

# Degenerate or Hierarchical Neutrinos in Supersymmetric Inflation

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**ABSTRACT:** Two moderate extensions of the minimal supersymmetric standard model are considered. The first one includes a  $U(1)_{B-L}$  gauge group, while the second is based on a left-right symmetric gauge group. In these models, hybrid inflation is ‘naturally’ realized and the  $\mu$  problem is solved via a Peccei-Quinn symmetry. Baryon number conservation is an automatic consequence of a R-symmetry. The baryon asymmetry of the universe is generated through a primordial leptogenesis. In the ‘ $B-L$ ’ case, neutrinos are assumed to acquire degenerate masses  $\approx 1.5$  eV by coupling to  $SU(2)_L$  triplet superfields, thereby providing the hot dark matter of the universe. In the ‘left-right’ model, light neutrinos acquire hierarchical masses by the seesaw mechanism. They are taken from the small angle MSW resolution of the solar neutrino puzzle and the SuperKamiokande data. Maximal  $\nu_\mu - \nu_\tau$  mixing, implied by the same data, is easily accommodated. The gravitino and baryogenesis constraints can be satisfied, in both models, with more or less ‘natural’ values of the relevant coupling constants.

Despite its compelling properties, the minimal supersymmetric standard model (MSSM) leaves a number of fundamental physical issues unanswered. This clearly indicates that it must be part of a more basic theory. Some of the shortcomings of MSSM, which are relevant for the discussion here, are in order:

- i) Inflation cannot be implemented.
- ii) There is no understanding of how the  $\mu$  term, with  $\mu \sim 10^2 - 10^3$  GeV, arises.
- iii) Neutrinos remain massless and, thus, there are no neutrino oscillations in contrast to recent experimental evidence [1].
- iv) Although the lightest supersymmetric particle (LSP) of MSSM is a promising candidate for cold dark matter, hot dark matter cannot be accommodated with purely MSSM fields. It has become increasingly clear, however, that a combination of both cold and hot dark matter is required [2] to

fit the data on large scale structure formation in the universe, especially in the case of zero cosmological constant ( $\Lambda = 0$ ).

- v) The observed baryon asymmetry of the universe (BAU) cannot be generated easily in MSSM (through the nonperturbative electroweak sphaleron processes).

All these problems can be simultaneously resolved in moderate extensions of MSSM. Two such extensions are based on the gauge groups:

- (a)  $G_S \times U(1)_{B-L} \equiv G_{B-L}$  ( $G_S$  being the standard model gauge group) [3].
- (b)  $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \equiv G_{LR}$  [4, 5].

In the  $G_{B-L}$  case, light neutrino masses can be generated by including [6, 7]  $SU(2)_L$  triplet pairs of superfields  $T_a, \bar{T}_a$  ( $a = 1, 2, \dots, n$ ). It is then not inconceivable that these masses are degenerate and we will assume them to be so. The hot dark matter of the universe can, in this case,

consist of light neutrinos without any incompatibility with atmospheric [1] and solar neutrino oscillations even with three neutrino species.

In the case of the left-right symmetric gauge group  $G_{LR}$ , right handed neutrino superfields,  $\nu^c$ , are present forming  $SU(2)_R$  doublets with the  $SU(2)_L$  singlet charged antileptons  $E^c$ . Light neutrino masses are then generated via the well-known seesaw mechanism and cannot be ‘naturally’ degenerate. We, thus, take hierarchical light neutrino masses in this case which, being unable to provide the hot dark matter, are more appropriate for a universe with nonzero cosmological constant ( $\Lambda \neq 0$ ) favored by recent observations [8]. In fact, it has been shown [9] that, for  $\Lambda \neq 0$ , cold dark matter alone can lead to a ‘good’ fit of the cosmic background radiation and both the large scale structure and age of the universe data. Moreover, the possibility of improving this fit by adding light neutrinos as hot dark matter appears [10] to be rather limited. Note that neutrino masses could be hierarchical even for  $\Lambda = 0$  provided that hot dark matter consists of some other particles (say axinos).

The spontaneous breaking of  $G_{B-L}$  to  $G_S$ , at a superheavy mass scale  $M \sim 10^{16}$  GeV, is achieved via the renormalizable superpotential

$$W = \kappa S(\phi\bar{\phi} - M^2), \quad (1)$$

where  $\phi, \bar{\phi}$  is a conjugate pair of standard model singlet left handed superfields with  $B-L$  charges equal to 1, -1 respectively, and  $S$  is a gauge singlet left handed superfield. The coupling constant  $\kappa$  and the mass parameter  $M$  can be made positive by suitable phase redefinitions. In the  $G_{LR}$  case,  $\phi\bar{\phi}$  in Eq.(1) is replaced by  $l^c\bar{l}^c$ , where  $l^c, \bar{l}^c$  is a conjugate pair of  $SU(2)_R$  doublet left handed superfields with  $B-L$  charges equal to 1, -1 respectively ( $\phi, \bar{\phi}$  correspond to the neutral components of  $l^c, \bar{l}^c$ ). The supersymmetric minima of the scalar potential lie on the D flat direction  $\phi = \bar{\phi}^*$  ( $l^c = \bar{l}^{c*}$ ) at  $\langle S \rangle = 0$ ,  $|\langle \phi \rangle| = |\langle \bar{\phi} \rangle| = M$  ( $|\langle l^c \rangle| = |\langle \bar{l}^c \rangle| = M$ ).

