

# Theoretical developments on Quartic Gauge Boson Couplings at LEP\*

---

**Fulvio Piccinini**<sup>† ‡</sup>

*Theory Division, CERN, CH-1211, Geneva 23, Switzerland*

*E-mail: Fulvio.Piccinini@cern.ch*

**Marco Osmo**

*Dipartimento di Fisica Nucleare e Teorica - Università di Pavia, and*

*INFN - Sezione di Pavia, Via A. Bassi 6, Pavia, Italy*

*E-mail: Marco.Osmo@pv.infn.it*

**ABSTRACT:** The search for quartic anomalous gauge couplings (QAGC) at LEP requires appropriate predictions for the radiative processes  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ ,  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  and  $e^+e^- \rightarrow 4 \text{ fermions} + \gamma$ . The current knowledge on dimension-six operators giving rise to QAGC is briefly reviewed, together with their implementation in event generators. The accuracy of calculations based on real approximations (used up to now for the LEP experimental analysis) is examined by comparing them with the available exact matrix element calculations.

---

## 1. Introduction

Despite the striking success of the Standard Model (SM) in accommodating the precision data collected at high-energy colliders, important tests of the theory, such as the non-abelian nature of the gauge symmetry and the mechanism of electroweak symmetry breaking, are still at a beginning stage. To this end, gauge-boson self interactions play a key role. At present, triple gauge couplings are being probed at LEP [1] and the Tevatron [2], while direct measurements of quartic couplings only very recently became available through the study of radiative events at LEP [3]–[6]. Actually, events with one or two isolated, hard photons are analysed at LEP, to search for anomalies in the sector of quartic gauge-boson couplings. Only vertices involving at least one photon can be constrained, since quadrilinear interactions containing four massive gauge bosons give rise to a final

---

\*Based on work done in collaboration with G. Montagna, M. Moretti and O. Nicrosini.

<sup>†</sup>Speaker.

<sup>‡</sup>On leave of absence from INFN - Sezione di Pavia.

state of three massive gauge bosons and are therefore beyond the potential of LEP, because of the lack of phase space. The processes considered in the experimental analyses are  $e^+e^- \rightarrow W^+W^-\gamma$ ,  $e^+e^- \rightarrow Z\gamma\gamma$  and  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$  [3]–[6]. The  $W^+W^-\gamma$  signature, which yields a four-fermion plus gamma final state, is interesting in order to test  $WW\gamma\gamma$  and  $WWZ\gamma$  vertices. For the  $Z\gamma\gamma$  events, the final states due to the hadronic decays of the  $Z$  boson with two jets and two visible photons are selected to probe the purely anomalous vertex  $ZZ\gamma\gamma$ , which is of particular interest, being absent from the SM at tree level. The final state with two neutrinos and two acoplanar photons allows a study of the quartic  $WW\gamma\gamma$  and  $ZZ\gamma\gamma$  interactions.

In the light of these experimental analyses, the aim of the present contribution is to review the state of the art of exact SM calculations for the processes  $e^+e^- \rightarrow 4$  fermions ( $4f$ ) +  $\gamma$ ,  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  and  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ , including the effects of quartic anomalous gauge couplings (QAGC).

## 2. Theoretical approach

The theoretical framework of interest is the formalism of electroweak chiral Lagrangians. In such a scenario, QAGC involving four massive gauge bosons emerge as operators of dimension four at next-to-leading order, while QAGC with at least one photon originate from six (or higher) -dimensional operators at next-to-next-to-leading order. They are said genuinely anomalous if they do not induce new trilinear gauge interactions.

