

## Pion-cloud effects in the BSE description of mesons

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We investigate the effect of including hadronic resonances contributions in the description of light quarks and mesons. To this end we take into account the back-coupling of the pion onto the quark propagator within the non-perturbative continuum framework of Schwinger-Dyson equations (SDE) and Bethe-Salpeter equations (BSE), in essence describing the so-called *pion-cloud*. As a result of our study we find that an unquenching of this form provides for considerable effects in the spectrum of light mesons.

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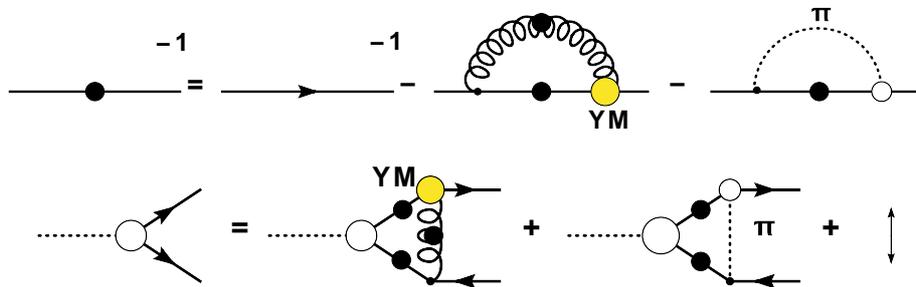
## 1. Introduction

In QCD it is not the quarks and gluons that are asymptotic states, directly observable in detectors, but rather colourless composites such as mesons and baryons. This entails the need to describe in detail the properties of our final states in terms of their constituent particles, an inherently complicated task due to the non-perturbative effects of confinement and dynamical chiral symmetry breaking.

The natural framework for this composite description of mesons in the continuum are the Schwinger-Dyson and Bethe-Salpeter equations (SDEs and BSEs). Such studies have been extensively performed in the Rainbow-Ladder approximation [1], which generally reduce to a quenched study with non-perturbative effects subsumed into an effective gluonic interaction. While there have been several studies attempting to improve upon this simplest of all truncations, in the form of including unquenching effects due to quarks, *e.g.* [2, 3, 4], we here take a different viewpoint. Instead of modelling the contribution from quarks directly, we rather consider the dominant contributions arising from the resultant *mesonic* degrees of freedom. Thus in this talk we focus upon pion-cloud contributions [5] to the light meson spectrum in a beyond the rainbow truncation scheme.

## 2. Including the Pion-Cloud

The prescription for including pion degrees of freedom in the SDEs and BSEs in a manner consistent with the axial-vector Ward-Takahashi identity (avWTI) was first presented in [6] and investigated within the real-value approximation. Further modifications were suggested in [7], with the resultant system of equations depicted in Fig. 1. The first loop diagram of both pictorial equations relates to the usual rainbow-ladder, where the infrared suppressed gluon is enhanced by the vertex dressing, indicated by *YM*. To satisfy the avWTI this enhancement is restricted to depend on the same momentum as the exchanged gluon; the two dressings are often combined into an effective gluon dressing which is consequently modelled. The second diagram that contributes to the quark-SDE and meson-BSE represents the back-reaction of the pion onto the quark, *i.e.* pion-cloud effects. This requires knowledge of the quark-pion vertex, which we parameterise in terms of the pion Bethe-Salpeter amplitude. Necessarily this couples the two equations (whose form is presented in [7, 8]), forming a highly non-trivial system of integral equations.



**Figure 1:** The approximated quark-SDE with effective one-gluon and one-pion exchange, together with the corresponding BSE. The up-down arrow indicates an averaging procedure of the pion-exchange diagram with respect to the dressed/undressed quark-pion vertex.

## 2.1 Yang-Mills Part

We need to specify the details of our gluon exchange and quark-gluon vertex that model the interaction. We choose two different model ansätze that have been employed already in previous works. The first model takes fits to the gluon as obtained from numerical solutions of the corresponding Schwinger-Dyson equations. This is taken in combination with results inspired from solutions of the quark-gluon vertex for a restricted kinematic section [9]. This has the virtue that it also provides for generation of a topological charge and hence an anomalous mass contribution to the  $\eta$ ,  $\eta'$  [10].

Thus, in choosing a particular kinematic configuration for our vertex dressing we employ a rainbow-ladder form of the interaction (thus preserving the avWTI), *i.e.*

$$\Gamma_v^{YM}(p_1, p_2, p_3) = \gamma_v Z_2 / \tilde{Z}_3 \Gamma^{YM}(p_3^2), \quad (2.1)$$

with quark momenta by  $p_1$  and  $p_2$  and the gluon momentum  $p_3$ . Note, however, that (2.1) involves only the  $\gamma_v$ -part of the full tensor structure of the vertex. It has been shown in the analysis of the full quark-gluon vertex of Ref. [11] that such a model cannot capture all essentials of dynamical chiral symmetry breaking. We refer to this as the soft-divergent model interaction.

The second model that we consider is that of Maris-Tandy [12]. In this case, the dressing of both the gluon and the vertex is modelled by a phenomenological ansatz that includes the correct one-loop UV running and provides dynamical chiral symmetry breaking. We include a study of this much used model in the context of pion unquenching for comparison.