Hybrid inflation [11] is ‘naturally’ and automatically realized [12, 13] in such supersymmetric schemes. The scalar potential possesses a built-in inflationary trajectory at  $|S| > M$ ,  $\phi = \bar{\phi} = 0$  ( $l^c = \bar{l}^c = 0$ ) with a constant tree-level potential energy density  $\kappa^2 M^4$  which causes

the exponential expansion of the universe. Moreover, since this constant energy density breaks supersymmetry and produces mass splitting in the supermultiplets  $\phi, \bar{\phi}$  ( $l^c, \bar{l}^c$ ), there are important radiative corrections [13] which provide a slope along the inflationary trajectory necessary for driving the inflaton towards the vacua. At one-loop, the cosmic microwave quadrupole anisotropy, in the  $G_{B-L}$  model, is given [3] by

$$\left(\frac{\delta T}{T}\right)_Q \approx 8\pi \left(\frac{N_Q}{45}\right)^{1/2} \left(\frac{M}{M_P}\right)^2 x_Q^{-1} y_Q^{-1} \Lambda(x_Q^2)^{-1}, \quad (2)$$

where  $N_Q \approx 50 - 60$  denotes the number of e-foldings experienced by our present horizon size during inflation,  $M_P \approx 1.22 \times 10^{19}$  GeV is the Planck scale and

$$\Lambda(z) = (z-1)\ln(1-z^{-1}) + (z+1)\ln(1+z^{-1}). \quad (3)$$

Also,

$$y_Q^2 = \int_1^{x_Q^2} \frac{dz}{z} \Lambda(z)^{-1}, \quad y_Q \geq 0, \quad (4)$$

with  $x_Q = |S_Q|/M$  ( $x_Q \geq 1$ ),  $S_Q$  being the value of the scalar field  $S$  when our present horizon scale crossed outside the inflationary horizon. The superpotential parameter  $\kappa$ , in the  $G_{B-L}$  model, can be evaluated [3] from

$$\kappa \approx \frac{8\pi^{3/2}}{\sqrt{N_Q}} y_Q \frac{M}{M_P}. \quad (5)$$

Note that, in the  $G_{LR}$  case, the right hand sides of Eqs.(2) and (5) should be divided by  $\sqrt{2}$  [5]. This is due to the fact that the replacement of  $\phi, \bar{\phi}$  by  $l^c, \bar{l}^c$  doubles the one-loop contribution to the effective ‘inflationary’ potential.

The  $\mu$  term can be generated [14] by adding the superpotential coupling

$$\delta W = \lambda S \epsilon^{ij} H_i^{(1)} H_j^{(2)} = \lambda S H^2 \quad (\lambda > 0), \quad (6)$$

where  $H = (H^{(1)}, H^{(2)})$  is the electroweak higgs superfield belonging, in the  $G_{LR}$  case, to a bidoublet  $(2, 2)_0$  representation of  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ . After gravity-mediated supersymmetry breaking,  $S$  develops [14] a vacuum expectation value  $\langle S \rangle \approx -m_{3/2}/\kappa$ , where  $m_{3/2} \sim$

(0.1–1) TeV is the gravitino mass, and generates a  $\mu$  term with  $\mu = \lambda \langle S \rangle \approx -(\lambda/\kappa)m_{3/2}$ .

This particular solution of the  $\mu$  problem is [3, 15], however, not totally satisfactory since it requires the presence of ‘unnaturally’ small coupling constants ( $\kappa \lesssim \times 10^{-5}$ ). This is due to the fact that the inflaton system decays predominantly into electroweak higgs superfields via the renormalizable superpotential coupling in Eq.(6). The gravitino constraint [16] on the ‘reheat’ temperature then severely restricts the corresponding dimensionless coupling constant and, consequently, the parameter  $\kappa$ . Moreover, for hierarchical neutrino masses from the seesaw mechanism, the requirement of maximal  $\nu_\mu - \nu_\tau$  mixing from the SuperKamiokande experiment [1] further reduces [15]  $\kappa$  to become of order  $10^{-6}$ .

