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# Refitting parameters including CM corrections in the Relativistic Mean Field Models

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Relativistic models for finite nuclei contain spurious center-of-mass motion due to the fact that in most applications the nuclear many-body problem is treated in the mean-field approximation, where invariably the nuclear wave function is taken as a single Slater determinant with wave functions always described in a space-fixed frame. We use the Peierls-Yoccoz projection method to restore the broken translational invariance and reparametrize the model. The consequences for the energy, charge radius and a simple application for the form factor calculation are presented.

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## 1. Introduction

Projection methods were widely applied to non-relativistic mean field solutions in the past, in order to restore the broken symmetries that inevitably arise when the mean field approach is implemented for a many body system. Here we apply one of those restoration methods, the translational symmetry restoration, to a Dirac-Hartree Slater determinant. The method was already developed for a $\sigma$-$\omega$ model of nuclei [1], where the mesonic degrees of freedom were explicitly taken in to account in the projection procedure. Here we use a simpler version of the method but include the $\rho$ and $\delta$ mesons in a density-dependent parametrization. As the center-of-mass (CM) energy should be included in the parametrization, we find a new set that fit the energy and charge radius well for spherically symmetric nuclei. Although the effect on other observables will be properly investigated in a forthcoming paper, a simple application to the $^4He$ charge form factor is shown at the end of this contribution.

## 2. Formalism and Model parametrization

The Lagrangian density for the model is:

$$\mathscr{L} = \sum_{i=p,n} \mathscr{L}_i + \mathscr{L}_\sigma + \mathscr{L}_\omega + \mathscr{L}_\rho + \mathscr{L}_\delta + \mathscr{L}_\gamma, \tag{2.1}$$

where the nucleon Lagrangian reads, $\mathscr{L}_i = \bar{\psi}_i \left[\gamma_\mu iD^\mu - M^*\right] \psi_i$, with $M^* = M - \Gamma_s\phi - \Gamma_\delta \tau \cdot \delta$ and $iD^\mu = \left(i\partial^\mu - \Gamma_\omega V^\mu - \frac{\Gamma_\rho}{2}\tau\cdot\mathbf{b}^\mu - e\frac{1+\tau_3}{2}A^\mu\right)$. The meson and electromagnetic Lagrangian densities are:

$$\mathscr{L}_\sigma = \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right) \;,\; \mathscr{L}_\omega = \frac{1}{2}\left(-\frac{1}{2}\Omega_{\mu\nu}\Omega^{\mu\nu} + m_v^2 V_\mu V^\mu\right)$$

$$\mathscr{L}_\rho = \frac{1}{2}\left(-\frac{1}{2}\mathbf{B}_{\mu\nu}\cdot\mathbf{B}^{\mu\nu} + m_\rho^2\mathbf{b}_\mu\cdot\mathbf{b}^\mu\right) \;,\; \mathscr{L}_\delta = \frac{1}{2}(\partial_\mu\delta\partial^\mu\delta - m_\delta^2\delta^2) \;,\; \mathscr{L}_\gamma = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu},$$

where $\Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ , $\mathbf{B}_{\mu\nu} = \partial_\mu\mathbf{b}_\nu - \partial_\nu\mathbf{b}_\mu - \Gamma_\rho(\mathbf{b}_\mu\times\mathbf{b}_\nu)$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The electromagnetic coupling constant is $e = \sqrt{4\pi/137}$ and $\tau$ is the isospin operator. The set of Dirac and Klein-Gordon equations which result from the above Lagrangian are solved self-consistently in the mean-field approximation imposing the conditions of static and spherically symmetric fields. For that purpose we specify the functional form of the various couplings as, $\Gamma_i(\rho) = \Gamma_i(\rho_0)h_i(x), \quad x = \rho/\rho_0$, with

$$h_i(x) = a_i\frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2},\; i=\sigma,\omega,\;\; h_i(x) = a_i\exp[-b_i(x-1)] - c_i(x-d_i),\; i=\rho,\delta\;. \tag{2.2}$$

