

## Experimental review on $V_{us}$ extraction from kaon decays

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A comprehensive review on the extraction of CKM matrix element  $V_{us}$  from  $K_{l3}$  decay rates is given. Recent experimental results on  $K_L$ ,  $K_S$  and  $K^\pm$  decays are considered, including the preliminary ones presented at this conference. From a global fit of all of the present kaon data a value of  $|V_{us}|f_+(0) = 0.21663 \pm 0.00047$  is obtained, where  $f_+(0)$  represents the hadronic matrix element at zero momentum transfer. Using the estimate  $f_+(0) = 0.961(8)$ , the result for  $V_{us}$  is compatible at one  $\sigma$  level with what expected from CKM unitarity.

*KAON International Conference*

*May 21-25 2007*

*Laboratori Nazionali di Frascati dell'INFN, Rome, Italy*

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## 1. Introduction

At present, the first-row constraint  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$  (with  $|V_{ub}|^2$  negligible) offers the most precise test of CKM unitarity. Up until 2002 (and for the 2004 PDG edition [1]), the evaluation of  $V_{us}$  from older  $K \rightarrow \pi l \nu$  ( $K_{l3}$ ) data gave  $2.3\sigma$  hint of unitarity violation in the first-row test. The 2003 measurement of  $\text{BR}(K_{e3}^+)$  by BNL E865 [2] gave a value for  $|V_{us}|$  consistent with unitarity. In the period 2004-2006, many new measurements of branching ratios (BR), lifetimes ( $\tau$ ), and form-factor slopes ( $\lambda$ ) were announced by KLOE, KTeV, ISTRA+ and NA48. Compared to old determinations, all of these new measurements are based on much higher statistics; moreover, the radiative corrections are applied consistently. The 2006 PDG review on  $|V_{us}|$  includes many, but not all, of these important developments [3]. I present an up-to-date evaluation that includes preliminary results presented at this conference. The combination of experimental results has been carried out by M. Moulson for the FlaviaNet Working Group on Kaon decays, and has been first presented at 2006 CKM Workshop in Nagoya [4].

$V_{us}$  is related to the kaon semileptonic decay rate through the following equation:

$$\Gamma(K_{l3}) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+(0)|^2 I_{Kl}(\lambda) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{Kl}^{EM}), \quad (1.1)$$

where  $K = K^0, K^\pm$ ,  $l = e, \mu$  and  $C_K^2$  is a Clebsch-Gordan coefficient, equal to  $1/2$  and  $1$  for  $K^\pm$  and  $K^0$ , respectively. In the above expression, the decay width  $\Gamma(K_{l3})$  is experimentally determined by measuring the kaon lifetime and the semileptonic BRs. A well determined treatment of the radiative decays is required in order to use the measured BR in the expression 1.1; usually, most recent experiments publish values of BR totally inclusive of radiation. The hadronic matrix element (form factor) for the  $K \rightarrow \pi$  transition is parameterized in terms of its value at zero momentum transfer for neutral kaon decays,  $f_+(0) \equiv f_+^{K^0\pi^-}(0)$ , which is determined from theory. Form factor dependence on the momentum is described by one or more slope parameters  $\lambda$ , which are measured from the decay spectra, and is integrated over the decay phase space, giving rise to the  $I_{Kl}(\lambda)$  integral in equation 1.1. On top of that, some higher order corrections have to be computed from theory:  $S_{EW} = 1.0232$  is the universal short-distance electroweak correction;  $\Delta_K^{SU(2)}$  and  $\Delta_{Kl}^{EM}$  are  $SU(2)$ -breaking and long-distance electromagnetic corrections, which depend on the kaon charge and on the lepton flavor.

In the following sections 2 to 4 most recent measurements for  $K_L$ ,  $K_S$  and  $K^\pm$  BRs and lifetimes will be firstly reviewed and the updated averages will be presented; in section 5 the measurement of the form factor slopes and the evaluation of the phase space integrals will be summarized; in sections 6 and 7 the extraction on  $V_{us}$  will be finally addressed.

## 2. $K_L$ branching ratios and lifetime

Many new measurements of  $K_L$  BRs were performed during the last three years, which contributed to clarify the  $K_L$  experimental picture with respect to the 2004 PDG compilation [5]. The KTeV experiment has measured accurately five ratios of  $K_L$  main decay widths from independent samples of  $10^5 - 10^6$  events collected with a single trigger. They obtain [7]:  $\Gamma(K_{\mu 3})/\Gamma(K_{e 3}) = 0.6640(26)$ ,  $\Gamma(\pi^+\pi^-\pi^0)/\Gamma(K_{e 3}) = 0.3078(18)$ ,  $\Gamma(\pi^+\pi^-)/\Gamma(K_{e 3}) = 0.004856(28)$ ,  $\Gamma(2\pi^0)/\Gamma(3\pi^0) =$