We adopt an alternative solution of the  $\mu$  problem constructed [17] by coupling the electroweak higgses to superfields causing the breaking of the Peccei-Quinn symmetry ( $U(1)_{PQ}$ ). We introduce two extra gauge singlet left handed superfields  $N$  and  $\bar{N}$  with  $PQ$  charges -1 and 1 respectively. The relevant superpotential couplings are  $\lambda N^2 \bar{N}^2 / 2m_P$  ( $m_P \equiv M_P / \sqrt{8\pi} \approx 2.44 \times 10^{18}$  GeV) and  $N^2 H^{(1)} H^{(2)}$  (or  $N^2 H^2$ ). After gravity-mediated supersymmetry breaking, the scalar potential generated by  $N^2 \bar{N}^2$  is [7]

$$\left( m_{3/2}^2 + \lambda^2 \left| \frac{N\bar{N}}{m_P} \right|^2 \right) [(|N| - |\bar{N}|)^2 + 2|N||\bar{N}|] + |A|m_{3/2}\lambda \frac{|N\bar{N}|^2}{m_P} \cos(\epsilon + 2\theta + 2\bar{\theta}), \quad (7)$$

where  $\epsilon$ ,  $\theta$ ,  $\bar{\theta}$  are the phases of  $A$ ,  $N$ ,  $\bar{N}$ . Minimization of this potential then requires  $\epsilon + 2\theta + 2\bar{\theta} = \pi$ ,  $|\langle N \rangle| = |\langle \bar{N} \rangle|$  and, for  $|A| > 4$ ,

$$|\langle N \rangle| = (m_{3/2} m_P)^{\frac{1}{2}} \left( \frac{|A| + (|A|^2 - 12)^{\frac{1}{2}}}{6\lambda} \right)^{\frac{1}{2}} \sim (m_{3/2} m_P)^{\frac{1}{2}} \sim 10^{11} \text{ GeV}. \quad (8)$$

This scale is identified with the symmetry breaking scale  $f_a$  of  $U(1)_{PQ}$ . Substitution of  $\langle N \rangle$  in the superpotential coupling  $N^2 H^{(1)} H^{(2)}$  (or  $N^2 H^2$ ) generates a  $\mu$  parameter of order  $m_{3/2}$ .

This resolution of the  $\mu$  problem avoids the direct coupling of the inflaton system  $S$ ,  $\phi$ ,  $\bar{\phi}$  (or

$S$ ,  $l^c$ ,  $\bar{l}^c$ ) to the electroweak higgses. Thus, the inflaton does not predominantly decay into higgses via renormalizable couplings as in the previous case. It decays to  $SU(2)_L$  triplets  $T_a$ ,  $\bar{T}_a$  (or right handed neutrino superfields  $\nu^c$ ) via non-renormalizable interactions, which are ‘naturally’ suppressed by  $m_P^{-1}$  (see below). The gravitino constraint can be satisfied with more ‘natural’ values of the dimensionless parameters [3, 5].

We now proceed to the detailed description of the two models [3, 5]. The superpotential  $W$  contains, in addition to the terms in Eq.(1), the following couplings in the two cases:

$$G_{B-L} : H^{(1)} Q U^c, H^{(2)} Q D^c, H^{(2)} L E^c, N^2 \bar{N}^2, N^2 H^{(1)} H^{(2)}, TLL, \bar{T}H^{(1)}H^{(1)}, \bar{\phi}^2 T\bar{T}; \quad (9)$$

$$G_{LR} : HQQ^c, HLL^c, N^2 \bar{N}^2, N^2 H^2, \bar{l}^c \bar{l}^c L^c L^c. \quad (10)$$

Here  $Q_i$  and  $L_i$  denote the  $SU(2)_L$  doublet left handed quark and lepton superfields, whereas the superfields  $Q_i^c = (U_i^c, D_i^c)$  and  $L_i^c = (\nu_i^c, E_i^c)$  are the  $SU(2)_L$  singlet ( $SU(2)_R$  doublet) antiquarks and antileptons ( $i=1,2,3$  is the family index). Of course, the right handed neutrino superfields  $\nu_i^c$  are absent in the  $G_{B-L}$  case, where two pairs of  $SU(2)_L$  triplets  $T_a$ ,  $\bar{T}_a$  ( $a=1,2$ ) with  $Y=1, -1$  and  $B-L=2, 0$  respectively are included.

The continuous global symmetries of the superpotential are  $U(1)_B$  (and, thus,  $U(1)_L$ ) with the extra chiral superfields  $S$ ,  $\phi$ ,  $\bar{\phi}$ ,  $N$ ,  $\bar{N}$ ,  $T$ ,  $\bar{T}$ ,  $l^c$ ,  $\bar{l}^c$  carrying zero baryon number, an anomalous Peccei-Quinn symmetry  $U(1)_{PQ}$ , and a non-anomalous R-symmetry  $U(1)_R$ . The  $PQ$  charges of the superfields, in the two cases, are as follows:

$$G_{B-L} : H^{(1)}(1), H^{(2)}(1), L(-1), E^c(0), Q(-1), U^c(0), D^c(0), S(0), \phi(0), \bar{\phi}(0), N(-1), \bar{N}(1), T(2), \bar{T}(-2); \quad (11)$$

$$G_{LR} : H(1), L(-1), L^c(0), Q(-1), Q^c(0), S(0), l^c(0), \bar{l}^c(0), N(-1), \bar{N}(1). \quad (12)$$