The parameters $a_i, b_i, c_i, d_i$ and $\Gamma_i(\rho_0)$ are usually adjusted to reproduce nuclear matter properties as well as binding energy and charge radius of selected nuclei along the periodic table. The nucleon and meson masses are taken as fit parameters or from experiment. From the model Lagrangian, eq.(2.1), one easily obtains the corresponding Hamiltonian $H$, which is obviously translationally invariant. In order to perform the linear momentum projection, it is then useful to construct the Hamiltonian operator in the nucleonic field space, which can be expanded as

$$\psi(x) = \sum_\alpha u_\alpha(\vec{r})e^{-iE_\alpha t}b_\alpha + \sum_\alpha v_\alpha(\vec{r})e^{iE_\alpha t}d_\alpha^\dagger \;,\; \psi^\dagger(x) = \sum_\alpha u_\alpha^\dagger(\vec{r})e^{iE_\alpha t}b_\alpha^\dagger + \sum_\alpha v_\alpha^\dagger(\vec{r})e^{-iE_\alpha t}d_\alpha \;,$$

where $u_\alpha(\vec{r})$ and $v_\alpha(\vec{r})$ form a complete set of Dirac spinors in the coordinate space, and $b_\alpha$ and $b^\dagger_\alpha$, $d_\alpha$ and $d^\dagger_\alpha$, denote the creation and the annihilation operators for the nucleons and anti-nucleons respectively in the state $\alpha$. Neglecting the anti-particles (no-sea approximation), we thus obtain [2]:

$$H = \sum_{\alpha\alpha'} \int u^\dagger_{\alpha'}(\vec{r})\, \hat{t}\, u_\alpha(\vec{r}) d\vec{r}\, b^\dagger_{\alpha'} b_\alpha + \frac{1}{2} \sum_{\alpha,\alpha',\beta,\beta'} \int u^\dagger_{\alpha'}(\vec{r}_1) u^\dagger_{\beta'}(\vec{r}_2) V_{\alpha,\alpha'}(\vec{r}_1,\vec{r}_2) \times u_\beta(\vec{r}_2) u_\alpha(\vec{r}_1)\, d\vec{r}_1\, d\vec{r}_2\, b^\dagger_{\alpha'} b^\dagger_{\beta'} b_\beta b_\alpha \tag{2.3}$$

with the kinetic and potential terms given by

$$\hat{t} = (-i\gamma^0 \vec{\gamma}.\vec{\nabla} + \gamma_0 M) \tag{2.4}$$

$$V_{\alpha,\alpha'}(\vec{r}_1,\vec{r}_2) = \sum_{i=\sigma,\delta,\omega,\rho,\gamma} \frac{1}{4\pi} \gamma_0(1)\gamma_0(2)\Lambda_i(1,2) \times \frac{\exp\{-r[m_i^2 - (E_\alpha - E_{\alpha'})^2]^{1/2}\}}{r}, \tag{2.5}$$

where $r = |\vec{r}_1 - \vec{r}_2|$, $\Lambda_\sigma(1,2) = -\Gamma_s(1)\Gamma_s(2)$, $\Lambda_\delta(1,2) = -\tau(1).\tau(2)\Gamma_\delta(1)\Gamma_\delta(2)$, $\Lambda_\omega(1,2) = \gamma_\mu(1)\gamma^\mu(2)\Gamma_v(1)\Gamma_v(2)$, $\Lambda_\rho(1,2) = \tau(1).\tau(2)\gamma_\mu(1)\gamma^\mu(2)\Gamma_\rho(1)\Gamma_\rho(2)$ and $\Lambda_\gamma(1,2) = e^2\gamma_\mu(1)\gamma^\mu(2)\delta_{\tau_3 1/2}$. Note the implicit dependence of the coupling constants on the position through the density, i.e., $\Gamma_s(1)$ means $\Gamma_s(\rho(\vec{r}_1))$, etc. The energy dependence $(E_\alpha - E_{\alpha'})$ is disregarded in the Hartree approximation. In order to restore the translational invariance we use a variation-before-projection procedure, which is executable in the context of relativistic nuclear structure calculations. We use the Pierls-Yoccoz projector[3] defined as:

$$\mathscr{P}_{\vec{p}} = \frac{1}{(2\pi)^3} \int \exp[\mathrm{i}(\hat{\vec{P}} - \vec{p}) \cdot \vec{a}]\, d^3\vec{a}\,, \tag{2.6}$$

where $\hat{\vec{P}} = \hat{\vec{P}}_A$ is the total linear momentum operator and $\vec{p}$ the corresponding eigenvalue. The center-of-mass projected energy is given by:

$$E_{\vec{p}=0} = \frac{\langle\Psi| H \mathscr{P}_{\vec{p}=0} |\Psi\rangle}{\langle\Psi| \mathscr{P}_{\vec{p}=0} |\Psi\rangle}\,, \tag{2.7}$$

