension of the Standard Model that solves three open questions: the strong CP problem, the origin of neutrino masses, and dark matter. The model has an accidental PQ symmetry and an axion field that solves the strong CP problem, and the same scalar v.e.v.s that break the PQ symmetry also give masses to the right-handed neutrinos and consequently the active neutrinos via the Type I seesaw mechanism. The RH neutrino masses are at the electroweak scale coming from the non-renormalisable operators. The model has two DM candidates: the axion and the stable RH neutrino. Depending on the field content, the DM density is provided by either or both candidates, and the direct detection signal is expected to be very weak due to the feeble interaction. The RH neutrino sector at the electroweak scale can provide additional signatures, including leptogenesis. The model predicts additional Higgs decays into axions and modifies the Higgs potential, but these effects are always suppressed by the PQ scale.

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