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$N_c = 2$ lattice gauge theories with adjoint Wilson fermions

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We investigate $N_c = 2$ gauge theories with adjoint fermions. Motivated by simulations with dynamical overlap fermions, we adopt two flavors of the Wilson-Dirac fermions together with twisted mass ghosts, that setup corresponds to the topology fixing term in the overlap simulations. In this paper we focus on the adjoint representation of Wilson fermion with the Iwasaki gauge action on a $8^3 \times 16$ lattice, and explore the Aoki phase structure in the β-M_0 plane. We observe the first order phase transition at $\beta = 0.8$–1.0 by increasing the bare quark mass (with negative sign convention) from the physical quark mass region. This first order transition seems to disappear at $\beta = 1.2$. We also investigate the phase structure around the 'second finger' of the Aoki phase so as to search for the range at which the overlap fermion is simulated with locality.

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1. Introduction

In a lattice gauge theory at finite lattice spacing, there may appear a phase structure which does not exist in the continuum counterpart. The Aoki phase of the Wilson-Dirac fermions is a well-known and important example of such a phase [1, 2]. In the case of $N_f = 2$ QCD at zero temperature, chiral symmetry is spontaneously broken and three massless Nambu-Goldstone modes (pions) appear accordingly. For the Wilson-Dirac operator, which explicitly breaks chiral symmetry, Aoki proposed a conjecture that the chiral limit is characterized by a second order phase transition to a phase in which the flavor-parity symmetry is spontaneously broken. In spite of numbers of numerical results in accord with this picture [3], the phase structure of the Wilson-Dirac operator is still in debate both in weak and strong coupling regimes [4, 5, 6].

In addition to studies of the Wilson-type fermions around the chiral limit, it is also important to explore the phase structure of the Wilson-Dirac operator for an application of the overlap fermion formulation which has an exact chiral symmetry on the lattice. The locality of the overlap operator is guaranteed only when its Wilson-Dirac kernel has a gap [7] or its near-zero modes are exponentially local [8, 9]. According to the argument by Golterman and Shamir [8, 9], the Aoki phase is characterized by extended near-zero modes. This implies that the locality of the overlap operator is satisfied when its Wilson kernel is out of the Aoki phase.

Our original motivation drawing this work is nonperturbative search for gauge theories which exhibit nontrivial infrared fixed point and a conformal window [10]. Such a theory would give insight on the so-called walking technicolor theory, which may provide alternative mechanism to the electroweak symmetry breaking. In this context, the $N_c = 2$ gauge theories draw much attention because the conformal-like (walking) behavior is expected with less numbers of flavors, in particular in higher representations of fermions. While the infrared fixed point is directly explored by computation of the running coupling, it may suffer from large statistical and systematic uncertainties. An alternative approach is to examine the dynamics of spontaneous chiral symmetry breaking with changing N_f. By adopting the overlap fermions, one can simulate the theory in the ε-regime and examine possible chiral symmetry breaking through the low-lying spectrum of the overlap-Dirac operator.

We investigate the SU(2) gauge theories with adjoint fermions. In present work, we focus on the phase structure of the Wilson-Dirac operator, as preparation to the simulations with dynamical overlap fermions, as well as of its own physical interests. This is an extension of our previous work in quenched approximation [13]. While the same analysis is performed also for the fundamental Wilson-Dirac operator, this paper concentrates on the case of the adjoint representation.

2. Lattice setup

Motivated by dynamical overlap simulations with fixed topological charge, we adopt the $N_f = 2$ Wilson fermions accompanied by the twisted mass ghosts [11, 12],

$$\det\left(\frac{H_W^2}{H_W^2 + \mu^2}\right) = \int \mathscr{D}\chi^\dagger \mathscr{D}\chi \exp[-S_E]. \tag{2.1}$$

The numerator, the standard Wilson-Dirac operator with negative bare mass M_0, represents physical fermions for $M_0 < M_{0c1}$ where M_{0c1} is the smallest critical value of M_0, *i.e.* the left edge of the

Aoki phase. In the case of M_0 in between the first and second fingers of Aoki phase, the numerator of (2.1) suppresses the near-zero modes of $H_W = \gamma_5 D_W(-M_0)$, and keep the topological charge fixed by prohibiting the lowest eigenvalue to cross zero. The twisted mass ghost operator in the denominator cancels the effect of the high frequency modes of H_W. The existence of the latter is main difference from previous works by other groups [14, 15, 16]. Since the low energy effect brought by the near-zero modes of H_W is not spoiled, the above action represents two flavors of Wilson fermions when the quark mass is small enough. However, when the gap of H_W is sizable, the system resembles the quenched system.