0.004446(25), and  $\Gamma(3\pi^0)/\Gamma(K_{e3}) = 0.4782(55)$ . The six decay modes involved cover 99.93% of  $K_L$  width, so KTeV combines the ratios to extract the BRs. The measured ratios are used instead in the present evaluation to perform a global fit to  $K_L$  BRs and lifetime, with correlations provided by the experiment. The NA48 experiment has measured the ratio of  $K_{e3}$  decays normalized to final states with two charged tracks, obtained from a sample of  $80 \times 10^6$  events. They find [8]  $\Gamma(K_{e3})/\Gamma(2 \text{ track}) = 0.4978(35)$ , and using  $\text{BR}(2 \text{ track}) \sim 1.0048 - \text{BR}(3\pi^0)$ , they evaluate  $\text{BR}(K_{e3})$ . The measured ratio is used in the present fit to  $K_L$  decay modes. Moreover, the NA48 experiment has produced also a preliminary result for the ratio  $\Gamma(K_L \rightarrow 3\pi^0)/\Gamma(K_S \rightarrow 2\pi^0)$  [9]. Using  $\Gamma(K_S \rightarrow 2\pi^0)$  as external input, they obtain  $\text{BR}(K_L \rightarrow 3\pi^0)/\tau_L = 3.795(58)$  MHz, which is used in the global fit as well. The KLOE experiment has measured the absolute BRs for the four main  $K_L$  decay channels from a sample of  $13 \times 10^6$   $\phi \rightarrow K_S K_L$  events with a  $K_S \rightarrow \pi^+ \pi^-$  decay reconstructed in the apparatus. The results depend on the  $K_L$  lifetime through the geometrical acceptance of the apparatus as  $d\text{BR}/\text{BR} = 0.67d\tau_L/\tau_L$ . Using as reference value  $\tau_L^{(0)} = 51.54$  ns they get [10]  $\text{BR}^{(0)}(K_{e3}) = 0.4049(21)$ ,  $\text{BR}^{(0)}(K_{\mu3}) = 0.2726(16)$ ,  $\text{BR}^{(0)}(3\pi^0) = 0.2018(24)$ , and  $\text{BR}^{(0)}(\pi^+ \pi^- \pi^0) = 0.1276(15)$ . In their paper, they constrain the BR sum to one, and solve for  $\tau_L$  by using the above relation; in this way they improve the measurement errors too. For the global  $K_L$  fit the reference values have been used instead, accounting for lifetime dependence and other experimental correlations. KLOE has provided also an independent measurement of  $\tau_L$ , obtained by fitting the proper decay time distribution for  $K_L \rightarrow 3\pi^0$  events, for which the reconstruction efficiency is high and uniform over a fiducial volume of  $\sim 0.4\lambda_L$ . They find [11]  $\tau_L = 50.92(30)$  ns.

All of the results discussed above, plus few more which are described in Ref. [4], are used in a PDG-like fit to evaluate the  $K_L$  main decay channels and lifetime. The only constraint used in this fit is  $\Sigma(\text{BR}) = 1$ . The fit converges successfully with  $\chi^2/ndf = 20.2/11$  (Prob= 4.3%), and the results are reported in table 1. Figure 1 shows a comparison between previous PDG averages and

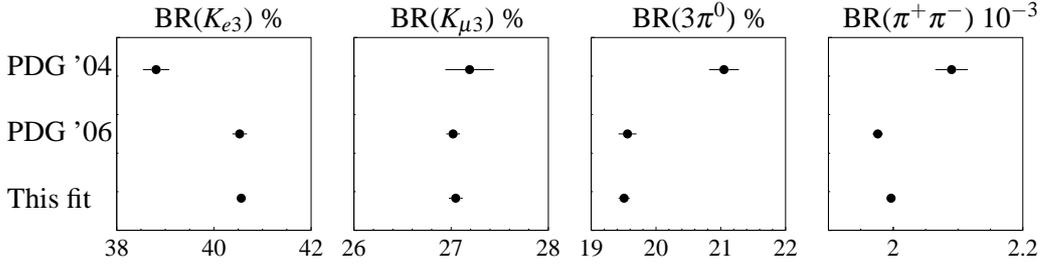
	$\text{BR}(K_{e3})$	$\text{BR}(K_{\mu3})$	$\text{BR}(3\pi^0)$	$\text{BR}(\pi^+ \pi^- \pi^0)$
Value	0.40563(74)	0.27047(71)	0.19507(86)	0.12542(57)
Scale factor	1.1	1.1	1.2	1.1
	$\text{BR}(\pi^+ \pi^-)$	$\text{BR}(2\pi^0)$	$\text{BR}(\gamma\gamma)$	$\tau_L$
Value	$1.9966(67) \times 10^{-3}$	$8.644(42) \times 10^{-4}$	$5.470(40) \times 10^{-4}$	51.173(200) ns
Scale factor	1.1	1.3	1.1	1.1

**Table 1:**  $K_L$  BRs and lifetime from a fit to recent data.

the present results for  $K_{e3}$ ,  $K_{\mu3}$ ,  $3\pi^0$  and  $\pi^+ \pi^-$  decay channels. Differences between this fit and 2006 PDG edition [6] are minor, while a substantial difference is observed with respect to the 2004 PDG review [5], due to a completely renewed set of measurements.

