PROCEEDINGS OF SCIENCE

Monte Carlo modeling of jets

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Monte Carlo approaches play a crucial role in collider physics as they enable theory-data comparisons for complex multi-particle observables that are otherwise challenging to calculate perturbatively. In the context of heavy-ion collisions, numerous Monte Carlo approaches have emerged to study jet quenching phenomena, which refer to the modifications experienced by high momentum particles and jets as they traverse the hot and dense medium produced in these collisions. These models are continuously evolving in conjunction with theoretical efforts aimed at understanding and accurately describing experimental results from both RHIC and the LHC.

This manuscript provides a comprehensive overview of the essential components that these tools must address in order to describe jets in heavy-ion collisions. Additionally, it presents a comparative analysis of the most recent results obtained from several jet quenching Monte Carlo models, focusing on jet and intra-jet observables. Finally, the manuscript concludes with a forward-looking perspective on future developments in this field.

PoS(HardProbes2023)016

HardProbes2023
26-31 March 2023
Aschaffenburg, Germany

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1. Introduction

The study of jets in heavy-ion collisions has made significant progress in understanding the fundamental processes occurring in the presence of a hot and dense medium, known as the Quark-Gluon Plasma (QGP) (see [1, 2] for recent reviews). Analytical description of jets, based on first principle calculations, have played a crucial role in this advancement. These approaches have surpassed static medium assumptions and limited kinematic approximations, providing a solid foundation for understanding elementary interactions (see [3] for a review at this conference). Additionally, experimental studies have focused on background-free observables, enabling direct comparisons between theoretical predictions and experimental data. However, the applicability of analytical approaches becomes limited when dealing with complex multi-particle observables. The intricate interactions and correlations between particles make it impractical to calculate such observables solely using analytical methods. Furthermore, it is still unknown how to define consistently the interplay between vacuum and medium-induced showers or how to accurately describe the low-momentum scales, where non-perturbative effects become significant. To address these challenges, Monte Carlo (MC) event generators have emerged as powerful computational tools. These generators offer a complementary approach by simulating the evolution of parton showers in the presence of the hot and dense medium. They incorporate a range of interactions and dynamics, using both theoretical models and phenomenological assumptions. By considering the full evolution of the jet shower, MC event generators provide a comprehensive framework for studying jet dynamics, including formation, fragmentation, and energy loss in the medium, leading to a more realistic representation of the experimental environment.

Despite being widely used in heavy-ion collisions, it is important to acknowledge some limitations associated with MC approaches. Although built upon analytical results, these models may not immediately include the most recent theoretical developments, thus resulting in discrepancies between theoretical predictions and experimental observations. Furthermore, MC approaches often require additional modelling beyond analytically-controlled phase-space regions to describe certain phenomena. This introduces additional uncertainties and assumptions in the model-data comparison.

This manuscript aims to provide an overview of the main ingredients used in MC modelling of jets. We aim to highlight the strengths and limitations of MC approaches to jet quenching studies, showcasing the successes achieved thus far. As such, this manuscript is organized as follows: in Section 2, we present a general overview of the different elements used in MC models, followed by some of their data comparisons in Section 3. Finally, an outlook towards new opportunities will be presented in Section 4.

2. Jet Quenching MC models

Jet quenching MC models are powerful tools for generating an N-particle system through a parton shower. These models incorporate both vacuum and medium-induced effects, contributing to the resulting jet's formation and fragmentation. Vacuum radiation represents the emission of particles from the parton shower in the absence of a medium, akin to a proton-proton system. On the other hand, medium-induced effects stem from interactions between partons and the medium, resulting in modifications to the jet shower. One important medium-induced effect is medium-induced gluon radiation. This process alters the jet's properties, such as energy and particle content. Another significant effect is the jet-induced medium response, which occurs as the high-energy jet interacts elastically with the surrounding medium, triggering a correlated response from the medium constituents. Additionally, medium response re-scattering may arise when particles from the jet-medium interaction experience further elastic interactions with the medium constituents, leading to energy and momentum redistribution between the jet and medium constituents.