The  $R$  charges of the superfields are ( $W$  carries one unit of  $R$  charge):

$$G_{B-L} : H^{(1)}(0), H^{(2)}(0), L(1/2), E^c(1/2), Q(1/2), U^c(1/2), D^c(1/2), S(1), \phi(0), \bar{\phi}(0), N(1/2), \bar{N}(0), T(0), \bar{T}(1); \quad (13)$$

$$G_{LR} : H(0), L(1/2), L^c(1/2), Q(1/2), Q^c(1/2), S(1), l^c(0), \bar{l}^c(0), N(1/2), \bar{N}(0). \quad (14)$$

Note that  $U(1)_B$  (and, thus,  $U(1)_L$ ) is automatically implied by  $U(1)_R$  even if all possible nonrenormalizable terms are included. This is due to the fact that the  $R$  charges of the products of any three color (anti)triplets exceed unity and cannot be compensated since there are no negative  $R$  charges available.

To avoid undesirable mixing of  $L$ 's with the higgs  $H^{(2)}$  or  $\bar{l}^c$  via the allowed superpotential couplings  $N\bar{N}LH^{(1)}\phi$ ,  $N\bar{N}LHl^c$ ,  $N\bar{N}L^c\bar{l}^c$ , we impose an extra discrete  $Z_2$  symmetry ('lepton parity') under which  $L$ ,  $L^c = (\nu^c, E^c)$  change sign. This symmetry is equivalent to 'matter parity' (under which  $L$ ,  $L^c = (\nu^c, E^c)$ ,  $Q$ ,  $Q^c = (U^c, D^c)$  change sign), since 'baryon parity' (under which  $Q$ ,  $Q^c = (U^c, D^c)$  change sign) is also present being a subgroup of  $U(1)_B$ .

The only superpotential terms which are permitted by the global symmetries  $U(1)_R$ ,  $U(1)_{PQ}$  and 'lepton parity' are the ones in Eqs.(1) and (9) (or (10)) as well as  $LLl^c\bar{N}^2l^c\bar{l}^c$  and  $LLl^cHH$ , in the  $G_{LR}$  case, modulo arbitrary multiplications by nonnegative powers of the combination  $\phi\bar{\phi}$  (or  $l^c\bar{l}^c$ ). The vevs of  $\phi$ ,  $\bar{\phi}$  (or  $l^c$ ,  $\bar{l}^c$ ) and  $N$ ,  $\bar{N}$  leave unbroken only the symmetries  $G_S$ ,  $U(1)_B$  and 'matter parity'.

We will first concentrate on the model based on the  $G_{B-L}$  gauge group and discuss in some detail neutrino mass generation and baryogenesis in its context. After  $B-L$  (and lepton number) breaking at the superheavy scale  $M$ , the last term in Eq.(9) generates intermediate scale masses for the  $SU(2)_L$  triplet superfields  $T_a$ ,  $\bar{T}_a$  ( $a=1,2$ ). These masses can be taken positive and diagonal by appropriate transformations and are given by  $M_a = \gamma_a M^2/m_P$  ( $\gamma_a$  ( $a=1,2$ ) are the dimensionless coupling constants of the terms  $m_P^{-1}\bar{\phi}^2 T_a \bar{T}_a$ ). Also, after the electroweak breaking, the last two terms in Eq.(9) give rise to terms linear with respect to  $T_a$ 's in the scalar potential. The  $T_a$ 's then acquire vevs given by  $\langle T_a \rangle = \beta_a \langle H^{(1)} \rangle^2 / M_a \sim M_W^2 / M \ll M_W$ , with  $\beta_a$  being the coupling constant of the term  $\bar{T}_a H^{(1)} H^{(1)}$ . These vevs violate lepton number and, substituted to the coupling  $TLL$  in Eq.(9), generate nonzero masses for light neutrinos. The neutrino mass matrix can be diagonalized by a suitable 'Kobayashi-Maskawa' rotation in its standard form (involving three angles and a CP vi-

olating phase) and the complex eigenvalues can be written as

$$m_i = \sum_{a=1,2} \alpha_{ai} \beta_a \frac{\langle H^{(1)} \rangle^2}{M_a}, \quad (15)$$

where  $\alpha_{ai}$  are the (complex) eigenvalues of the complex symmetric coupling constant matrix of the term  $T_a L_i L_j$ . Note that the  $m_i$ 's, being in general complex, carry two extra CP violating phases (an overall phase factor is irrelevant) which appear in some processes like neutrinoless double-beta decay.

We take degenerate light neutrino masses, which are not inconceivable here as in the see-saw case. Neutrinos can then provide the hot dark matter of the universe needed for explaining [2] its large scale structure for  $\Lambda = 0$  without any conflict with atmospheric/solar neutrino oscillations even within a three neutrino scheme.