### 3. $K_S$ branching ratios and lifetime

Present knowledge of  $K_S$  main BRs is dominated by two recent KLOE results on  $K_{e3}$  and  $2\pi$  events, based on a sample of  $1.2 \times 10^8$   $\phi \rightarrow K_S K_L$  events. They measure  $\Gamma(K_{e3})/\Gamma(\pi^+ \pi^-) = 10.19(13) \times 10^{-4}$  [14], and  $\Gamma(\pi^+ \pi^-)/\Gamma(2\pi^0) = 2.2549(54)$  [15]. These two ratios completely determine the value of  $K_S$  main BRs, and as far as the value of  $V_{us}$  is concerned we use  $\text{BR}(K_{e3}) = 7.046(91) \times 10^{-4}$ .



**Figure 1:** BR evolution for some representative  $K_L$  channels:  $K_{e3}$ ,  $K_{\mu 3}$ ,  $3\pi^0$ , and  $\pi^+\pi^-$ .

The value of  $K_S$  lifetime used in the present determination of  $|V_{us}|f_+(0)$  is  $\tau_S = 0.08958(5)$  ns, which is taken from PDG fit to  $CP$  parameters [6]. This result is highly constrained by measurements from NA48 [16] and KTeV [17].

#### 4. $K^\pm$ branching ratios and lifetime

Most of the present experimental efforts focus on charged kaon decays, and particularly on  $K_{l3}^\pm$  decays. The NA48 experiment has published recently (and updated at this conference) precise measurements of  $\Gamma(K_{l3}^\pm)/\Gamma(\pi^\pm\pi^0)$ , obtained with simultaneous  $K^+K^-$  beams. The  $K_{l3}^\pm$  samples amount to  $30 - 50 \times 10^3$  events for  $K^-$  and  $K^+$ , respectively, and the final accuracy is limited by statistics. They find [18][19]  $\Gamma(K_{e3}^\pm)/\Gamma(\pi^\pm\pi^0) = 0.2470(9)(4)$ , and  $\Gamma(K_{\mu 3}^\pm)/\Gamma(\pi^\pm\pi^0) = 0.1637(6)(3)$ , where the first error is statistical and the second one refers to systematic effects. The  $K_{e3}$  ratio has been measured recently also by the ISTRA+ Collaboration, using a sample of  $2.2 \times 10^6$   $K_{e3}^-$  decays. Their result [20],  $\Gamma(K_{e3}^-)/\Gamma(\pi^- \pi^0) = 0.2449(4)(14)$ , is in good agreement with the NA48 one. The three ratios presented above are used in the global fit for BRs and lifetime. The last contribution is from the KLOE experiment, which has presented at this conference [21] the absolute measurement of  $BR(K_{l3}^\pm)$  with  $\sim 1\%$  total error. This result is achieved by selecting  $\phi \rightarrow K^+K^-$  events in which one of the two kaons decays to  $\mu^\pm\nu$  or  $\pi^\pm\pi^0$  final state, giving both a normalization for BR evaluation and a tag for the signal search. Using  $\tau_\pm^{(0)} = 12.385(25)$  ns [6] in order to account for acceptance dependence from kaon lifetime, they obtain  $BR^{(0)}(K_{e3}^\pm) = 0.04965(53)$ , and  $BR^{(0)}(K_{\mu 3}^\pm) = 0.03233(39)$ . The error matrix and the lifetime dependence are provided by the experiment, in order to use these measurements in a global fit.

As far as the  $K^\pm$  lifetime is concerned, the world average [6] is very precise, but presents a poor consistency between measurements performed with different techniques; some confirmation is therefore needed. The KLOE experiment has presented two new independent determination of  $\tau_\pm$  at this conference [22]. The first measurement,  $\tau_\pm = 12.367(44)(65)$  ns, is obtained from the kaon track decay length; the second one,  $\tau_\pm = 12.391(49)(25)$  ns, is obtained from the kaon decay time in  $K^\pm \rightarrow \pi^\pm\pi^0$  decays. The combined result is  $\tau_\pm = 12.384(48)$  ns, which is in perfect agreement with PDG average.