where $|\Psi\rangle$ is the A-particle Slater determinant obtained from our Hartree variational approach. Defining now $U(\pm\vec{a}/2) = \exp[\pm i\vec{a}/2 \cdot \hat{\vec{P}}]$ and noting that $U$ is the product of one-body operators, it follows that $\left|\tilde{\Psi}^{(\pm)}\right\rangle \equiv U(\pm\frac{\vec{a}}{2})|\Psi\rangle$ is also a Slater determinant with single particle states $\tilde{\phi}^{(\pm)}_\alpha(\vec{r}) = \exp[\pm i\frac{\vec{a}}{2} \cdot \hat{\vec{P}}]\phi_\alpha(\vec{r}) = \phi_\alpha(\vec{r} \pm \frac{\vec{a}}{2})$ . The one-body (kinetic) kernel term reads:

$$T(\vec{a}) = \left\langle \tilde{\Psi}^{(-)} \right| \hat{t} \left| \tilde{\Psi}^{(+)} \right\rangle = \frac{1}{2} N(\vec{a}) \sum_{\alpha\beta} \left\langle \tilde{\phi}^{(-)}_\alpha \right| \hat{t} \left| \tilde{\phi}^{(+)}_\beta \right\rangle (B^{-1})_{\beta\alpha} \tag{2.8}$$

The potential kernel is written as a sum, $V(\vec{a}) = V_D(\vec{a}) + V_E(\vec{a})$ where,

$$V_D(\vec{a}) = \frac{1}{2} N(\vec{a}) \sum_{\alpha\beta\gamma\delta} \left\langle \tilde{\phi}^{(-)}_\alpha \tilde{\phi}^{(-)}_\beta | \hat{V} | \tilde{\phi}^{(+)}_\gamma \tilde{\phi}^{(+)}_\delta \right\rangle (B^{-1})_{\gamma\alpha} (B^{-1})_{\delta\beta} \tag{2.9}$$

and:

$$V_E(\vec{a}) = -\frac{1}{2} N(\vec{a}) \sum_{\alpha\beta\gamma\delta} \left\langle \tilde{\phi}^{(-)}_\alpha \tilde{\phi}^{(-)}_\beta | \hat{V} | \tilde{\phi}^{(+)}_\gamma \tilde{\phi}^{(+)}_\delta \right\rangle (B^{-1})_{\gamma\beta} (B^{-1})_{\delta\alpha}\,. \tag{2.10}$$



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$\hat{t}$ and $\hat{V}$ are given in equations (2.4) and (2.5) and $N(\vec{a}) = \det\left\{B_{\alpha\beta}\right\}$ is the overlap kernel. The details of the calculation of the above kernels for single-particle Dirac functions were presented in previous publications [1],[4]. The above results can be substituted in Eq. (2.7) to obtain the total energy. In our fits, the translationally-invariant charge radius has also been used. From the center-of-mass coordinate definition, $\vec{r}_{CM} = \frac{1}{A}\sum_{i=1}^{A}\vec{r}_i$ , the projected charge radius is given by:

$$R_c^2 = \frac{1}{Z\langle\Psi|\,\mathscr{P}_{\vec{p}=0}\,|\Psi\rangle}\,\langle\Psi|\sum_{i=1}^{Z}(\vec{r}_i-\vec{r}_{CM})^2\mathscr{P}_{\vec{p}=0}\,|\Psi\rangle\ . \tag{2.11}$$

Finally, for further applications, we define the translationally-invariant nuclear many-body charge form factor by [5]:

$$F(q) = \frac{f_p(q^2)}{\langle\Psi|\,\mathscr{P}_{\vec{p}=0}\,|\Psi\rangle}\,\langle\Psi|\sum_{i=1}^{Z}exp(i\vec{q}.(\vec{r}_i-\vec{R}_{CM}))\mathscr{P}_{\vec{p}=0}\,|\Psi\rangle\ , \tag{2.12}$$

where $\vec{q}$ is the momentum transfer and $f_p$ is the proton electric form factor. This can be compared to the expression:

$$F(q) = \frac{1}{Z}f_p(q^2)\ f_{CM}(q^2)\,\langle\Psi|\sum_{i=1}^{Z}exp(i\vec{q}.(\vec{r}_i))\,|\Psi\rangle\ , \tag{2.13}$$

with the usual Tassie center-of-mass correction [6] factor, $f_{CM}$.