For definiteness, we can take the model of neutrino masses and mixing discussed in Ref.[18], although the precise values of mixing angles and square-mass differences are not relevant for our discussion. This scheme has almost degenerate neutrino masses and employs the bimaximal neutrino mixing [19] which is consistent with the vacuum oscillation explanation [20] of the solar neutrino puzzle. Moreover, all three neutrino masses are real, but the CP parity of one of them (say the second one) is opposite to the CP parities of the other two. This is important for satisfying the experimental constraints [21] from neutrinoless double-beta decay. The neutrino scheme of Ref.[18] can be obtained in our model provided the coupling constants  $\alpha_{ai}$  ( $a=1,2$ ;  $i=1,2,3$ ) satisfy the relations  $\alpha_{a1} = -\alpha_{a2} = \alpha_{a3} \equiv \alpha_a$  to a very good approximation and this will be the only information we will use from this scheme.

We now turn to the discussion of the decay of the inflaton, which consists of the two complex scalar fields  $S$  and  $\theta = (\delta\phi + \delta\bar{\phi})/\sqrt{2}$ , where  $\delta\phi = \phi - M$ ,  $\delta\bar{\phi} = \bar{\phi} - M$ , with mass  $m_{infl} = \sqrt{2}\kappa M$ . The scalar  $\theta$  ( $S$ ) can decay into a pair of fermionic (bosonic)  $T_a$ ,  $\bar{T}_a$ 's as one easily deduces from the last coupling in Eq.(9) and the coupling  $\kappa S\phi\bar{\phi}$  in Eq.(1). The decay width is the same for both scalars and equals

$$\Gamma = \frac{3}{8\pi} \gamma_a^2 \left( \frac{M}{m_P} \right)^2 m_{infl}. \quad (16)$$

Of course, decay of the inflaton into  $T_a, \bar{T}_a$  is possible provided that the corresponding triplet mass  $M_a \leq m_{infl}/2$ . The gravitino constraint [16] on the ‘reheat’ temperature,  $T_r$ , then implies strong bounds on the  $M_a$ ’s which satisfy this inequality. Consequently, the corresponding dimensionless coupling constants,  $\gamma_a$ , are restricted to be quite small.

To minimize the number of small couplings, we then take  $M_2 < m_{infl}/2 \leq M_1 = M^2/m_P$  ( $\gamma_1 = 1$ ) so that the inflaton decays into only one (the lightest) triplet pair with mass  $M_2$ . Using Eq.(5), the requirement  $m_{infl}/2 \leq M_1$  becomes  $y_Q \leq \sqrt{N_Q}/2\pi \approx 1.2$ , for  $N_Q = 56$ , and Eq.(4) gives  $x_Q \lesssim 1.6$ . As an example, we choose  $x_Q = 1.2$  which corresponds to  $y_Q = 0.61$  (see Eq.(4)). Eqs.(2), (5) with  $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$  from the cosmic background explorer (COBE) [22] then give  $M \approx 4.43 \times 10^{15}$  GeV and  $\kappa \approx 1.32 \times 10^{-3}$ . Also, the inflaton mass is  $m_{infl} \approx 8.27 \times 10^{12}$  GeV, the  $SU(2)_L$  triplet masses are  $M_1 \approx 8.04 \times 10^{12}$  GeV,  $M_2 \approx 8.04\gamma_2 \times 10^{12}$  GeV, and the ‘reheat’ temperature is  $T_r \approx 8.19\gamma_2 \times 10^{11}$  GeV. The gravitino constraint [16] for MSSM spectrum ( $T_r \approx (1/7)(\Gamma M_P)^{1/2} \lesssim 10^9$  GeV) then implies  $\gamma_2 \lesssim 1.11 \times 10^{-3}$ .

In this scheme, baryon number is violated only by ‘tiny’ nonperturbative  $SU(2)_L$  instanton effects. So the only way to produce the observed BAU is to first generate a primordial lepton asymmetry [23] which is then partially converted to baryon asymmetry by sphalerons. The primordial lepton asymmetry is produced via the decay of the superfields  $T_2, \bar{T}_2$  which emerge as decay products of the inflaton. This mechanism for leptogenesis has been discussed in Refs.[7, 24]. The  $SU(2)_L$  triplet superfields decay either to a pair of  $L_i$ ’s or to a pair of  $H^{(1)}$ ’s. The relevant one-loop diagrams are [24] of the self-energy type [25] with a s-channel exchange of  $T_1, \bar{T}_1$ . The resulting lepton asymmetry is [24]

$$\frac{n_L}{s} \approx -1.33 \frac{3}{8\pi} \frac{T_r}{m_{infl}}$$

$$\frac{M_1 M_2}{M_1^2 - M_2^2} \frac{\text{Im}(\beta_1^* \beta_2 \text{Tr}(\hat{\alpha}_1^\dagger \hat{\alpha}_2))}{\text{Tr}(\hat{\alpha}_2^\dagger \hat{\alpha}_2) + \beta_2^* \beta_2}, \quad (17)$$

where  $\hat{\alpha}_a = \text{diag}(\alpha_a, -\alpha_a, \alpha_a)$ . Note that this formula holds provided [26] the decay width of