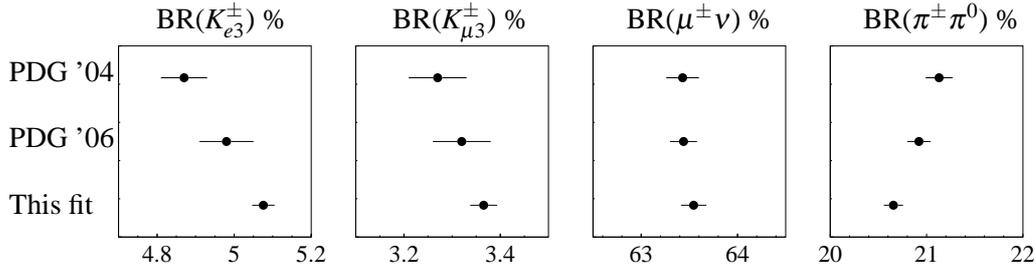
When combining experimental results for  $K^\pm$  decays, it is not possible to rest on recent data only, like for  $K_L$ . Indeed, while  $K_{l3}^\pm$  have been measured recently by NA48, ISTRA+, KLOE and E865, for channels like  $\pi^\pm\pi^0$  and  $\pi^\pm\pi^\pm\pi^\mp$  the BR determination is still based essentially on

measurements by Chiang '72 [23], which are provided without accounting for radiative corrections and correlations between channels. The results of a fit to the available data, both preliminary and published, are reported in table 2. The fit quality is very poor ( $\chi^2/ndf = 52/25$ , Prob= 0.11%), due to the above mentioned tension between the old lifetime measurements, and to a  $3\sigma$  inconsistency between KLOE and NA48 results on  $K_{l3}^\pm$  decays. Figure 2 shows a comparison between previous

	BR( $\mu^\pm\nu$ )	BR( $\pi^\pm\pi^0$ )	BR( $\pi^\pm\pi^\pm\pi^\mp$ )	BR( $K_{e3}$ )
Value	0.63545(132)	0.20656(100)	0.055962(303)	0.050758(290)
Scale factor	1.2	1.3		1.3
	BR( $K_{\mu 3}$ )	BR( $\pi^\pm\pi^0\pi^0$ )	$\tau_\pm$	
Value	0.033656(280)	0.017614(226)	12.3840(193) ns	
Scale factor	1.7	1.1	1.7	

**Table 2:**  $K^\pm$  BRs and lifetime from a fit to the whole available data.

PDG averages and the present results for  $K_{e3}^\pm$ ,  $K_{\mu 3}^\pm$ ,  $\mu^\pm\nu$ , and  $\pi^\pm\pi^0$  decay channels. Differences between 2006 PDG compilation [6] and the present fit are appreciable, the latter including 2007 results on  $K_{l3}^\pm$  decays.



**Figure 2:** BR evolution for some representative  $K^\pm$  channels:  $K_{e3}^\pm$ ,  $K_{\mu 3}^\pm$ ,  $\mu^\pm\nu$ , and  $\pi^\pm\pi^0$ .

## 5. Form factor slopes and determination of $K_{l3}$ decay phase space

Since  $K \rightarrow \pi$  is a  $0^- \rightarrow 0^-$  transition, only the vector part of the weak current has a nonvanishing contribution. The matrix element can be expressed as

$$\langle \pi | J_\mu^V | K \rangle = [(P+p)_\mu f_+(t) + (P-p)_\mu f_-(t)] \quad (5.1)$$

where  $P$ ,  $p$ , are the kaon, pion momenta, and  $t = (P-p)^2$  is the only  $L$ -invariant variable. The term proportional to  $f_-(t)$  is only relevant for  $K_{\mu 3}$  decays, since it is multiplied by the lepton mass. It is customary to expand the vector form factor  $f_+(t)$  as

$$f_+(t) = f_+(0) \left[ 1 + \lambda'_+ \frac{t}{m^2} + \frac{1}{2} \lambda''_+ \left( \frac{t}{m^2} \right)^2 + \dots \right], \quad (5.2)$$

where  $m$  is the mass of the charged pion, and only linear and quadratic terms are retained. In the above expression, the form factor at zero momentum transfer,  $f_+(0)$ , is evaluated from theory, while the form factor slopes,  $\lambda'_+$ ,  $\lambda''_+$  are experimentally determined from semileptonic decay spectra. A

scalar form factor  $f_0(t)$  is introduced in the parametrization of  $f_-(t)$ , defined as  $f_-(t) = (f_0(t) - f_+(t))(M_K^2 - m^2)/t$  with  $f_0(0) = f_+(0)$ . As in the case of the vector form factor, the scalar form factor is expanded in powers of momentum transfer  $t$ :

$$f_0(t) = f_0(0) \left[ 1 + \lambda_0 \frac{t}{m^2} + \dots \right], \quad (5.3)$$

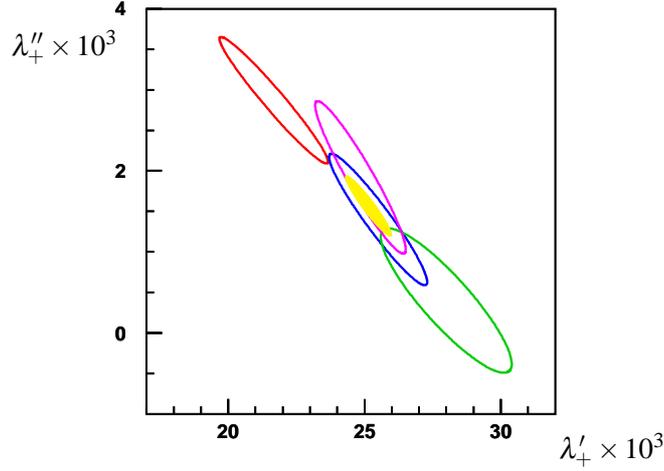
where only the linear term is retained.