Anomalous  $WW\gamma\gamma$  and  $ZZ\gamma\gamma$  vertices were originally introduced in ref. [7]. In this paper, the authors show that, by assuming  $C$  and  $P$  conservation and further imposing  $U(1)_{em}$  gauge invariance and  $SU(2)_c$  custodial symmetry, two independent Lorentz structures contribute to  $WW\gamma\gamma$  and  $ZZ\gamma\gamma$  interactions according to the following Lagrangians

$$\begin{aligned}\mathcal{L}_0 &= -\frac{e^2}{16} \frac{a_0}{\Lambda^2} F_{\mu\nu} F^{\mu\nu} \vec{W}^\alpha \cdot \vec{W}_\alpha \\ \mathcal{L}_c &= -\frac{e^2}{16} \frac{a_c}{\Lambda^2} F_{\mu\alpha} F^{\mu\beta} \vec{W}^\alpha \cdot \vec{W}_\beta,\end{aligned}\tag{2.1}$$

where  $F_{\mu\nu}$  is the electromagnetic field tensor, and  $\vec{W}$  is a  $SU(2)$  triplet describing the  $W$  and  $Z$  physical fields,  $\cos\theta_w$  being the cosine of the weak mixing angle. In eq. (2.1)  $a_0$  and  $a_c$  are (dimensionless) anomalous couplings, divided by an energy scale  $\Lambda$ , which has the meaning of scale of new physics. Generally speaking,  $\Lambda$  is in principle unknown and model-dependent. However, the ratios  $a_i/\Lambda^2$  entering the phenomenological Lagrangians can be meaningfully extracted from the data in a model-independent way.

The anomalous  $WWZ\gamma$  vertex has been analysed in ref. [8], showing that, under the assumption of  $C$ ,  $P$  and  $U(1)_{em}$  invariance, five additional Lorentz structures with respect to eq. (2.1) contribute. It is further demonstrated that, by embedding all the structures related to  $WW\gamma\gamma$ ,  $ZZ\gamma\gamma$  and  $WWZ\gamma$  vertices in  $SU(2) \times U(1)$  gauge-invariant and  $SU(2)_c$  symmetric combinations, fourteen  $C$ - and  $P$ - conserving operators are allowed, with  $k_j^i$  parameters, which parametrize the strength of anomalous couplings.

By allowing the violation of at least one discrete symmetry, but retaining  $U(1)_{em}$  invariance and global custodial  $SU(2)_c$  symmetry, additional terms can be introduced in

the Lagrangian. In particular, in the literature three different contributions have been considered:  $\mathcal{L}_n$  [9]–[11], which violates  $C$  and  $CP$ ,  $\tilde{\mathcal{L}}_0$  [12, 13], which violates  $P$  and  $CP$ , and  $\tilde{\mathcal{L}}_n$  [12, 13], which violates both  $C$  and  $P$ , thus conserving  $CP$ .

In the approach of ref. [14], all the different operatorial structures contributing to the vertices analysed at LEP have been implemented directly at the Lagrangian level in the Monte Carlo codes `NUNUGPV` [15] and `WRAP` [16] with arbitrary  $a_i$  coefficients. By means of appropriate relations between the  $a_i$  parameters, both parametrizations available in the literature for QAGC, namely the parametrization in terms of  $a_0, a_c, a_n$  couplings and the one in terms of  $k^i$  coefficients, can be obtained, as shown explicitly in ref. [14].

By means of the above event generators, theoretical predictions for the processes  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ ,  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  and  $e^+e^- \rightarrow 4f + \gamma$  can be obtained. As an option of `WRAP`, predictions for the inclusive final states  $WW\gamma$  and  $Z\gamma\gamma$  can also be obtained, especially in order to compare them with results existing in the literature treating the  $W$  and  $Z$  bosons in the on-shell approximation, as those of refs. [8, 11]. Recently the code `RacoonWW` has been upgraded to include QAGC for the set of  $4f + \gamma$  processes [12, 13].

### 3. Discussion

A detailed numerical investigation of the potentialities of the above-mentioned processes in the search for QAGC at LEP can be found in refs. [12, 14]. Here only the main results are summarized.

The options of  $WW\gamma$  and  $Z\gamma\gamma$  real production allowed a comparison of the predictions given by the Monte Carlo `WRAP` with the results already present in the literature. In particular, such a comparison showed a very satisfactory agreement with the numerical results of ref. [8], while discrepancies were found with the results published in ref. [11], for the dependence of the  $WW\gamma$  and  $Z\gamma\gamma$  cross sections on  $a_0$  and  $a_c$  parameters. More precisely, an opposite sign is present for the relative effects of  $a_0$  and  $a_c$  on the  $WW\gamma$  cross section, as noted in ref. [12], whereas this is not the case for the  $Z\gamma\gamma$  final state.