In simulations with overlap fermions, it is convenient to employ the action (2.1) to suppress the near-zero modes of H_W which may deteriorate the locality of the overlap operator and make numerical simulation costly. For this purpose we need to choose the value of M_0 in between the first and second fingers of Aoki phase, and sufficiently away from its edges. In this case our present setup corresponds to the quenched approximation, since the action (2.1) does not represent physical fermion degrees of freedom. The Wilson fermion term in Eq. (2.1) plays dynamical role only when the corresponding quark mass is sufficiently small compared to the twisted mass of the ghosts.

In this work, we adopt the Iwasaki gauge action and the above Wilson fermion action in the adjoint representation. We survey the phase structure in β-M_0 plane over the range of M_0 from physical region to the second finger of the Aoki phase.

As a probe of the Aoki phase, we first analyze the pseudoscalar (PS) meson correlator following Ref. [9]. We introduce the twisted mass term as an external field in the Wilson-Dirac fermion action as

$$S_{Wtm} = \bar{\psi}[D_W - im_1\tau_3\gamma_5]\psi. \tag{2.2}$$

Then in the Aoki phase, in the limit of $m_1 \to 0$, π_3 becomes massive while $\pi_\pm$ remain massless. Thus we measure the PS correlator in the broken direction,

$$\Gamma(x,y) = \langle \pi_+(x)\pi_-(y)\rangle, \qquad \pi_\pm(x) = i\bar{\psi}(x)\gamma_5\tau_\pm\psi(x). \tag{2.3}$$

The extracted PS meson masses with standard fitting procedure are extrapolated to vanishing twisted mass. Vanishing m_π signals the Aoki phase.

3. Numerical result

In this work we use a lattice of size $8^3 \times 16$. We adopt β =0.80, 0.90, 1.00, and 1.20 for the Iwasaki gauge action and $\mu = 0.2$ for the twisted mass ghost. For each ensemble, the HMC algorithm is used to generate 500 configurations each separated by 10 trajectories of unit length after 500 trajectories for thermalization. The first 100 configurations are used for the analysis of the meson correlators. We measure the PS correlator (2.3) as well as that of the vector (V) meson with the point source and sink by varying bare valence quark mass M_{0V} with $m_1 = 0.01$–0.08. The correlators are fitted to the hyperbolic cosine form at $t = 6-10$, and then the extracted values of meson mass squared at smallest three values of m_1 are linearly extrapolated to $m_1 = 0$.

Figure 1 shows the partially quenched result of the PS and V meson masses around the 'first finger' of the Aoki phase structure at $\beta = 0.90$. For $M_0 \le 1.51$, the meson masses decrease as the bare valence mass M_{0V} increases as shown in the top panels. On the other hand, for $M_0 \ge 1.52$,