New precise determinations of  $\lambda'_+$ ,  $\lambda''_+$ , and  $\lambda_0$  have been published during the last three years by KTeV [26], NA48 [27][28] and KLOE [29][30] on  $K_L l3$  decays, and by ISTRA+ [24][25] on  $K^- l3$  decays. All of these results are reported in table 3, for  $K_{e3}$  and  $K_{\mu 3}$  decays separately. The

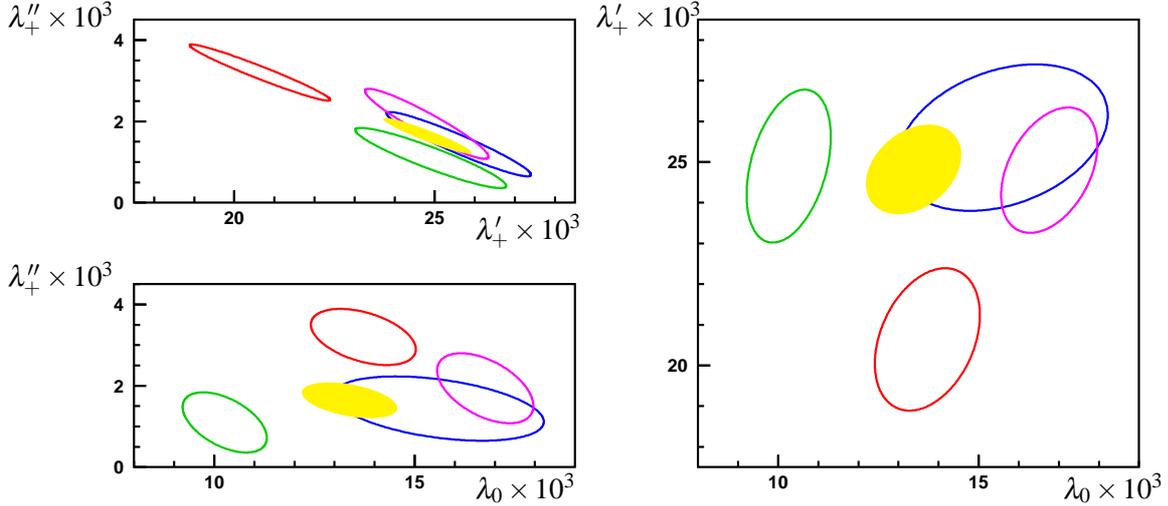
$K_{e3}$ form-factor slopes			$K_{\mu 3}$ form-factor slopes			
	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$		$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	$\lambda_0 \times 10^3$
ISTRA+ [24]	24.9(1.7)	1.9(0.9)	ISTRA+ [25]	23.0(6.4)	2.3(2.3)	17.1(2.2)
KTeV [26]	21.7(2.0)	2.9(0.8)	KTeV [26]	17.0(3.7)	4.4(1.5)	12.8(1.8)
NA48 [27]	28.0(2.4)	0.4(0.9)	NA48 [28]	20.5(3.3)	2.6(1.3)	9.5(1.4)
KLOE [29]	25.5(1.8)	1.4(0.8)	KLOE prel. [30]	25.6(1.8)	1.4(0.8)	15.6(2.6)

**Table 3:** Current data on  $K_{l3}$  form-factor slopes; ISTRA+ results are rescaled by  $(m_{\pi^+}/m_{\pi^0})^2$ ; KLOE preliminary result on  $K_{\mu 3}$  is obtained from a combined fit to  $K_{e3}$  and  $K_{\mu 3}$  spectra.

