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Why once-subtracted dispersion relations can lead to a precise description of $\pi\pi$ scattering and the $f_0(600)$ parameters?

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We review how theoretical constraints from dispersion relations imposed on amplitudes fitted to $\pi\pi$ experimental data significantly help in the determination of these amplitudes below 1 GeV. These constraints lead to scalar amplitudes with threshold behavior in agreement with Chiral Perturbation Theory predictions and allow for a very precise determination, from data, of the position of the $f_0(600)$ (sigma) pole in the complex energy plane. In this short report we concentrate on dispersion relations with crossing symmetry constraints. We explain how their errors propagate and we compare how twice and once subtracted dispersion relations (the Roy's and GKPY equations respectively) constrain the $\pi\pi$ amplitudes. We conclude that the latter ones provide a more stringent consistency check for our parameterizations of the $\pi\pi$ amplitudes above around 450 MeV. We show that these once-subtracted dispersion relations, together with forward dispersion relations (FDR) and sum rules (SR), lead to precise determinations of the $\pi\pi$ partial waves $JI = S0$, $S2$ and P below 1 GeV. Our data analysis is model independent and based only on unitarity, analyticity and crossing symmetry.

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1. Comparison of the Roy's and GKPY equations

The idea of implementing the crossing symmetry condition into the twice subtracted dispersion relations in the description of $\pi\pi \to \pi\pi$ scattering amplitudes was formulated and applied for the first time by Roy in 1971 [1]. This set of coupled dispersion relations has been used later in order to eliminate the long standing "up-down" ambiguity in scalar-isoscalar $\pi\pi$ amplitudes below 1 GeV [2]. Recently, several other analyses of $\pi\pi$ scattering have appeared [3, 4] combining different model independent approaches and data.

In the series of papers [4], a careful analysis of $\pi\pi$ amplitudes for many partial waves (from S to G with isospin 0, 1 and 2) fitted to old and new data sets (e.g. to data from K_{e4} decays [5]), to FDR, several SR and the Roy's equations was presented. It was shown that these constraints on the data allow for precise predictions of threshold parameters, phase shifts up to the two-kaon threshold, and the sigma ($f_0(600)$ meson) pole position.

Continuing our work on the dispersive analysis of the $\pi\pi$ amplitudes, our preliminary results [6] show that once subtracted dispersion relations (GKPY equations) can be very helpful in the analysis of the $\pi\pi$ amplitudes. Both Roy's and GKPY equations can be expressed as a sum of so called subtracting, kernel and driving terms (ST, KT and DT respectively - for details see [4]). The main difference between these two sets of equations is in ST, which for the Roy's equations are first order polynomials in the s variable, whereas for GKPY eqs. are constant. The values of these two ST terms are given by combinations of the scattering lengths for the $S0$- and $S2$-waves.

In Figure 1 we show the real part of the amplitudes obtained from the Roy's and GKPY equations (called "out") together with the corresponding real part of the amplitudes coming into ST, KT and DT (called "in"). In our calculations we minimize the difference between the input and output amplitudes for the $S0$, $S2$ and P waves below 1 GeV. It is clearly seen that above about $s^{1/2} = 400$ MeV the errors of the GKPY equations are significantly smaller than the errors of the Roy's equations. This monotonous increase of the latter ones is caused mainly by the linear propagation of the errors from the scattering lengths in ST.

The continuation of the amplitudes into the complex energy plane allows us to calculate the position of the sigma pole. Our preliminary values are $459^{+36}_{-33} - i257^{+17}_{-18}$ MeV from Roy's equations and $461^{+14.5}_{-15.5} - i257 \pm 16$ MeV from GKPY. Although the central values may change in our final results it is evident that errors are considerably smaller when obtained from GKPY equations. Preliminary values for scattering lengths are (in pion mass units) 0.223 ± 0.009 for the S0 wave and -0.0444 ± 0.0045 for the $S2$.

One can therefore use both types of equations (together with FDR and SR) to constrain the studied $\pi\pi$ amplitudes and extract precise physical observables related to meson spectroscopy.

References

[1] S. M. Roy, Phys. Lett. B **36**, 353 (1971).

[2] R. Kaminski, L. Leśniak and B. Loiseau, Phys. Lett. B **551**, 241 (2003).

[3] B. Ananthanarayan Phys. Rept. 353, 207 (2001); S. Descotes-Genon *et al.*, Eur. Phys. J. C24, 469 (2002); G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B 603, 125 (2001); I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. 96, 132001 (2006).