$T_1, \bar{T}_1$  is much smaller than  $(M_1^2 - M_2^2)/M_2$ , which is well satisfied here since  $M_2 \ll M_1$ . For MSSM spectrum, the observed BAU is given [27] by  $n_B/s = -(28/79)(n_L/s)$ . It is important to ensure that the primordial lepton asymmetry is not erased by lepton number violating  $2 \rightarrow 2$  scattering processes at all temperatures between  $T_r$  and 100 GeV. This gives [27]  $m_{\nu_\tau} \lesssim 10$  eV which is readily satisfied.

The parameters  $\alpha_a, \beta_a, \gamma_a$  ( $a=1,2$ ) are constrained by the requirement that the hot dark matter of the universe consists of light neutrinos. We take the ‘relative’ density of hot dark matter  $\Omega_{HDM} \approx 0.2$ , which is favored by the structure formation in cold plus hot dark matter models [2] with  $\Lambda = 0$ , and  $h \approx 0.5$ , where  $h$  is the present value of the Hubble parameter in units of  $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ . The common mass of the three light neutrinos is then about 1.5 eV and Eq.(15) gives the constraint

$$\left| \sum_{a=1,2} \frac{\alpha_a \beta_a}{\gamma_a} \right| \approx \left( \frac{M}{7.02 \times 10^{15} \text{ GeV}} \right)^2 \equiv \xi, \quad (18)$$

where  $|\langle H^{(1)} \rangle|$  was taken  $\approx 174$  GeV. Lepton asymmetry is maximized, under this constraint, for  $|\alpha_1 \beta_1 / \gamma_1| = |\alpha_2 \beta_2 / \gamma_2| \equiv \delta$  and  $\sqrt{3}|\alpha_2| = |\beta_2|$ . Substituting  $T_r \approx (1/7)(\Gamma M_P)^{1/2}$  with  $\Gamma$  from Eq.(16), Eq.(17) gives

$$\left| \frac{n_L}{s} \right| \lesssim \frac{0.107}{\pi} \frac{M}{\sqrt{m_{infl} M_P}} \gamma_1 \gamma_2^2 \xi \left( 1 - \frac{\xi^2}{4\delta^2} \right)^{1/2}. \quad (19)$$

which is further maximized at  $\alpha_1 = \beta_1 = 1$ . This gives  $\delta = 1$  (for  $\gamma_1 = 1$ ). For  $x_Q = 1.2$ ,  $\xi \approx 0.4$  and the maximal lepton asymmetry becomes  $\approx 5.86 \gamma_2^2 \times 10^{-3}$ . The low deuterium abundance constraint [28] on the BAU,  $\Omega_B h^2 \approx 0.019$ , can then be satisfied provided  $\gamma_2 \gtrsim 1.88 \times 10^{-4}$ . So, for  $\gamma_2$  in the range  $1.9 \times 10^{-4} - 1.1 \times 10^{-3}$ , both the gravitino and baryogenesis restrictions can be met. We see that, in the  $G_{B-L}$  model, the required values of the relevant coupling constants  $\kappa$  and  $\gamma_2$  are more or less ‘natural’ ( $\sim 10^{-3}$ ).

We now turn to the discussion of the second model based on the left-right symmetric gauge group  $G_{LR}$ . After  $B-L$  breaking by  $\langle l^c \rangle, \langle \bar{l}^c \rangle$ , the last term in Eq.(10) generates intermediate scale masses for the right handed neutrino superfields  $\nu_i^c$  ( $i=1,2,3$ ). The dimensionless coupling

constant matrix of this term can be made diagonal with positive entries  $\gamma_i$  ( $i=1,2,3$ ) by a rotation on  $\nu_i^c$  's. The right handed neutrino mass eigenvalues are then  $M_i = 2\gamma_i M^2/m_P$  (with  $\langle l^c \rangle$ ,  $\langle \bar{l}^c \rangle$  taken positive by a  $B-L$  transformation).

The light neutrino masses are generated via the seesaw mechanism and, therefore, cannot be 'naturally' degenerate. We will, thus, assume hierarchical light neutrino masses. Analysis [29] of the CHOOZ experiment [30] shows that the oscillations of solar and atmospheric neutrinos decouple. This fact allows us to concentrate on the two heaviest families ignoring the first one. We will denote the two positive eigenvalues of the light neutrino mass matrix by  $m_2 (=m_{\nu_\mu})$ ,  $m_3 (=m_{\nu_\tau})$ . We take  $m_{\nu_\mu} \approx 2.6 \times 10^{-3}$  eV which is the central value of the  $\mu$ -neutrino mass coming from the small angle MSW resolution of the solar neutrino problem [31]. The  $\tau$ -neutrino mass is taken to be  $m_{\nu_\tau} \approx 7 \times 10^{-2}$  eV which is the central value implied by SuperKamiokande [1].

