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Wave functions of the nucleon and its parity partner from lattice QCD

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We compute moments of distribution amplitudes using gauge configurations with two flavors of clover fermions from QCDSF/DIK and operators which are optimized with respect to their behavior under the lattice symmetries. The knowledge of these quantities helps in understanding the internal structure of hadrons and in the analysis of (semi-)exclusive processes. We present results for the nucleon distribution amplitude which suggest that the asymmetries (the deviations from the asymptotic form) are smaller than indicated by sum rule calculations. Using the same approach we were also able to calculate the same quantities for the $N^\star(1535)$, the parity partner of the nucleon. These results show a stronger deviation from the asymptotic form.

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1. Introduction

Distribution amplitudes [1, 2, 3] describe the structure of hadrons in terms of valence quark Fock states at small transverse separation and are required in the calculation of hard (semi)exclusive processes. A simple picture is obtained at very large values of the momentum transfer. In this limit the process factorizes and, for example, the magnetic Sachs form factor of the nucleon $G_M(Q^2)$ can be expressed as a convolution of the hard scattering kernel $h(x_i, y_i, Q^2)$ and the leading-twist quark distribution amplitude in the nucleon $\varphi(x_i, Q^2)$ [3],

$$G_M(Q^2) = f_N^2 \int_0^1 [\mathrm{d}x] \int_0^1 [\mathrm{d}y]\ \varphi^\star(y_i, Q^2) h(x_i, y_i, Q^2) \varphi(x_i, Q^2), \tag{1.1}$$

where $[\mathrm{d}x] = \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}x_3 \delta(1 - \sum_{i=1}^{3} x_i)$, and $-Q^2$ is the squared momentum transfer in the hard scattering process. However, in the kinematic region $1\,\mathrm{GeV}^2 < Q^2 < 10\,\mathrm{GeV}^2$, which has attracted a lot of interest recently due to the JLAB data [4, 5] for G_M, the situation is more complicated. Here calculations are possible, e.g., within the light-cone sum rule approach [6, 7]. They indicate that higher-twist distribution amplitudes become important while higher Fock states do not play a significant role. In any case, the distribution amplitudes are needed as input.

As advocated in the pioneering work [8], lattice QCD is well suited to calculate such non-perturbative quantities. Our recent calculation [9] of moments of the nucleon distribution amplitudes shows that they can be determined reasonably well on the lattice. Furthermore, using the same methods we were able to determine distribution amplitudes of the nucleon parity partner $N^\star(1535)$ with comparable accuracy. We find that the asymmetry of the leading-twist amplitude of the nucleon is smaller than in QCD sum rule calculations, in agreement with light-cone sum rules [10] and quark models [11], which suggest a less asymmetric form. On the other hand, our results for $N^\star(1535)$ suggest that the asymmetry for the parity partner of the nucleon is considerably enhanced.

2. Basics

In this section we work in Minkowski space. The leading-twist distribution amplitudes for octet baryons and in particular nucleon distribution amplitudes were introduced within the classical framework of hard exclusive processes in [1, 2, 3] The starting point for baryons is the matrix element of a trilocal quark operator, which can be written to leading-twist accuracy in terms of three invariant functions V, A and T [12]:

$$\begin{aligned} &\langle 0| \left[P\exp\left(ig \int_{z_1}^{z_3} A_\mu(\sigma) \mathrm{d}\sigma^\mu \right) f_\alpha(z_1) \right]^a \left[P\exp\left(ig \int_{z_2}^{z_3} A_\nu(\tau) \mathrm{d}\tau^\nu \right) g_\beta(z_2) \right]^b h_\gamma^c(z_3)\, |p\rangle \varepsilon^{abc} \\ &= \frac{1}{4} \left\{ (p\cdot\gamma C)_{\alpha\beta} (\gamma_5 N)_\gamma f_V V(z_i\cdot p) + (p\cdot\gamma\gamma_5 C)_{\alpha\beta} N_\gamma f_A A(z_i\cdot p) + (i\sigma_{\mu\nu} p^\nu C)_{\alpha\beta} (\gamma^\mu \gamma_5 N)_\gamma f_T T(z_i\cdot p) \right\}. \end{aligned}$$

Here a, b, c are color indices, $|p\rangle$ denotes a baryon state with momentum p and f, g, h are quark fields. We consider these matrix elements for the space time separation of the quarks on the light cone with $z_i = a_i z$ ($z^2 = 0$) and $\sum_i a_i = 1$. On the r.h.s. C is the charge conjugation matrix, $f_{(V,A,T)}$ are normalization constants of the leading-twist distribution amplitudes and N is the baryon spinor.