To describe medium-induced effects in the parton shower, two different approaches are commonly employed, each with its own advantages and considerations. The first approach involves modifying the parton shower's evolution throughout its formation by accommodating parton-medium interactions and resulting medium-induced radiation (see, e.g., [4, 5]). This approach allows medium-induced modifications to occur at any stage during the partonic cascade evolution, primarily dominating momentum scales above the non-perturbative region. Implementing this approach requires selecting or developing a specific vacuum parton shower, fixing the ordering variable and parton shower accuracy. The second approach focuses on modifying a semi-developed partonic cascade, keeping the vacuum parton structure unmodified, particularly the hard and collinear components (see, e.g. [6–8]). In this case, medium-induced effects are more prominent at lower momentum scales. Both approaches rely on analytical descriptions of medium-induced effects, inheriting their kinematical restrictions. It is important to note that there is no definitive "correct" answer when choosing between these approaches. The first approach covers the entire jet evolution and provides modifications at all momentum scales, offering insight into the interplay between vacuum and medium showers. However, it requires careful selection or development of a specific vacuum parton shower and may present challenges in implementing the medium-induced effects consistently. On the other hand, the second approach minimizes changes to the vacuum parton shower and is easier to integrate with vacuum physics. Nevertheless, it can be challenging to accurately describe the low-momentum region, which involves QCD processes close to the non-perturbative QCD phase-space.

Another crucial aspect of MC models is their ability to include the evolving medium simultaneously with the developing parton shower. MC approaches encompass a range of evolution scenarios, ranging from simplified one-dimensional expansions to more comprehensive three-dimensional simulations. In the case of simplified one-dimensional expansions, models such as the Bjorken longitudinal expansion [9] assume a power-law scaling to characterize the energy density. Although these models provide valuable insights, they are limited in capturing the full complexity of the evolving medium. Alternatively, three-dimensional non-ideal relativistic hydrodynamics [10] can be employed to achieve a more realistic representation. By considering factors like viscosity and pressure gradients, these models account for the hydrodynamic flow, offering a deeper understanding of medium evolution and enabling the exploration of phenomena like collective behaviour and thermalization.

Even with the continuous advancements in medium evolution modelling, the onset of jet-medium interactions remains a significant source of uncertainty(e.g, see [11, 12]). Determining the precise moment when the interaction between the jet and the medium begins is an ongoing area of research. This onset plays a crucial role in accurately capturing the effects of jet quenching and understanding the subsequent modifications in the jet properties.

With all the possibilities to build a Jet Quenching MC model, it is no surprise to see the plethora of different approaches that have emerged in these recent years.

3. The successes of MC approaches

3.1 Transport coefficient

Medium-induced radiation and momentum broadening are closely connected phenomena. In the multiple soft-scattering approximation, partons experience multiple interactions with the medium, leading to an accumulation of momenta. This enhanced momentum accumulation results in increased gluon radiation and transverse momentum broadening of the partons. To quantify this effect, one can define a transport coefficient, $\hat{q}$, representing the average transverse momentum broadening, $\langle k_\perp^2 \rangle$ per mean free path, λ. It is intimately related to the medium scattering potential $V(\mathbf{q})$ as it can be written as:

$$\hat{q} = \frac{\langle k_\perp^2 \rangle}{\lambda} \propto \int d^2\mathbf{q}\, q^2 \frac{d\sigma(q)}{d^2\mathbf{q}} \tag{1}$$

where $\mathbf{q}$ represents the two-dimensional transverse momentum transfer and

$$\sigma(\mathbf{r}) = \int \frac{d^2q}{(2\pi)^2} V(\mathbf{q})(1 - e^{i\mathbf{q}\cdot\mathbf{r}}) \tag{2}$$

the dipole cross-section in coordinate space.

Recovering the transport coefficient from single-particle or jet suppression measurements is possible but comes with large uncertainties. These uncertainties arise due to several factors. Firstly, the QGP initialization conditions play a significant role, and there are currently several uncertainties on how to describe parton-medium interaction during the pre-QGP phase (see Section 2). Naturally, considering energy loss during the entire parton shower evolution versus energy loss only during the hydrodynamic phase leads to a compensation of effects, which can influence the derived transport coefficient. Furthermore, improved Bayesian analysis techniques have revealed a stronger temperature dependence of the transport coefficient, emphasizing the need for refined calculations. Incorporating different data sets, such as boson-hadron correlations dominated by quark contributions and inclusive particle spectra that contain a mixture of quarks and gluons, adds further complexity and uncertainties to the $\hat{q}$ extraction as an exact quantification of each parton flavour is difficult. Moreover, comparing measurements of hadrons and jets introduces model-dependent descriptions of the medium response, which must be considered when interpreting the results. Despite these challenges and uncertainties, efforts are being made towards a quantitative assessment of the characteristics of the QGP using not only jets but also high-momentum particles. The result is illustrated in Fig 1, where several $\hat{q}$ extractions out of experimental data by different groups are drawn, each with their coloured markers or bands.