An important issue, which can be addressed with complete matrix element calculations, is the reliability of the calculations performed in the narrow-width approximation, used so far in the experimental search for QAGC.

As far as the neutral QAGC sector is concerned, the cross sections obtained with complete calculations for the final states  $q\bar{q}\gamma\gamma$  and  $\nu\bar{\nu}\gamma\gamma$  have been compared in ref. [14] with the  $Z\gamma\gamma$  approximation as functions of the parameters  $a_0$  and  $a_c$  at the c.m. energy of 200 GeV. In the case of the  $q\bar{q}\gamma\gamma$  channel, a cut on the invariant mass of the jet–jet system around the  $Z$  mass ( $80 \text{ GeV} \leq M_{q\bar{q}} \leq 100 \text{ GeV}$ ) has been imposed, while for the  $\nu\bar{\nu}\gamma\gamma$  final state a cut on the recoil mass again around the  $Z$  mass ( $80 \text{ GeV} \leq M_{recoil} \leq 120 \text{ GeV}$ ) has been adopted, in order to perform a consistent comparison with the  $Z\gamma\gamma$  approximation. It is worth noting that these selection criteria are also adopted by the real experimental analysis. This comparison allows the effects due to the  $\gamma$ – $Z$  interference in the  $q\bar{q}\gamma\gamma$  channel and to  $W$ – $Z$  interference in the  $\nu\bar{\nu}\gamma\gamma$  one to be quantified, as well as the effects of the off-shellness of the  $Z$  boson, in the extraction of limits on the QAGC. The numerical investigation of ref. [14] shows an agreement between the integrated cross

sections at the per cent level as a function of  $a_0$  and  $a_c$  variations inside the currently allowed experimental constraints, thus illustrating the reliability of the  $Z\gamma\gamma$  approximation in view of the expected experimental precision. This conclusion also holds true for the differential distributions mostly sensitive to QAGC and considered in the experimental studies.

A similar analysis has been performed in ref. [14] for a  $4f + \gamma$  final state ( $e^+e^- \rightarrow \mu\bar{\nu}_\mu u\bar{d}\gamma$ ), as computed by means of the exact calculation of WRAP, in comparison with the  $WW\gamma$  approximation considered in the literature [8, 11]. The sensitivity to the anomalous couplings  $a_0, a_c, a_n$  has been studied for the  $4f + \gamma$  final state according to two different event selections: no cuts on the invariant masses of the decay products, and cuts on the invariant masses of the decay products around the  $W$  mass, i.e.  $75 \text{ GeV} \leq M_{u\bar{d}, \mu\bar{\nu}_\mu} \leq 85 \text{ GeV}$ , in order to disentangle, as much as possible, the contributing Feynman graphs with two final-state resonant  $W$  bosons. The  $WW\gamma$  approximation predicts a quite different sensitivity with respect to the complete  $4f + \gamma$  calculation.

By considering variations of the anomalous couplings within the allowed experimental bounds, differences at the ten per cent level are registered between the  $WW\gamma$  approximation and the  $4f + \gamma$  prediction, when invariant mass cuts are imposed in the  $4f + \gamma$  calculation. Notice that, if invariant mass cuts are not considered, the  $4f + \gamma$  cross section grows up by a factor of 2 with respect to the cross section in the presence of cuts. Therefore, the  $WW\gamma$  approximation should be employed with due caution in QAGC studies, especially if we take into account that exact  $4f + \gamma$  generators, such as WRAP [16] and RacoonWW [12], incorporate the effects of QAGC and are at our disposal for such experimental studies.

As a last remark, it is worth mentioning that, in realistic experimental analyses, the effects of ISR (properly simulated by means of the available event generators) should be considered, since they tend to diminish the sensitivity on the QAGC at the level of some per cent.