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In momentum space we have

$$V(x_i) \equiv \int V(z_i \cdot p) \prod_{i=1}^{3} \exp\left(ix_i(z_i \cdot p)\right) \frac{\mathrm{d}(z_i \cdot p)}{2\pi}, \qquad V(x_i) \equiv V(x_1, x_2, x_3), \tag{2.1}$$

and similarly for $A(x_i)$ and $T(x_i)$. The distribution amplitudes $V(x_i)$, $A(x_i)$ and $T(x_i)$ describe the quark distribution inside the baryon as functions of the longitudinal momentum fractions x_i. They also depend on the renormalization scale.

The moments of distribution amplitudes $V^{lmn} = \int_0^1 [\mathrm{d}x]\, x_1^l x_2^m x_3^n\, V(x_1, x_2, x_3)$ are related to matrix elements of local operators such as

$$\mathcal{V}_\tau^{\rho\bar{l}\bar{m}\bar{n}}(0) \equiv \mathcal{V}_\tau^{\rho(\lambda_1\cdots\lambda_l)(\mu_1\cdots\mu_m)(\nu_1\cdots\nu_n)}(0) = \varepsilon^{abc}\left[i^l D^{\lambda_1}\ldots D^{\lambda_l} f(0)\right]_\alpha^a (C\gamma^\rho)_{\alpha\beta}$$
$$\times\, [i^m D^{\mu_1}\ldots D^{\mu_m} g(0)]_\beta^b\; [i^n D^{\nu_1}\ldots D^{\nu_n}(\gamma_5 h(0))]_\tau^c, \tag{2.2}$$

by $P_{LTW}\ \langle 0|\mathcal{V}_\tau^{\rho\bar{l}\bar{m}\bar{n}}(0)|p\rangle = -f_V V^{lmn} p^\rho p^{\bar{l}} p^{\bar{m}} p^{\bar{n}} N_\tau(p)$, with similar relations for A and T, see, e.g., [8]. The multiindex $\bar{l}\bar{m}\bar{n}$ with $\bar{l} \equiv \lambda_1 \ldots \lambda_l$ and similarly for $\bar{m}$ and $\bar{n}$ denotes the Lorentz structure given by the covariant derivatives $D_\mu = \partial_\mu - igA_\mu$ on the r.h.s. of eq. (2.2). The indices l, m, n (without bars) are the total number of derivatives acting on the first, second and third quark, respectively. The index ρ corresponds to the uncontracted Lorentz index of the gamma matrices in the operators. The leading-twist projection, P_{LTW}, can be achieved, e.g., by symmetrization in Lorentz indices and subtraction of traces.

Since in the nucleon the two quarks f and g are of the same flavor we have additional relations for the moments of the distribution amplitudes,

$$V^{lmn} = V^{mln}, \quad A^{lmn} = -A^{mln}, \quad T^{lmn} = T^{mln}, \tag{2.3}$$

which can be restored from the combination

$$\phi^{lmn} = \frac{1}{3}(V^{lmn} - A^{lmn} + 2T^{lnm}) \tag{2.4}$$

by taking into account the isospin symmetry. Hence we have only one independent distribution amplitude. In particular the normalization constants are equal, $f_N = f_V = f_A = f_T$, where f_N is the nucleon wave function normalization constant defined by the choice $\phi^{000} = 1$. For the parity partner of the nucleon $N^\star(1535)$ we have exactly the same relations. The combination $\varphi^{lmn} = V^{lmn} - A^{lmn}$, often used in sum rule calculations, can easily be obtained by the relation $\varphi^{lmn} = 2\phi^{lmn} - \phi^{nml}$. Due to momentum conservation we have additional relations between lower and higher moments of the distribution amplitudes: $\phi^{lmn} = \phi^{(l+1)mn} + \phi^{l(m+1)n} + \phi^{lm(n+1)}$. In particular this implies

$$1 = \phi^{000} = \phi^{100} + \phi^{010} + \phi^{001} = \phi^{200} + \phi^{020} + \phi^{002} + 2(\phi^{011} + \phi^{101} + \phi^{110}) = \ldots \tag{2.5}$$