3.2 The elusive medium response

The quest to directly observe the response of the medium in high-energy nuclear collisions remains a challenging endeavour. To achieve a more accurate description of jet observables, such as the radial profile and jet mass, the inclusion of soft components appears to be highly necessary,

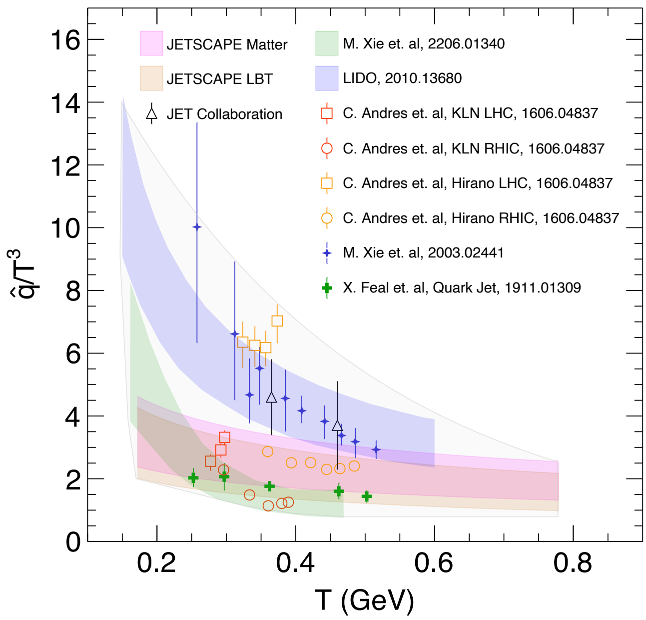

Figure 1: A snapshot of $\hat{q}$ extracted from charged hadron spectra, di-hadron, γ-hadron correlations and jet inclusive spectra as a function of temperature. A gray area is also shown to cover the central values of the extracted $\hat{q}$ from different models. Fig. taken from [1].

regardless of the specific MC model employed. An example is illustrated in Fig. 2, where it is shown the ratio of the jet radial profile as obtained from MARTINI [13] (left panel) and LBT [14] (right panel) with respect to their proton-proton *vacuum* reference. In both panels, it is possible to see sizable differences just by considering the effect of medium response, although the magnitude of this contribution differs between the models. In MARTINI, the energy loss effects on the parton shower induce a strong suppression of the particle yield and momentum (red dashed line on the left panel) that are transferred to the medium. As such, this energy is only recovered when medium recoils are again considered part of the jet constituents (green dashed line on the left panel). LBT, on the other hand (right panel), induces small modifications to the jet's internal structure (blue lines). Once the medium recoiling particles are included as being part of the jet, it can qualitatively reproduce the observed increase in the jet radial profile radial distances around 0.3. However, while introducing medium recoiling constituents can lead to a satisfactory description of these jet observables, it also results in an accumulation of energy at large radial distances. This outcome seems to conflict with experimental results on large-radius jets [15], urging for a better theoretical control over medium response with respect to the poorly understood non-perturbative physics.