## 2.2 Pionic Contribution

The decomposition of a Bethe-Salpeter vertex function  $\Gamma(p; P)^{(\mu)}$  is well-established in the literature, with its form constrained by transformation properties under CPT [13]. In particular the pion is given by the following form

$$\Gamma_\pi^j(p; P) = \tau^j \gamma_5 \left[ F_1(p; P) - i \not{P} F_2(p; P) - i \not{p} (p \cdot P) F_3(p; P) - [\not{P}, \not{p}] F_4(p; P) \right]. \quad (2.2)$$

This is of particular relevance for our pionic part of the interaction. We approximate the full pion Bethe-Salpeter wave function in the quark-SDE and the kernel of the BSE by the leading amplitude in the chiral limit given by

$$\Gamma_\pi^j(p; P) = \tau^j \gamma_5 \frac{B_\chi(p^2)}{f_\pi}. \quad (2.3)$$

Here  $B_\chi(p^2)$  is the scalar dressing function of the quark propagator in the chiral limit. The effects of neglecting the three sub-leading amplitudes have been quantified for a real-value approximation in Ref. [6]. The great advantage of the approximation (2.3) compared to the full back-coupling performed in Ref. [6] is that we can then fully take into account the quark propagator in the complex plane as necessary input into the Bethe-Salpeter equation. Whilst proving to be a simple prescription in itself, it gives rise to the technical challenge of having to evaluate the full normalisation condition of the BSE due to the non-trivial momentum dependence of the exchange kernel. We discuss this in the next section.

### 3. Normalisation

Since we solve homogeneous equations for the Bethe-Salpeter amplitude, their subsequent renormalisation comes as an auxiliary condition derived from the inhomogeneous BSE:

$$\delta^{ij} = 2 \frac{\partial}{\partial P^2} \text{tr} \int \frac{d^4 k}{(2\pi)^4} \left[ 3 \left( \bar{\Gamma}_\pi^i(k, -Q) S(k+P/2) \Gamma_\pi^j(k, Q) S(k-P/2) \right) + \int \frac{d^4 q}{(2\pi)^4} [\bar{\chi}_\pi^i]_{sr}(q, -Q) K_{tu,rs}^{\text{pion}}(q, k; P) [\chi_\pi^j]_{ut}(k, Q) \right],$$

where  $Q^2 = -M^2$  is fixed to the on-shell meson mass, the trace is over Dirac matrices and the Bethe-Salpeter wave-function  $\chi$  is defined by  $\chi_\pi^j(k; P) = S(k+P/2) \Gamma_\pi^j(k, P) S(k-P/2)$ . The conjugate vertex function  $\bar{\Gamma}$  is given by  $\bar{\Gamma}(p, -P) = C \Gamma^T(-p, -P) C^{-1}$ , with the charge conjugation matrix  $C = -\gamma_2 \gamma_4$ .

The first term of (3.1), independent of the kernel, is easily evaluated since one needs only derivatives of the quark propagator. The second term is substantially more complicated due to the double integration over the interaction kernel with respect to two four-momenta,  $k$  and  $q$ . We evaluate the integral of (3.1) and employ finite difference methods to compute the derivative. We find that the contribution from the kernel provides important contributions.

### 4. Results

The parameters of for both model interactions were fit to pion observables, with the additional constraint of the topological charge for our soft-divergent interaction. In constraining the parameter set to meson observables, we find for our model a quark mass of  $m_{\overline{\text{MS}}} = 3.4$  MeV at  $\mu = 2$  GeV, whilst for Maris-Tandy we have  $m_{\overline{\text{MS}}} = 4.4$  MeV. The remaining parameters of the interaction are:

Soft-Divergent Interaction				Maris-Tandy Interaction			
$d_1$ (GeV <sup>2</sup> )	$d_2$ (GeV <sup>2</sup> )	$d_3$ (GeV <sup>2</sup> )	$\Lambda_{\text{QCD}}^2$ (GeV <sup>2</sup> )	$\omega$ (GeV)	$D$ (GeV <sup>2</sup> )	$m_t$ (GeV)	$\Lambda_{\text{QCD}}^2$ (GeV <sup>2</sup> )
1.45	0.1	3.95	0.52	0.37	1.45	0.5	0.234 <sup>2</sup>

We calculated a range of meson observables, detailed in Table 1. We observe that the effect of the pion back-reaction has only a small impact on the pion mass itself, resulting in a small positive or negative shift depending upon the form of the interaction. The impact of including pion-cloud effects on the leptonic decay constant is fairly large, with effects of the order of 10%.

For the remaining heavier mesons, the common trend is that the inclusion of such an unquenching gives rise to negative mass shifts of 100–200 MeV. Most notable of these are for the rho, where we predict that unquenching from the pion-cloud yields a bound-state  $\sim 100$  MeV lighter than in the quenched theory, in line with recent lattice simulations [14].

It is clear, however, that in order to reproduce the rich spectrum of light mesons that we need to include spin dependent contributions from the Yang-Mills part of the quark-gluon vertex. It is envisaged that this will indeed have a strong impact on the calculated masses of bound-states and is the object of future research.

Model Employed		$M_\pi$	$f_\pi$	$M_\sigma$	$M_\rho$	$f_\rho$	$M_{a_1}$	$M_{b_1}$	$M_\eta$	$M_{\eta'}$
Maris-Tandy	w/o pi	140	104	746	821	160	979	820		
	inc. pi	138 <sup>†</sup>	93.2 <sup>†</sup>	598	720	167	913	750		
Our Model	w/o pi	125	102	638	795	159	941	879	493	949
	inc. pi	138 <sup>†</sup>	93.8 <sup>†</sup>	485	703	162	873	806	497	963
Experiment		138	92.4	400–1200	776	156	1230	1230	548	948

**Table 1:** BSE results for a range of mesons in the Maris-Tandy and soft-divergence models employed with (‘incl.’) and without (‘w/o’) the pion back-reaction. Model parameters are tuned such that values marked by † are reproduced when the pion-exchange kernel is switched on. Results for rainbow-ladder without pion effects are for the same parameter set.

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