The determinant and the trace invariance of the light neutrino mass matrix imply [32] two constraints on the (asymptotic) parameters:

$$m_2 m_3 = \frac{(m_2^D m_3^D)^2}{M_2 M_3}, \quad (20)$$

$$m_2^2 + m_3^2 =$$

$$\frac{(m_2^D)^2 c^2 + (m_3^D)^2 s^2}{M_2^2} + \frac{(m_3^D)^2 c^2 + (m_2^D)^2 s^2}{M_3^2} + \frac{2(m_3^D)^2 - (m_2^D)^2}{M_2 M_3} c^2 s^2 \cos 2\delta. \quad (21)$$

Here,  $m_{2,3}^D$  ( $m_2^D \leq m_3^D$ ) are the 'Dirac' neutrino masses considered diagonal, and  $c = \cos \theta$ ,  $s = \sin \theta$  with  $\theta$  and  $\delta$  being the rotation angle and phase which diagonalize the Majorana mass matrix of the right handed neutrinos.

The  $\nu_\mu - \nu_\tau$  mixing angle  $\theta_{\mu\tau}$  lies in the range

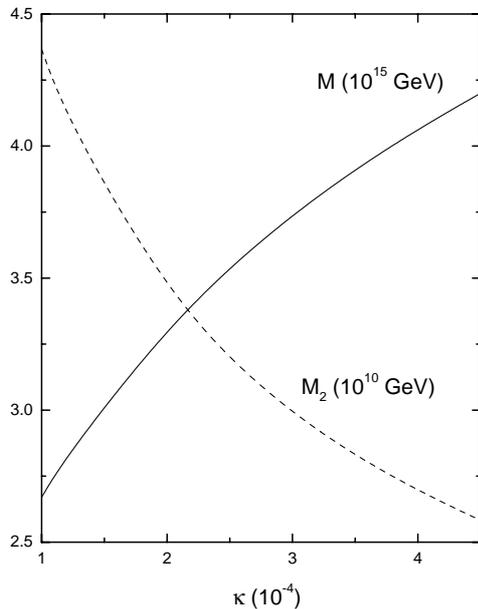
$$|\varphi - \theta^D| \leq \theta_{\mu\tau} \leq \varphi + \theta^D, \text{ for } \varphi + \theta^D \leq \pi/2, \quad (22)$$

where  $\varphi$  is the rotation angle diagonalizing the light neutrino mass matrix and  $\theta^D$  the 'Dirac' (unphysical) mixing angle defined in the absence of right handed neutrino Majorana masses [32].

We will now discuss the 'reheating' process in the  $G_{LR}$  model. The inflaton again consists of the two complex scalar fields  $S$  and  $\theta$  ( $\phi, \bar{\phi}$  are now the neutral components of  $l^c, \bar{l}^c$ ). In this case, however, the scalar  $S$  ( $\theta$ ) decays into a pair of bosonic (fermionic)  $\nu_i^c$  's via the last coupling in Eq.(10) and  $\kappa S l^c \bar{l}^c$  with a decay width

$$\Gamma = \frac{1}{8\pi} \left( \frac{M_i}{M} \right)^2 m_{infl}, \quad (23)$$

provided that  $M_i < m_{infl}/2$ . The gravitino constraint implies strong bounds on these  $M_i$  's and, consequently, on the corresponding  $\gamma_i$  's.



**Figure 1:** The mass scale  $M$  (solid line) and the Majorana mass of the second heaviest right handed neutrino  $M_2$  (dashed line) as functions of  $\kappa$ .

We minimize the number of small couplings by taking  $M_2 < m_{infl}/2 \leq M_3 = 2M^2/m_P$  ( $\gamma_3 = 1$ ) so that the inflaton decays into only one (the second heaviest) right handed neutrino with mass  $M_2$ . The second inequality implies  $y_Q \leq \sqrt{2N_Q}/\pi \approx 3.34$  (for  $N_Q = 55$ ) and, thus,  $x_Q \lesssim 3.5$ . The parameters  $M$  and  $\kappa$  are calculated for each value of  $x_Q$  in this range. Eliminating  $x_Q$ , we obtain  $M$  as a function of  $\kappa$  depicted in Fig.1. The inflaton mass  $m_{infl}$  and the heaviest right handed neutrino mass  $M_3$  are readily evaluated. The mass of the second heaviest right