Recently, γ+jet events have emerged as a promising avenue for detecting the formation of a Mach-cone wake, a distinctive characteristic of a hydrodynamic QGP. Several models (e.g [16]) have suggested that in such events, the presence of the QGP would manifest as a depletion of soft hadrons in the away-side direction of the jet. This phenomenon is visualized in the right panel of Fig. 3, where only the jet-induced medium excitation is depicted. However, identifying this signature is challenging due to multiple particle interactions (MPI) associated with triggered hard processes. In triggered jet events, the production of multiple minijets is enhanced as the

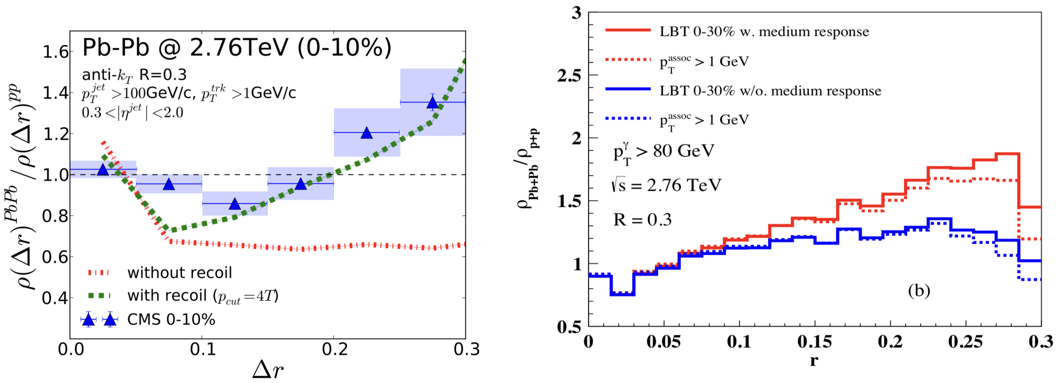

Figure 2: (Left) Ratio of the jtransverse jet profile of inclusive $[0-10]\%$ PbPb to pp collisions at $\sqrt{s_N N}$ = 2.76 TeV as obtained by MARTINI with (green dashed line) and without (red dashed line) recoils. CMS data (blue triangles) is also included. Fig. taken from [13]. (Right) Ratio of the transverse jet profile of γ-jets in central [0˜30]% PbPb and pp collisions from LBT simulations with (red) and without (blue) contributions from jet-induced medium response at $\sqrt{s_{NN}}$ = 2.76 TeV. Dashed lines correspond to including a trasverse momentum cut for the associated partons of $p_T^{ass} > 1$ GeV. Fig. taken from [14].

collision energy and trigger jet p_T increase. The interaction of these minijets with the medium leads to an enhancement of soft hadrons, effectively obscuring the signal of the diffusion wake. The right panel of Fig. 3 explores the possibility of enhancing the depletion by selecting events with enhanced jet quenching effects. By selecting events where the jet lost more than 20% of its initial energy (blue solid line, considering the transverse momentum of the γ, p_T^γ as a proxy for the jet-initiating parton), the depletion would still reduce the contribution from MPI (grey band), as opposed to a scenario in which no selection on the events is made (red dashed line). Since the QGP wake is directly connected to the QCD equation-of-state adopted by the hydrodynamic model, this example demonstrates how the continuous development of MC models can contribute to a better understanding of QGP characteristics and its response to high-momentum objects.

3.3 Understanding biases

One crucial aspect in the study of jet quenching is understanding biases that arise when comparing quenched and unquenched jets. To this aim, PbPb and pp jets are usually contrasted using a specific jet selection criterion, such as the jet's reconstructed transverse momentum (p_T). This comparison is quantified through the nuclear modification factor, a ratio between the high-momentum objects (jets or hadrons) yield of the two collision systems assuming a superposition scenario. At high p_T, this observable is mostly sensitive to QGP-induced effects, thus providing insights into the energy lost by a jet in the medium. However, due to quenching effects, jets initially belonging to a specific p_T bin will likely shift towards smaller p_T values. Accurately estimating the energy loss solely from this observable thus becomes challenging as the resulting surviving jet population experienced less in-medium effects. Proposals based on quantile analysis of the cross-section [17] allow a more accurate estimation of the average energy loss by mitigating this survival bias effect.