In the case of next-to-leading twist distribution amplitudes we restrict ourselves to operators without derivatives. Then only two additional constants, V_3^0 and T_3^0, appear [13], which determine the normalization of the twist-four distribution amplitudes. The combinations $\lambda_1 = V_1^0 - 4V_3^0$ and $\lambda_2 = 6(V_1^0 - 4T_3^0)$ are also used in the literature. They describe the coupling to the nucleon of two independent nucleon interpolating fields used in the QCD sum rule approach. One of the operators, $\mathcal{L}_\tau$, was suggested in [14] and the other, $\mathcal{M}_\tau$, in [15]:

$$\mathcal{L}_\tau(0) = \varepsilon^{abc}\left[u^{aT}(0)C\gamma^\rho u^b(0)\right] \times (\gamma_5\gamma_\rho d^c(0))_\tau, \quad \mathcal{M}_\tau(0) = \varepsilon^{abc}\left[u^{aT}(0)C\sigma^{\mu\nu} u^b(0)\right] \times (\gamma_5\sigma_{\mu\nu} d^c(0))_\tau.$$

Their matrix elements are given by

$$\langle 0|\mathscr{L}_\tau(0)|p\rangle = \lambda_1 m_N N_\tau, \qquad \langle 0|\mathscr{M}_\tau(0)|p\rangle = \lambda_2 m_N N_\tau. \tag{2.6}$$

3. Calculation on the Lattice

It is straightforward to translate the relevant operators to Euclidean space. However, due to the discretization of space-time, the mixing pattern of the operators on the lattice is more complicated than in the continuum. It is determined by the transformation behavior of the operators under the (spinorial) symmetry group of our hypercubic lattice. As operators belonging to inequivalent irreducible representations cannot mix, we derive our operators from irreducibly transforming multiplets of three-quark operators [16, 17] in order to reduce the amount of mixing to a minimum. These irreducible multiplets constitute also the basis for the renormalization of our operators, which is performed nonperturbatively in an RI-MOM-like scheme. In this procedure, the mixing with "total derivatives" is automatically taken into account. The numerical results presented in this work were obtained using QCDSF/DIK configurations generated with the standard Wilson gauge action and two flavors of nonperturbatively improved Wilson fermions (clover fermions). The gauge coupling used was $\beta = 5.40$, which corresponds to $a \approx 0.067\,\mathrm{fm}$ via a Sommer parameter of $r_0 = 0.467\,\mathrm{fm}$ [18, 19]. The lattice size was $24^3 \times 48$ with pion masses down to 420MeV.

In the case of the moments considered in this work we can avoid the particularly nasty mixing with lower-dimensional operators completely. Note that the operators $\mathscr{V}_\tau^{\rho\bar{l}\bar{m}\bar{n}}$, $\mathscr{A}_\tau^{\rho\bar{l}\bar{m}\bar{n}}$ and $\mathscr{T}_\tau^{\rho\bar{l}\bar{m}\bar{n}}$ with different multi-indices $\rho\bar{l}\bar{m}\bar{n}$ but the same lmn are related to the same moments V^{lmn}, A^{lmn} and T^{lmn}, and we make use of this fact not only in order to minimize the mixing problems but also in order to reduce the statistical noise by considering suitable linear combinations.

For the operators without derivatives, i.e., the matrix elements λ_1, λ_2 and f_N, we have performed a joint fit of all contributing correlators to obtain the values at the simulated quark masses. As these are larger than the physical masses a chiral extrapolation to the physical point is required in the end. To the best of our knowledge there are no results from chiral perturbation theory to guide this extrapolation. Therefore we have adopted a more phenomenological approach aiming at linear (in m_π^2) fits to our data. It turns out that the ratios f_N/m_N^2 and λ_i/m_N are particularly well suited for this purpose. In order to estimate the systematic error due to our linear extrapolation, we also consider a chiral extrapolation including a term quadratic in m_π^2 and take the difference as the systematic error. The results in the $\overline{\mathrm{MS}}$ scheme at a scale of $2\,\mathrm{GeV}$ are given in Table 1. Note that $2\lambda_1 \approx -\lambda_2$ for nucleon, a relation that is expected to hold in the nonrelativistic limit due to Fierz identities. However, for $N^\star(1535)$ we observe a strong deviation from this relation.