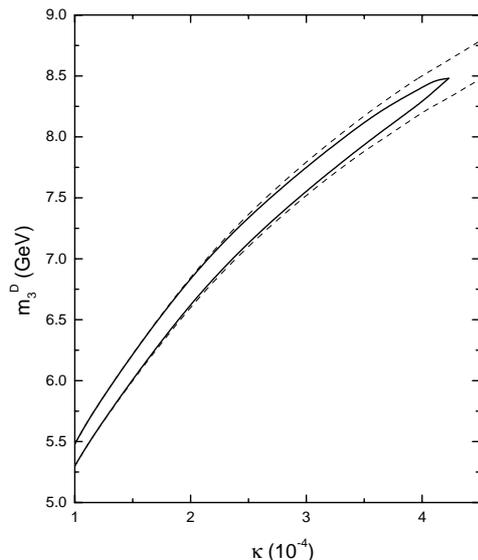
handed neutrino  $M_2$  is restricted by the gravitino constraint. We take it to be equal to its maximal allowed value in order to maximize  $\gamma_2$ . The value of  $M_2$  is also depicted in Fig.1.

Baryogenesis proceeds through a primordial leptogenesis [23] in this model too. The lepton asymmetry, however, is now produced through the decay of the superfield  $\nu_2^c$  which emerges as decay product of the inflaton. This mechanism for leptogenesis has been discussed in Ref.[23]. The  $\nu_2^c$  superfield decays into electroweak higgs and (anti)lepton superfields. The relevant one-loop diagrams are both of the vertex and self-energy type [25] with an exchange of  $\nu_3^c$ . The resulting lepton asymmetry is [32]

$$\frac{n_L}{s} \approx 1.33 \frac{9T_r}{16\pi m_{infl}} \frac{M_2}{M_3} \frac{c^2 s^2 \sin 2\delta (m_3^{D^2} - m_2^{D^2})^2}{|(H^{(1)})|^2 (m_3^{D^2} s^2 + m_2^{D^2} c^2)}. \quad (24)$$

Note that this formula holds [26] provided that  $M_2 \ll M_3$  and the decay width of  $\nu_3^c$  is much smaller than  $(M_3^2 - M_2^2)/M_2$ , and both conditions are well satisfied here. The ‘dangerous’ lepton number violating processes are well out of equilibrium in this case too.

For definiteness, we assume that the  $\nu_\mu - \nu_\tau$  mixing is about maximal ( $\theta_{\mu\tau} \approx \pi/4$ ) in accordance with the recent SuperKamiokande data [1]. We will also make the plausible assumption that the ‘Dirac’ mixing angle  $\theta^D$  is negligible ( $\theta^D \approx 0$ ). Under these circumstances, the rotation angle  $\varphi \approx \pi/4$ . Using the ‘determinant’ and ‘trace’ constraints in Eqs.(20) and (21) and diagonalizing the light neutrino mass matrix, we can determine the range of  $m_3^D$  which allows maximal  $\nu_\mu - \nu_\tau$  mixing for each value of  $\kappa$ . These ranges are depicted in Fig.2 for all relevant values of  $\kappa$  and constitute the area in the  $\kappa - m_3^D$  plane consistent with maximal mixing. For each allowed pair  $\kappa, m_3^D$ , the value of the phase  $\delta$  leading to maximal mixing can be determined from the ‘trace’ condition. The corresponding lepton asymmetry is then found from Eq.(24). The line consistent with the low deuterium abundance constraint [28] on the BAU ( $\Omega_B h^2 \approx 0.019$ ) is also depicted in Fig.2. We see that the required values of  $\kappa$  ( $\lesssim 4.2 \times 10^{-4}$ ),



**Figure 2:** The area (bounded by the dashed lines) on the  $\kappa - m_3^D$  plane consistent with maximal  $\nu_\mu - \nu_\tau$  mixing and the gravitino constraint. Along the thick solid line the low deuterium abundance constraint on the BAU is also satisfied.

although somewhat small, are much more ‘natural’ than the ones encountered in previous models [15] that solved the  $\mu$  problem and achieved maximal  $\nu_\mu - \nu_\tau$  mixing. For these values of  $\kappa$ ,  $\gamma_2 \gtrsim 1.9 \times 10^{-3}$  which is quite satisfactory.

In conclusion, we have presented two moderate extensions of MSSM based on the gauge groups  $G_{B-L}$  and  $G_{LR}$ . In the  $G_{B-L}$  case, neutrinos acquire degenerate masses, thereby providing the hot dark matter in the universe needed for explaining its large scale structure especially for zero cosmological constant. In the case of the left-right symmetric gauge group  $G_{LR}$ , neutrino masses are generated via the seesaw mechanism and are taken hierarchical. The recent SuperKamiokande restrictions on  $m_{\nu_\tau}$  and  $\nu_\mu - \nu_\tau$  mixing can be accommodated, in this model, with  $m_{\nu_\mu}$  from the small angle MSW resolution of the solar neutrino puzzle. Hybrid inflation is ‘naturally’ realized and the  $\mu$  problem is easily resolved by a Peccei-Quinn symmetry in both models. Also, the BAU is generated through a primordial leptogenesis and the gravitino and baryogenesis constraints are easily met with more or less ‘natural’ values of the parameters.

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