Another feature that emerges when looking at the survivor population of jets is that they exhibit a different jet substructure compared to pp collisions, thus endangering possible interpretation of

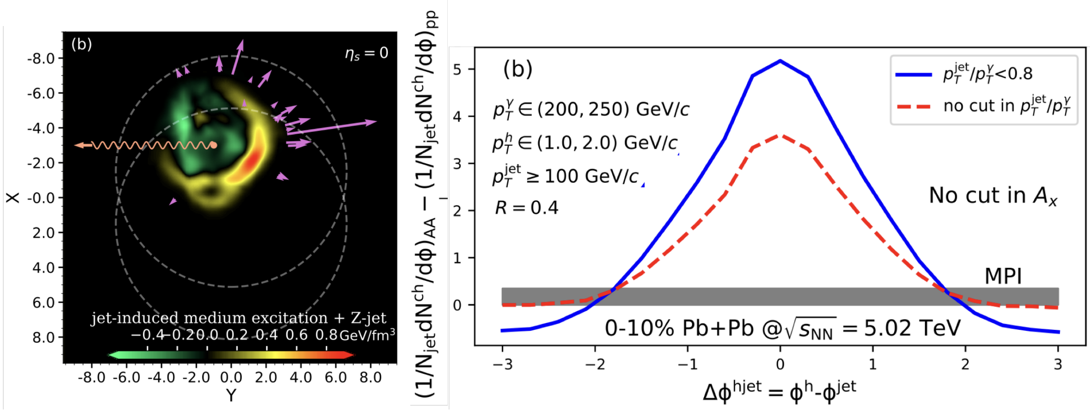

Figure 3: (Left) A snapshot of the jet-induced energy density distribution in the transverse plane of a semi-central Pb+Pb collision at $\sqrt{s}_{NN}$ = 5.02 TeV from the Co-LBT hydro hydrodynamic simulations of a Z (wavy lines) + jet event at a proper time $\tau = 4.6$ fm/c. The hadrons' transverse momenta are represented by straight lines and the colliding nuclei by the dashed circles. (Right) Difference in γ-hadron yields between $[0-10]\%$ PbPb and pp events at $\sqrt{s}_{NN}$ = 5.02 TeV with a cut on the γ-jet momentum asymmetry $p_T^{jet}/p_T^{\gamma} < 0.8$ (blue curve) and without (red dashed line). The contribution from multiple parton interaction is indicated by the gray band. Figs. taken from [16].

quenching effects on the jet fragmentation pattern. It was verified (e.g, in [18]) via grooming techniques, in which jets are reclustered with C/A followed by the iterative unclustering procedure along the primary branch until the first SoftDrop approved clustering step is found [19]. With the resulting two subjets, it is possible to withdraw the jet splitting function or the relative radial separation ΔR. Notably, if we exclude medium response effects, jets that survive a given p_T cut tend to be more narrow when compared to pp, as shown in Fig. 4 (left panel, blue dashed line for PbPb without medium response and the solid blue line for the pp reference). Nonetheless, since medium recoils are deflected towards larger radial distances, this component can produce a contribution of soft fragments at large ΔR, which seemingly results in no modification due to quenching (dotted blue line).

To address the challenge of pinpointing jet quenching effects, various strategies, including the application of machine learning techniques, have been suggested [20–22] to select a jet population where quenching effects are enhanced. While these techniques proved useful, they tend to bias the jet sample towards lower p_T where the effects of energy loss become more visible. A recent alternative involves using a different reclustering algorithm, namely τ [23], followed by a SoftDrop procedure to identify the two leading subjets. This methodology not only provides a good correlation between the formation time obtained via Soft-Drop approved clustering steps and those in the parton shower, as it allows to select a population of jets that experienced significant energy loss effects. This technique has been applied to Z+jet events [24]. By selecting on formation time, $\tau_{\rm form}$, it is possible to choose strongly quenched jets on a jet-by-jet basis effectively. Some results illustrating this methodology are presented in Fig. 4 (right panel), depicting the Z+jet momentum asymmetry ($x_{j,Z} = p_T^{jet}/p_T^{Z}$) represented by the black line, along with the respective contributions from the *early*-developing (blue) and *late*-developing (red) jets. These correspond to jets whose $\tau_{\rm form}$ values fall within the lower or higher 50% of the $\tau_{\rm form}$ distribution. Interestingly, the same selection performed in pp collisions does not exhibit any correlation with $x_{j,Z}$ (see [24] for further details), thus showing a promising avenue to increase the precision of future jet quenching studies.