experimental picture for  $K_{e3}$  decays is summarized in figure 3, where the  $1\sigma$  contours measured by each experiment in the  $\lambda'_+ - \lambda''_+$  plane are reported. The compatibility between the four experiments is excellent, the average slopes being  $\lambda'_+ = 25.15(0.87) \times 10^{-3}$  and  $\lambda''_+ = 1.57(0.38) \times 10^{-3}$ , with  $\chi^2/ndf = 5.3/6$  (Prob= 51%). The average correlation between slopes is  $\rho(\lambda'_+, \lambda''_+) = -0.941$ , and it determines the angular inclination of the ellipses, which is only marginally affected by the measurement systematic error. Integrating these slope parameters over the decay spectrum, the following values for the phase space integrals are obtained:  $I(K^0 e3) = 0.15465(21)$ , and  $I(K^\pm e3) = 0.15901(22)$ . The average slopes resulting from a global fit to  $K_{e3}$  and  $K_{\mu 3}$  are  $\lambda'_+ = 24.82(1.10) \times 10^{-3}$ ,  $\lambda''_+ = 1.64(0.44) \times 10^{-3}$ , and  $\lambda_0 = 13.38(1.19) \times 10^{-3}$ , with  $\chi^2/ndf = 53/13$  (Prob=  $10^{-6}$ ). The compatibility between measurements is very poor, and the inconsistency is parametrized by large scale factors (1.4, 1.3, and 1.9 for  $\lambda'_+$ ,  $\lambda''_+$ , and  $\lambda_0$ , respectively), which produce larger errors on  $\lambda'_+$  and  $\lambda''_+$  respect to the ones obtained from  $K_{e3}$  data alone. Such an inconsistency is entirely due to the value of the scalar slope measured by NA48, as it is evident by looking at figure 4, where the average  $K_{l3}$  contours for the single experiments are shown. Moreover, excluding the NA48  $K_{\mu 3}$  result from the slope fit one gets  $\chi^2/ndf = 12.2/10$  (Prob= 27.1%). The phase space integrals for the four decay modes are:  $I(K^0 e3) = 0.15454(29)$ ,  $I(K^\pm e3) = 0.15889(30)$ ,  $I(K^0 \mu 3) = 0.10209(31)$ , and  $I(K^\pm \mu 3) = 0.10504(32)$ . These integrals are computed without excluding any experimental result, and they will be used to evaluate  $|V_{\text{us}}|f_+(0)$ . The  $K_{e3}$  values differ by less than one per mill respect to the ones from  $K_{e3}$  fit. The  $K_{\mu 3}$  integrals change by 0.6% if the NA48 result is excluded from the fit, but this would shift the final average for  $|V_{\text{us}}|f_+(0)$  only by 0.08%.



**Figure 3:**  $1\sigma$  contours measured in the  $\lambda'_+-\lambda''_+$  plane for  $K_{e3}$  decays by KTeV (red), KLOE (blue), ISTRA+ (purple), and NA48 (green); yellow ellipse represents the average result.



**Figure 4:**  $1\sigma$  contours measured in the  $\lambda'_+-\lambda''_+$ ,  $\lambda_0-\lambda''_+$ , and  $\lambda_0-\lambda'_+$  planes for  $K_{\mu 3}$  decays by KTeV (red), KLOE (blue), ISTRA+ (purple), and NA48 (green); yellow ellipses represent the average results.

### 6. Extraction of $|V_{us}|f_+(0)$

$SU(2)$ -breaking and  $EM$  corrections which are used to extract  $|V_{us}|f_+(0)$  are summarized in table 4. The  $SU(2)$ -breaking correction is evaluated with ChPT to  $O(p^4)$ , as described in [31]. The long distance  $EM$  corrections to the full inclusive decay rate are evaluated with ChPT to  $O(e^2p^2)$  [31], and using low-energy constants from Ref. [33]. The quoted results have been evaluated recently [32], and include for the first time the  $K_{\mu 3}$  channels for both neutral and charged kaons. Using all of the experimental and theoretical inputs discussed above, the values of  $|V_{us}|f_+(0)$  have been evaluated for  $K_L e3$ ,  $K_L \mu3$ ,  $K_S e3$ ,  $K^\pm e3$ , and  $K^\pm \mu3$  decay modes, as shown in table 5 and figure 5. The five decay modes agree well within the quoted errors, and average to

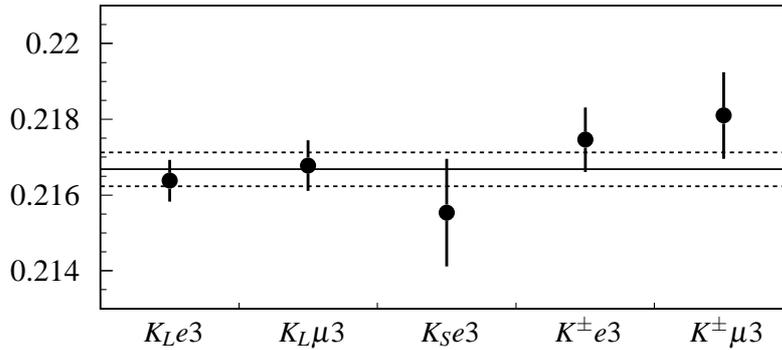
Mode	$\Delta_K^{SU(2)}$	$\Delta_{KI}^{EM}$
$K^0 e3$	0	0.57(15)%
$K^0 \mu3$	0	0.80(15)%
$K^\pm e3$	2.36(22)%	0.08(15)%
$K^\pm \mu3$	2.36(22)%	-0.12(15)%