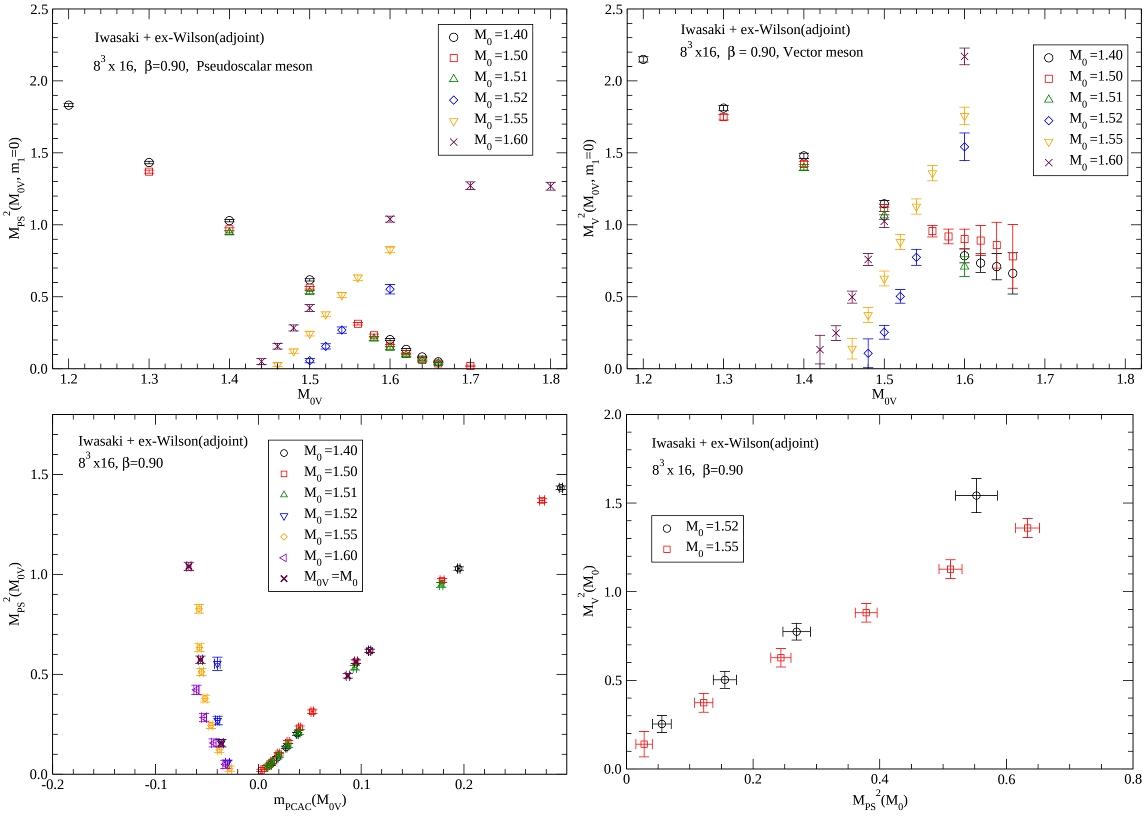

Figure 1: The meson masses and PCAC quark mass results around the 'first finger' of the Aoki phase at $\beta = 0.90$. The top panels show the partially quenched results for the PS and V meson masses vs valence bare mass M_{0V}. The bottom-left panel shows the PS meson mass vs PCAC quark mass. The bottom-right panel plots the PS and V meson masses for the negative PCAC quark mass region.

the slope changes discontinuously, and the meson masses with M_{0V} crosses at $M_0 \simeq 1.52$. This suggests that the first order phase transition occurs around $M_0 = 1.52$ and width of the Aoki phase shrinks to zero. The first order phase transition is also indicated by the plaquette values displayed in Fig. 2. The left panel of Fig. 2 shows the average plaquette values, and for $\beta = 0.9$ there appears a discontinuous change around $M_0 = 1.52$. The right panel of Fig. 2 shows hysteresis between $M_0 = 1.51$ and 1.52, which also indicates that the transition is of the first order. This behavior is consistent with other works with standard plaquette gauge action and without twisted mass ghost [14, 15]. For definite conclusion, however, we need to investigate the volume dependence, which is in progress as shown in the plot of the average plaquette values in Fig. 2.

The bottom-left panel of Fig. 1 plots the PS meson mass against PCAC quark mass. For $M_0 \leq 1.51$, the partially quenched data behave as $m_{PS}^2 \propto m_q$. The top-right panel of Fig. 1 also shows that the V meson mass stays finite at the transition when approaching from the small M_{0V} region. These partially quenched results show the QCD-like behavior, i.e. indicate the broken chiral symmetry, while the PCAC mass may not be sufficiently light. More detailed analysis is necessary to clarify the dynamics in the vicinity of the massless limit.

For $M_0 \geq 1.52$ the PCAC quark mass becomes negative. The sea quark mass nearest to zero is about 0.04 at $M_0 = 1.52$. The bottom-right panel of Fig. 1 displays the V and PS meson masses for