For the higher moments one can proceed in the same way and the constraint (2.5) is satisfied very well. However, the statistical errors in this approach are too large to allow an accurate determination of the (particularly interesting) asymmetries. We achieved smaller errors by calculating the ratios $R^{lmn} = \phi^{lmn}/S_i$ where

$$S_1 = \phi^{100} + \phi^{010} + \phi^{001} \qquad \text{for } l+m+n = 1, \tag{3.1}$$

$$S_2 = 2(\phi^{011} + \phi^{101} + \phi^{110}) + \phi^{200} + \phi^{020} + \phi^{002} \qquad \text{for } l+m+n = 2. \tag{3.2}$$

These ratios are extrapolated linearly to the physical masses. Again, we extrapolate all results also quadratically in m_π^2 and take the difference as an estimate for the systematic error of the chiral extrapolation. Requiring that the constraint (2.5) be satisfied for the renormalized values we can finally extract the moments from the ratios. These results are summarized in Table 1. In Table 2 we compare the moments φ^{lmn} as obtained from ϕ^{lmn} with some other estimates. From these values we see that the asymmetry of the $N^\star(1535)$ distribution amplitude is more pronounced compared to the nucleon and is mostly driven by the first moments. However, in both cases we have the approximate symmetry $\varphi^{lmn} \approx \varphi^{lnm}$.

	Asympt.	LAT N	LAT $N^\star(1535)$
$f_N \cdot 10^3\,[\mathrm{GeV}^2]$		3.144(61)(29)(54)	4.417(114)(215)(2)
$-\lambda_1 \cdot 10^3\,[\mathrm{GeV}^2]$		38.72(70)(43)(106)	40.88(110)(778)(57)
$\lambda_2 \cdot 10^3\,[\mathrm{GeV}^2]$		76.23(139)(84)(207)	208.9(47)(384)(42)
ϕ^{100}	$\frac{1}{3} \approx 0.333$	0.3638(11)(68)(3)	0.4007(14)(48)(12)
ϕ^{010}	$\frac{1}{3} \approx 0.333$	0.3023(10)(42)(5)	0.2610(18)(13)(16)
ϕ^{001}	$\frac{1}{3} \approx 0.333$	0.3339(9)(26)(2)	0.3384(11)(35)(4)
ϕ^{011}	$\frac{1}{7} \approx 0.143$	0.0724(18)(82)(70)	0.0706(22)(37)(66)
ϕ^{101}	$\frac{1}{7} \approx 0.143$	0.1136(17)(32)(21)	0.1213(23)(21)(16)
ϕ^{110}	$\frac{1}{7} \approx 0.143$	0.0937(16)(3)(38)	0.0943(24)(17)(38)
ϕ^{200}	$\frac{2}{21} \approx 0.095$	0.1629(28)(7)(68)	0.1825(35)(4)(56)
ϕ^{020}	$\frac{2}{21} \approx 0.095$	0.1289(27)(37)(51)	0.0962(37)(68)(119)
ϕ^{002}	$\frac{2}{21} \approx 0.095$	0.1488(32)(77)(73)	0.1429(34)(29)(75)

Table 1: Comparison of our lattice results (LAT) as obtained from QCDSF/DIK configurations at $\beta = 5.40$ for the nucleon (N) and $N^\star(1535)$ at $\mu^2 = 4\,\mathrm{GeV}^2$. The first error is statistical, the second (third) error represents the uncertainty due to the chiral extrapolation (renormalization). The systematic errors should be considered with due caution, see the text for their determination.

	Asympt.	QCD-SR	BK	BLW	LAT N	LAT $N^\star(1535)$
$f_N \cdot 10^3\,[\mathrm{GeV}^2]$		5.0(5)	6.64	5.0(5)	3.234(63)(86)	4.544(117)(223)
$-\lambda_1 \cdot 10^3\,[\mathrm{GeV}^2]$		27(9)		27(9)	35.57(65)(136)	37.55(101)(768)
$\lambda_2 \cdot 10^3\,[\mathrm{GeV}^2]$		54(19)		54(19)	70.02(128)(268)	191.9(44)(391)
φ^{100}	$\frac{1}{3} \approx 0.333$	0.560(60)	0.38	0.415	0.3999(37)(139)	0.4765(33)(155)
φ^{010}	$\frac{1}{3} \approx 0.333$	0.192(12)	0.31	0.285	0.2986(11)(52)	0.2523(20)(32)
φ^{001}	$\frac{1}{3} \approx 0.333$	0.229(29)	0.31	0.300	0.3015(32)(106)	0.2712(41)(136)
φ^{200}	$\frac{1}{7} \approx 0.143$	0.350(70)	0.18	0.212	0.1816(64)(212)	0.2274(89)(307)
φ^{020}	$\frac{1}{7} \approx 0.143$	0.084(19)	0.13	0.123	0.1281(32)(106)	0.0915(45)(224)
φ^{002}	$\frac{1}{7} \approx 0.143$	0.109(19)	0.13	0.132	0.1311(113)(382)	0.1034(160)(584)
φ^{011}	$\frac{2}{21} \approx 0.095$	−0.030(30)	0.08	0.053	0.0613(89)(319)	0.0398(132)(497)
φ^{101}	$\frac{2}{21} \approx 0.095$	0.102(12)	0.10	0.097	0.1091(41)(152)	0.1281(56)(131)
φ^{110}	$\frac{2}{21} \approx 0.095$	0.090(10)	0.10	0.093	0.1092(67)(219)	0.1210(101)(304)