**Table 4:** Summary of  $SU(2)$ -breaking and  $EM$  corrections.

Mode	$ V_{us} f_+(0)$	% err	Approx. contrib. to % err			
			BR	$\tau$	$\Delta$	$I_{KI}(\lambda)$
$K_L e3$	0.21627(60)	0.28	0.09	0.19	0.15	0.09
$K_L \mu3$	0.21678(67)	0.31	0.10	0.18	0.15	0.15
$K_S e3$	0.21544(144)	0.67	0.65	0.03	0.15	0.10
$K^\pm e3$	0.21725(89)	0.41	0.29	0.09	0.26	0.10
$K^\pm \mu3$	0.21800(114)	0.52	0.42	0.09	0.26	0.15

**Table 5:** Values of  $|V_{us}|f_+(0)$  extracted from  $K_{l3}$  decay rates; all sources contributing to the total fractional error are reported separately.

$|V_{us}|f_+(0) = 0.21663(47)$ , with  $\chi^2/ndf = 2.62/4$  (Prob= 62%). To evaluate the reliability of the  $SU(2)$ -breaking correction, a comparison is made between separate averages of  $|V_{us}|f_+(0)$  for the neutral and the charged channels, which are 0.21638(52) and 0.21740(86), respectively: they agree within  $1.1\sigma$ . Alternatively, an experimental estimate of  $\Delta_K^{SU(2)}$  is obtained by comparing the neutral result with the charged one evaluated without correcting for  $SU(2)$ -breaking: we get  $\Delta_{exp}^{SU(2)} = 2.84(40)\%$ , which is in good agreement with the value estimated from theory.



**Figure 5:** Values of  $|V_{us}|f_+(0)$  extracted from  $K_{l3}$  decay rates; the average between decay modes is indicated by a continuous line, dashed lines representing the  $1\sigma$  band.

## 7. Determination of $|V_{us}|$ and CKM unitarity test

In the previous section a determination of  $|V_{us}|f_+(0)$  from  $K_{l3}$  decays has been presented,

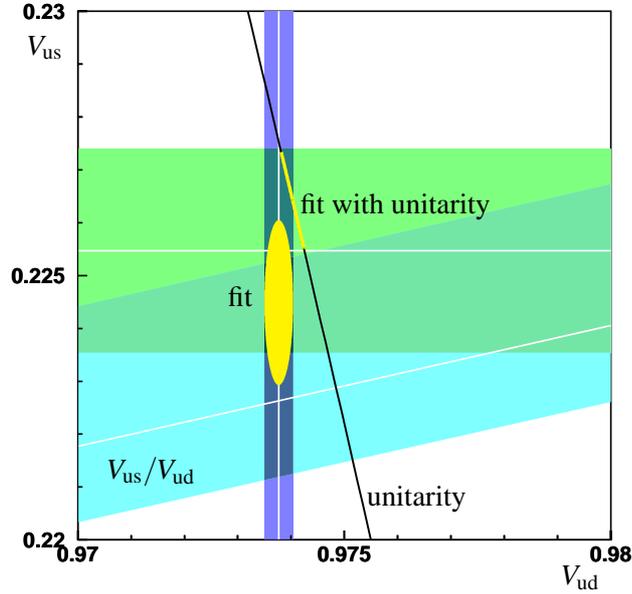
with  $\sim 2 \times 10^{-3}$  fractional accuracy. Assuming for  $f_+(0)$  the standard Leutwyler-Roos evaluation [34],  $f_+(0) = 0.961(8)$ , we get  $|V_{us}| = 0.2254(19)$ . To test the CKM unitarity, we use  $V_{ud} = 0.97372(26)$  [35], which is an average from  $0^+ \rightarrow 0^+$  nuclear beta decays, and includes a recent evaluation of electroweak radiative corrections. Combining the two results we get

$$V_{ud}^2 + V_{us}^2 - 1 = -0.0011(10), \quad (7.1)$$

which is consistent with unitarity to  $\sim 1\sigma$ . A value of  $V_{us}$  can also be obtained from a comparison of the radiative inclusive decay rates of  $K^\pm \rightarrow \mu^\pm \nu(\gamma)$  and  $\pi^\pm \rightarrow \mu^\pm \nu(\gamma)$  combined with a lattice calculation of  $f_K/f_\pi$  via [36]

$$\frac{\Gamma(K^\pm \rightarrow \mu^\pm \nu(\gamma))}{\Gamma(\pi^\pm \rightarrow \mu^\pm \nu(\gamma))} = \frac{|V_{us}|^2 f_K^2 m_K (1 - m_\mu^2/m_K^2)^2}{|V_{ud}|^2 f_\pi^2 m_\pi (1 - m_\mu^2/m_\pi^2)^2} \times 0.9930(35), \quad (7.2)$$