#### 4. Conclusions

The search for QAGC in radiative events at LEP demands precise predictions for the processes  $e^+e^- \rightarrow \nu\bar{\nu}\gamma\gamma$ ,  $e^+e^- \rightarrow q\bar{q}\gamma\gamma$  and  $e^+e^- \rightarrow 4 \text{ fermions} + \gamma$ . To this end, the available exact calculations for such processes, including the contribution of QAGC (and related event generators), have been reviewed.

After a brief description of the possible Lagrangians giving rise to QAGC in vertices involving at least one photon, and of their implementation in existing event generators, comparisons between exact calculations and approximate results existing in the literature have been discussed. It turns out that, for the  $\nu\bar{\nu}\gamma\gamma$  and  $q\bar{q}\gamma\gamma$  final states, the  $Z\gamma\gamma$  approximation works well, the interference effects present in the complete calculations being confined at the per cent level, if appropriate cuts around the  $Z$ -boson mass are required. The  $\nu\bar{\nu}\gamma\gamma$  final state with appropriate cuts on the recoil mass can be successfully exploited to extract limits on neutral QAGC, as a complementary channel to the  $q\bar{q}\gamma\gamma$  one. As far as the  $4 \text{ fermions} + \gamma$  final states are concerned, significant differences are seen between the exact calculation and the  $WW\gamma$  approximation, even in the presence of invariant mass cuts around the  $W$ -boson mass.

The authors are grateful to G. Montagna for his careful reading of the manuscript. F. P. thanks the Conveners of the Parallel Session “Test of the Standard Model” for their kind invitation.

## References

- [1] A. Heister et al., *Measurement of triple gauge boson couplings at LEP energies up to 189 GeV*, hep-ph/0104034, R. Barate et al., *Phys. Lett. B* **462** (1999) 389; P. Abreu et al., *Phys. Lett. B* **502** (2001) 9 and **459** (1999) 382; M. Acciarri et al., *Phys. Lett. B* **489** (2000) 55, **487** (2000) 229, and **467** (1999) 171; G. Abbiendi et al., *Eur. Phys. J. C* **19** (2001) 1 and **17** (2000) 553.
- [2] B. Abbott et al., *Phys. Rev. D* **62** (2000) 052005 and **60** (1999) 072002; K. Gounder, hep-ex/9903038.
- [3] M. Acciarri et al., *Phys. Lett. B* **490** (2000) 187; G. Abbiendi et al., *Phys. Lett. B* **471** (1999) 293.
- [4] M. Acciarri et al., *Phys. Lett. B* **505** (2001) 47 and **478**(2000) 39.
- [5] M. Musy, *Quartic Gauge Boson Couplings Results at LEP*, talk given at the XXXVI Rencontres de Moriond, March 2001.
- [6] M. Biglietti, these proceedings.
- [7] G. Belanger and F. Boudjema, *Phys. Lett. B* **288** (1992) 201, and 210.
- [8] G. Belanger et al., *Eur. Phys. J. C* **13** (2000) 283.
- [9] O.J.P. Eboli, M.C. Gonzalez-Garcia and S.F. Novaes, *Nucl. Phys. B* **411** (1994) 381.
- [10] G. Abu Leil and J.W. Stirling, *J. Phys. G* **21** (1995) 517.
- [11] J.W. Stirling and A. Werthenbach, *Eur. Phys. J. C* **14** (2000) 103.
- [12] A. Denner, S. Dittmaier, M. Roth and D. Wackerth, *Eur. Phys. J. C* **20** (2001) 201.
- [13] M. Roth, these proceedings.
- [14] G. Montagna, M. Moretti, O. Nicosini, M. Osmo and F. Piccinini, *Phys. Lett. B* **515** (2001) 197.
- [15] G. Montagna, O. Nicosini and F. Piccinini, *Comput. Phys. Commun.* **98** (1996) 206; G. Montagna, M. Moretti, O. Nicosini and F. Piccinini, *Nucl. Phys. B* **541** (1999) 31.
- [16] G. Montagna, M. Moretti, M. Osmo, O. Nicosini and F. Piccinini, *Eur. Phys. J. C* **21** (2001) 291.