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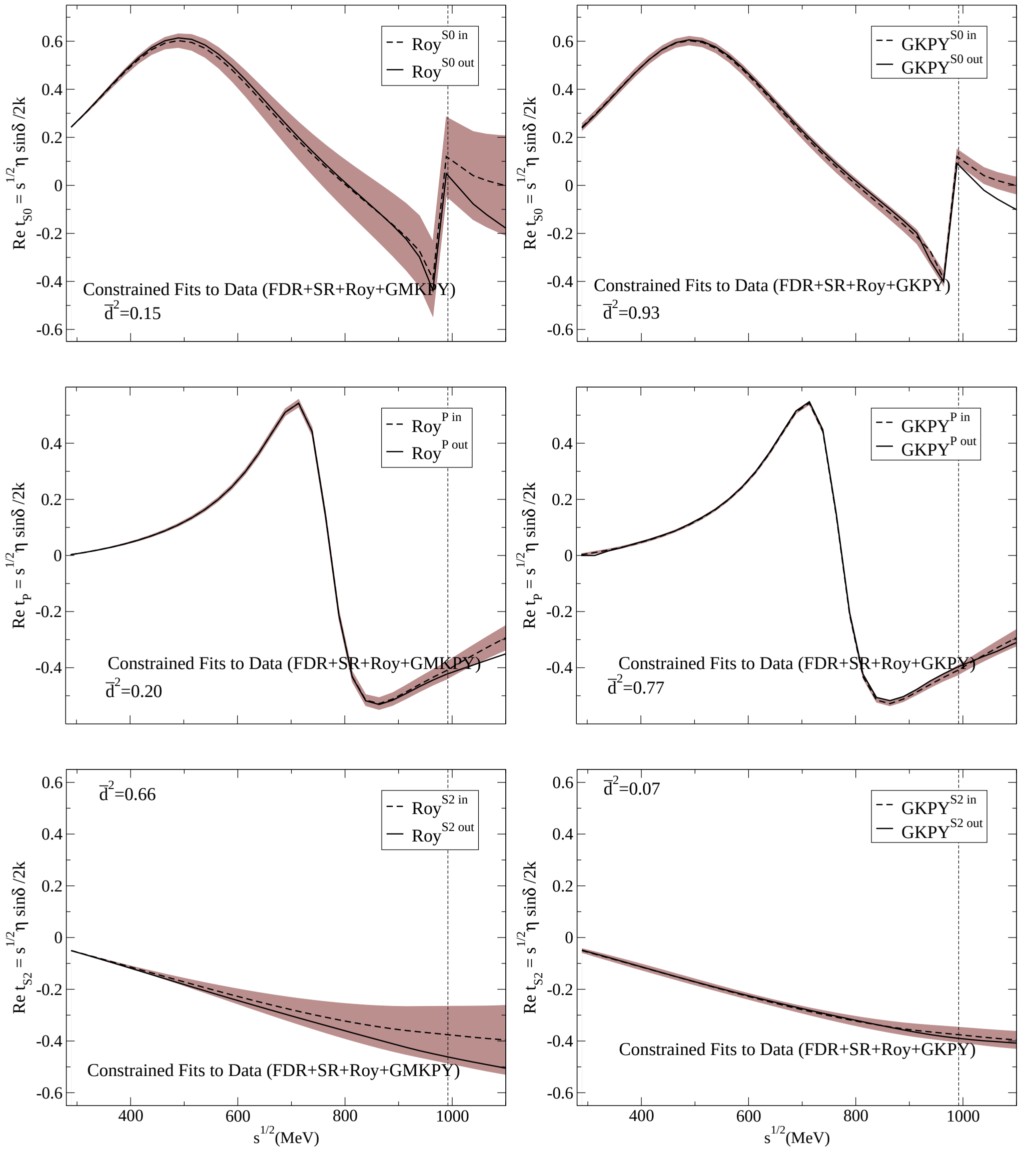

Figure 1: ‘In” amplitudes (dashed lines) versus “out” amplitudes (continuous lines). The “in” amplitudes correspond to a data fit on which FDR, SR, Roy’s eqs. and GKPY eqs. are imposed within errors. The “out” amplitudes are obtained using the “in” amplitudes inside Roy’s eqs. (left panel) and GKPY eqs. (right panel). The shaded bands cover the uncertainties in the difference between “in” and “out” amplitudes.

[4] R. Kaminski, J.R. Pelaez and F.J. Yndurain, Phys. Rev. D **77**, 054015(2008). J.R. Pelaez and F.J. Yndurain, Phys. Rev. D **71**, 074016 (2005). Nucl. Phys. B 603, 125 (2001). F. J. Yndurain, R. Garcia-Martin and J. R. Pelaez, Phys. Rev. D **76**, 074034 (2007).

[5] B. Bloch-Devaux (NA48/2 Collaboration), Nucl. Phys. Proc. Suppl. 174,(2007).

[6] R. Garcia-Martin, R. Kaminski and J. R. Pelaez, Int. J. Mod. Phys. A **24**, 590 (2009); R. Kaminski *et al* Int. J. Mod. Phys. A **24**, 402 (2009).