with the uncertainty on the multiplicative factor coming from the electroweak radiative corrections. To solve equation 7.2 for  $V_{us}/V_{ud}$ , we use  $\text{BR}(K^\pm \rightarrow \mu^\pm \nu) = 0.6366(17)$  from KLOE [37], and the preliminary lattice result  $f_K/f_\pi = 1.208(2)_{(-14)}^{(+7)}$ , from the MILC Collaboration [38]. From the above results we get  $V_{us}/V_{ud} = 0.2286_{(-15)}^{(+27)}$ . This ratio can be used in a fit together with the measurements of  $V_{us}$  and  $V_{ud}$ , as shown in figure 6. Results of this fit are  $V_{ud} = 0.97372(26)$ , and  $V_{us} = 0.2245(16)$ , with  $\chi^2/ndf = 0.73/1$  (Prob= 39%). If we add the constraint of unitarity we get  $V_{ud} = 0.97398(21)$ , and  $V_{us} = 0.2266(9)$ , with  $\chi^2/ndf = 3.6/2$  (Prob= 17%), which is again compatible with unitarity at  $1\sigma$  level.



**Figure 6:** Results of fits to  $|V_{ud}|$ ,  $|V_{us}|$ , and  $|V_{us}|/|V_{ud}|$ .

## References

- [1] F. Gilman, K. Kleinknecht, and B. Renk, in *Review of Particle Physics*, Phys. Lett. B **592**, 130 (2004).
- [2] A. Sher et al., Phys. Rev. Lett. **91**, 261802 (2003).
- [3] E. Blucher and W. Marciano, in *Review of Particle Physics*, J. Phys. G **33**, 677 (2006).
- [4] M. Moulson for the FlaviaNet Working Group on Kaon decays,  $V_{us}$  from kaon decays, , hep-ex/0703013.
- [5] S. Eidelman et al., *Review of Particle Physics*, Phys. Lett. B **592**, 1 (2004).
- [6] W.-M. Yao et al., *Review of Particle Physics*, J. Phys. G **33**, 1 (2006).
- [7] T. Alexopoulos et al., Phys. Rev. D **70**, 092006 (2004).
- [8] A. Lai et al, Phys. Lett. B **602**, 41 (2004).
- [9] L. Litov for the NA48 Collab., in *Proc. 32<sup>nd</sup> Int. Conf. on High-Energy Physics (ICHEP '04)* (Beijing, 2004), p. 817, hep-ex/0501048.
- [10] F. Ambrosino et al., Phys. Lett. B **632**, 43 (2006).
- [11] F. Ambrosino et al, Phys. Lett. B **626**, 15 (2005).
- [12] F. Ambrosino et al., Phys. Lett. B **638**, 140 (2006).
- [13] A. Lai et al., Phys. Lett. B **645**, 26 (2007).
- [14] F. Ambrosino et al., Phys. Lett. B **636**, 173 (2006).
- [15] F. Ambrosino et al., Eur. Phys. J. C **48**, 767 (2006).
- [16] A. Lai et al., Phys. Lett. B **537**, 28 (2002).
- [17] A. Alavi-Harati et al., Phys. Rev. D **67**, 012005 (2005).
- [18] J.R. Batley et al., Eur. Phys. J. C **50**, 329 (2007).
- [19] A. Dabrowski, these proceedings.
- [20] V.I. Romanovsky et al., arXiv:0704.2052.
- [21] B. Sciascia, these proceedings.
- [22] P. Massarotti, these proceedings.
- [23] I.H. Chiang et al., Phys. Rev. D **6**, 1254 (1972).
- [24] O.P. Yushchenko et al., Phys. Lett. B **589**, 111 (2004).
- [25] O.P. Yushchenko et al., Phys. Lett. B **581**, 31 (2004).
- [26] T. Alexopoulos et al., Phys. Rev. D **70**, 092007 (2004).
- [27] A. Lai et al., Phys. Lett. B **604**, 1 (2004).
- [28] A. Lai et al., Phys. Lett. B **647**, 341 (2007).
- [29] F. Ambrosino et al., Phys. Lett. B **636**, 166 (2006).
- [30] C. Gatti, these proceedings.
- [31] V. Cirigliano et al., Eur. Phys. J. C **23**, 121 (2002).

- [32] V. Cirigliano, these proceedings.
- [33] S. Descotes-Genon and B. Moussallam, *Eur. Phys. J. C* **42**, 403 (2005).
- [34] H. Leutwyler and M. Roos, *Z. Phys. C* **25**, 91 (1984).
- [35] W.J. Marciano, these proceedings.
- [36] W.J. Marciano, *Phys. Rev. Lett.* **93**, 231803 (2004).
- [37] F. Ambrosino et al., *Phys. Lett. B* **632**, 76 (2006).
- [38] C. Bernard et al., in *Proc. 5<sup>th</sup> Int. Workshop on Chiral Dynamics (Chiral '06)* (Chapel Hill NC, USA, 2006), hep-lat/0611024.