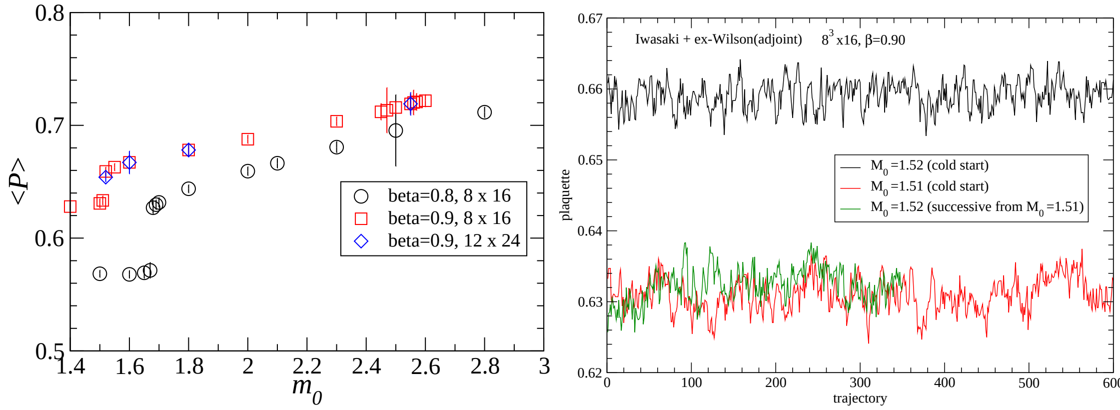

Figure 2: The left panel shows the M_0 dependence of the plaquette value at $\beta = 0.90$ and 0.80. The result at $\beta = 0.90$ on $12^3 \times 24$ is also displayed to examine the finite volume effect. The right panel shows the hysteresis of the plaquette value between $M_0 = 1.51$ and 1.52 at $\beta = 0.90$.

$M_0 = 1.52$ and 1.55. The ratio of the V and PS meson masses is almost constant. In Fig. 3 we show the potential of static quarks in fundamental representation. At $M_0 = 1.52$, fit of the result to the form of $V(r) = C - A/r + \sigma r$ results in the string tension σ consistent with zero. For $M_0 \geq 1.52$ the values of σ are much smaller than those at $M_0 \leq 1.51$ where clear confining feature is observed. While these features are consistent with the near-conformal behavior, careful analysis is required to distinguish from the deconfined phase, as pointed out in Refs. [14, 15, 16]. An unexpected result is that the linear term of the static potential is much smaller even at $M_0 = 2.0$, which is away from the transition, since the contribution of Eq. (2.1) is expected to be small in such a region. This may be explained by insufficient cancellation of the numerator and the denominator of Eq. (2.1) with our choice $\mu = 0.2$ which causes that the low-lying modes of H_W significantly affect the dynamics. Indeed preliminary result of simulations at smaller values of μ supports this explanation.

As for the second finger, we observe quite different behavior from the first finger, as shown in left panels of Fig. 4. While at $M_0 = 2.30$ and 2.45 the partially quenched result for the PS meson mass tend to vanish around $M_{0V} = 2.46$–2.50, the result at $M_0 = 2.50$ and 2.55 does not show linear behavior nor approach to zero. This behavior may be partly explained by a following conjecture: While this region corresponds to the Aoki phase, the Wilson fermion determinant suppresses the near-zero modes and prevents from massless behavior with our small lattice size. To clarify the dynamical mechanism of this behavior, the low-lying modes of the Wilson-Dirac operator are now under investigation.

At $\beta = 0.80$ and 1.0, we observed similar behavior as $\beta = 0.90$ around the first finger of the Aoki phase, while at $\beta = 1.0$ the signature of the first order phase transition becomes much milder. Figure 5 shows the partially quenched results of the PS meson mass $\beta = 1.0$ and 1.2. At $\beta = 1.2$, the indication of the first order transition disappear, while changing the lattice volume is necessary to draw definite conclusion.

4. Summary

We are studying SU(2) gauge theories with two flavors of adjoint fermions. As a preparation

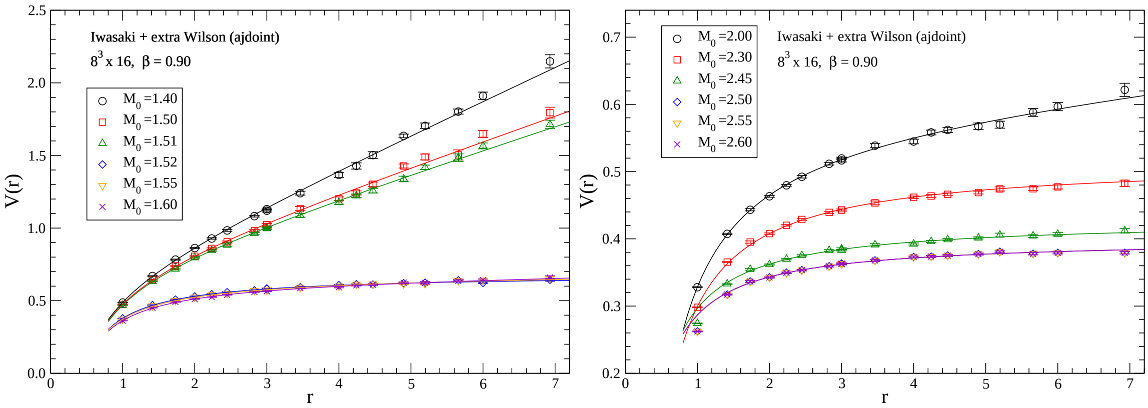

Figure 3: The static quark potential in the fundamental representation at $\beta = 0.90$.