Table 2: Comparison of our lattice results (LAT) for the nucleon N and $N^\star(1535)$ as obtained from QCDSF/DIK configurations at $\beta = 5.40$ using ϕ^{010}, ϕ^{001}, ϕ^{110}, ϕ^{200} and ϕ^{020} in Table 1 to selected sum rule results [20] (QCDSR) and the phenomenological estimates [10] (BLW) and [11] (BK) at the scale $\mu^2 = 1\,\mathrm{GeV}^2$.

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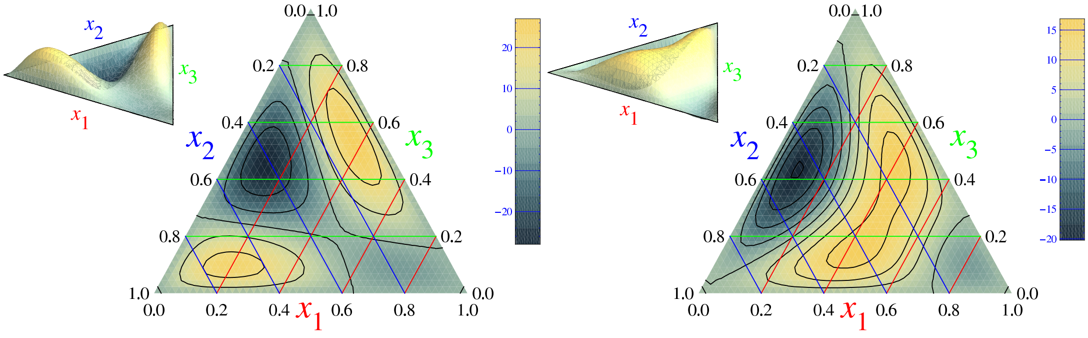

(a) (b)

Figure 1: Barycentric plot of the models of the distribution amplitudes for nucleon (a) and $N^{\star}(1535)$ (b) at $\mu = 1\text{GeV}$ using the central values of the lattice results. The lines of constant x_1, x_2 and x_3 are parallel to the sides of the triangle labelled by x_2, x_3 and x_1, respectively.

4. Model for Distribution Amplitudes

Let us now expand the distribution amplitude in terms of orthogonal polynomials P_{nj} chosen such that the one-loop mixing matrix is diagonal [21]:

$$\varphi(x_i,\mu) = 120x_1x_2x_3 \sum_{n=0}^{N}\sum_{j=0}^{n} c_{nj}(\mu_0)P_{nj}(x_i)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\omega_{nj}}.$$

Taking $N = 2$ and calculating the coefficients $c_{nj}(\mu_0)$ from an independent subset of the moments $\phi^{lmn}(\mu_0 = 2\,\text{GeV})$, we obtain a model function for the distribution amplitude presented in Fig. 1(a). While the (totally symmetric) asymptotic amplitude $120x_1x_2x_3$ has a maximum for $x_1 = x_2 = x_3 = 1/3$, inclusion of the first moments (i.e., choosing $N = 1$) moves this maximum in the case of the nucleon to $x_1 \approx 0.46$, $x_2 \approx 0.27$, $x_3 \approx 0.27$ giving the first quark substantially more momentum than the others. The second moments then turn this single maximum into the two local maxima in Fig. 1(a). This also happens in the case of $N^{\star}(1535)$. However the influence of the second moments on the form of the DA is reduced compared to the nucleon due to the stronger asymmetry in the first moments. The approximate symmetry in x_2 and x_3 seen in both cases is due to the approximate symmetry $\varphi^{lmn} \approx \varphi^{lnm}$ of our results. It is also seen in QCD sum rule calculations for the nucleon as well as in several models such as BLW and BK. Since higher-order polynomials have been disregarded, Figs. 1(a) and 1(b) should be interpreted with due caution.

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