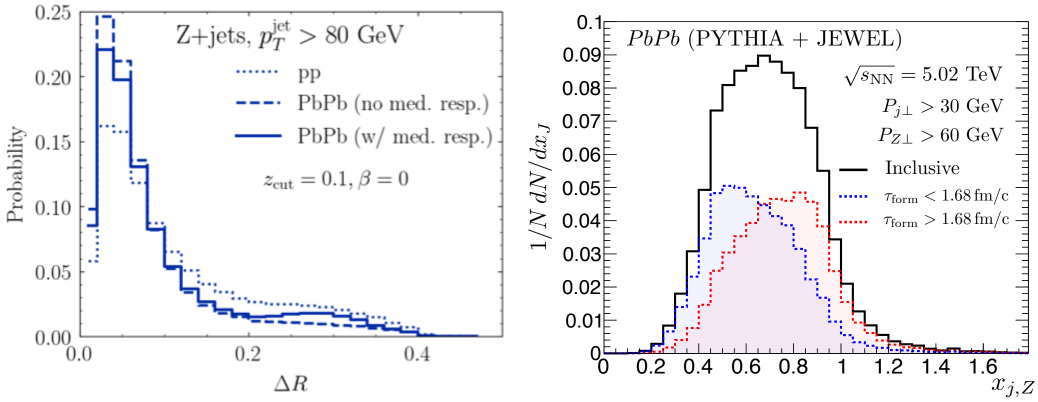

Figure 4: (Left) ΔR distributions obtained with the Hybrid model [7] for jets with $p_T > 80$ GeV in Z+jet events in vacuum (dotted blue line) and in heavy-ion collisions with (solid blue line) and without (dashed blue line) accounting for the hadrons coming from the response of the medium that are reconstructed as a part of the jets. Fig. taken from [18]. (Right) Z+jet momentum assymmetry, $x_{j,Z} = p_T^{\text{jet}}/p_T^Z$ in PbPb collisions as obtained with JEWEL [4] (black line) and the contributions coming from the 50% of jets with smaller formation time τ_{form} (in blue) and the 50% with larger τ_{form} (in red). Fig. taken from [24].

4. New opportunities and Outlook

Jet quenching MC models have significantly contributed to our understanding of high-energy nuclear collisions. However, they still encounter certain limitations. In the presence of a dense nuclear medium, partons undergo interactions and lose energy through induced radiation. Existing MC simulations struggle to fully capture the interplay between the evolution of parton showers and the medium. Another challenge lies in accurately describing low momentum scales. Jets originating from low-momentum partons are particularly sensitive to the presence of the medium. However, conventional approaches used in MC simulations fail to represent these low-energy phenomena consistently throughout experimental observables. Altogether, our understanding of the detailed modification of jet substructure remains incomplete.

Continuous improvement and integration of the latest analytical results are necessary to overcome these limitations. Incorporating findings from theoretical calculations and experimental measurements related to medium-induced effects can enhance the precision of MC models. This iterative process allows us to refine our tools and better understand the early-stage dynamics in heavy-ion collisions.

Jet quenching phenomena is still undergoing extensive theoretical development, and their implementation in MC event generators requires extrapolations and phenomenological extensions to fully describe jet evolution in heavy-ions. Even though much of our understanding of heavy-ion jets is derived from proton-proton results, medium-modified parton showers cannot currently achieve the same level of accuracy as in a proton-proton environment. However, heavy-ion collisions present a qualitatively distinct problem: a quantum system evolving within an evolving medium.

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In fact, recent works attempt to perform a tomographic analysis of the QGP [25] and provide a spacetime interpretation to jet quenching phenomena [23]. This offers a unique laboratory to probe the interplay between parton showers and the evolving medium while gaining insight into the fundamental constituents of the produced QGP.

MC event generators serve as invaluable tools for studying QGP physics. Their ability to incorporate phenomenological extensions continuously tested and refined by analytical input enables us to probe novel QCD-related phenomena. These generators provide a unique opportunity to test new observables under more realistic conditions, leading to a deeper understanding of the underlying physics. Additionally, by comparing simulated data with experimental results, they help uncover biases and improve the accuracy of our interpretations.

Acknowledgements

This work was supported by OE Portugal, Fundação para a Ciência e a Tecnologia (FCT), I.P., projects EXPL/FIS-PAR/0905/2021 and CERN/FIS-PAR/0032/2021; by European Research Council project ERC-2018-ADG-835105 YoctoLHC. L.A. was funded by FCT under contract 2021.03209.CEECIND.

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