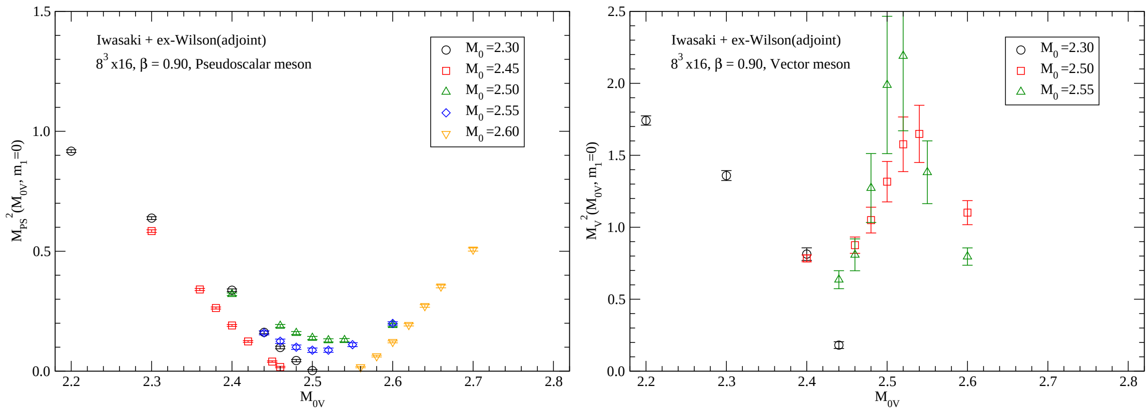

Figure 4: The partially quenched meson masses around the 'second finger' of the Aoki phase at $\beta = 0.90$.

to simulations with overlap fermions, we adopt the Wilson fermions accompanied by the twisted mass ghosts, and explore its phase structure in particular the location of the Aoki phase. At smaller β values, the first order phase transition is found for the first finger of the Aoki phase. We also identified the second finger of the Aoki phase, and explore the feature of the region in between the first and second fingers. For definite understanding of the phase structure of this theory, we need more detailed study in particular with larger lattice volumes. Such studies are in progress, as well as for the fundamental fermions. Simulations of SU(2) gauge theories with overlap fermions are also underway.

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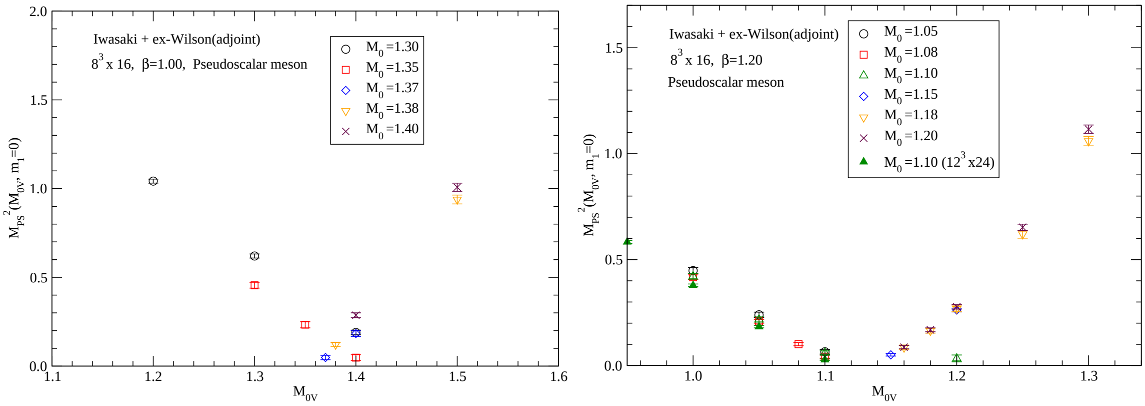

Figure 5: The partially quenched results for the PS meson masses vs M_{0V} at $\beta = 1.0$ (left panels) and 1.2 (right).

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