## 3. Results and discussion

In table 1 we show, for each meson, the values of the parameters after($a$) and before($b$) the fitting, including our CM correction. First of all we have compared basic properties of symmetric nuclear matter and have found respectively, $25.7MeV$ and $32.7MeV$ for the symmetry energy and $241.0MeV$ and $244.4MeV$ for the nuclear compressibility, before and after our re-parametrization. For finite nuclei, our main results are shown in table 2. Besides the binding energy and root mean square charge radius before and after the refitting, we display the center-of-mass energy calculated using the projection procedure presented above and two other commonly used approximate prescriptions: the mean value $\frac{<p^2>}{2M}$ (where M is the nucleus total mass and the mean value is taken within the Hartree solution), which we call here the *Hartree approximation* and the well known *Harmonic correction*. It is worthwhile to note that the $^4He$ nucleus was not included in our fitting.

Although the parameter values show very tiny modifications, these modifications can have a great impact on the values of observables, due to the fact that we now have translationally-invariant wave functions. As a simple example we have calculated the charge form factor for the $^4He$ nucleus, using equations (2.12) and (2.13). The results are presented in figure (1). As discussed elsewhere [8], valuable information for halos and skins of light nuclei can be obtained from elastic electron scattering, particularly close to and beyond the first diffraction minima of the cross section, where explicit center-of-mass corrections can play an important role.

## ACKNOWLEDGMENTS

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**Table 1:** Parameters for the DDH model before ($b$) and after ($a$) the re-fitting procedure. We take the values from [7] before the fitting.

| i | $m_i$(MeV) | $\Gamma_i(\rho_0)$ | $a_i$ | $b_i$ | $c_i$ | $d_i$ |
|---|---|---|---|---|---|---|
| $\sigma_b$ | 550 | 10.72854 | 1.36547 | 0.22606 | 0.40970 | 0.90199 |
| $\sigma_a$ | 550 | 10.72585 | 1.37380 | 0.22288 | 0.40953 | 0.90190 |
| $\omega_b$ | 783 | 13.29015 | 1.40249 | 0.17258 | 0.34429 | 0.98396 |
| $\omega_a$ | 783 | 13.28819 | 1.40334 | 0.17137 | 0.34296 | 0.98400 |
| $\rho_b$ | 763 | 11.7270 | 0.095268 | 2.1710 | 0.053360 | 17.84310 |
| $\rho_a$ | 763 | 12.76802 | 0.17424 | 1.61884 | 0.049148 | 17.80109 |
| $\delta_b$ | 980 | 7.58963 | 0.01984 | 3.47320 | -0.09080 | -9.81100 |
| $\delta_a$ | 980 | 7.58355 | 0.01992 | 3.74086 | -0.090789 | -9.79701 |

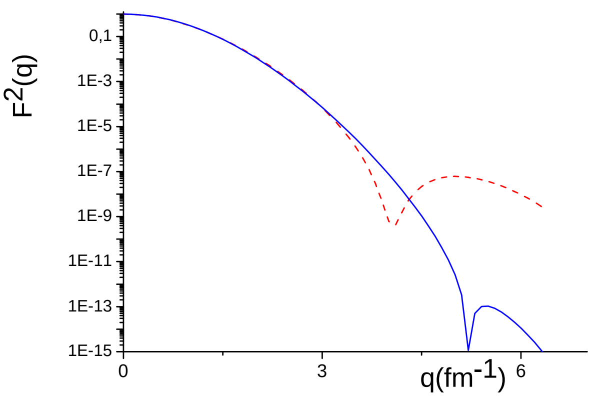


**Figure 1:** Elastic charge form factor for $^4He$ with a full CM calculation (full line) and with approximated (dashed line) CM corrections.

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**Table 2:** Charge radius and binding energy of the nuclei included in this work calculated before and after the refitting procedure. Also shown are the CM corrections in the Hartree and Harmonic approximations (see text for details). The $^4He$ nucleus was not included in our fit.

| Nuclei | Model | $R_c$ (fm) | $B(MeV)$ | Model | CM energy $(MeV)$ |
|---|---|---|---|---|---|
| | before | 1.909 | -31.082 | exact | 16.890 |
| $He^4$ | after | 1.928 | -31.103 | Hartree | 12.574 |
| | Exp. | 1.676 | -28.296 | Harmonic | 19.371 |
| | before | 2.641 | -126.501 | exact | 13.264 |
| $O^{16}$ | after | 2.675 | -128.079 | Hartree | 10.289 |
| | Exp. | 2.730 | -127.619 | Harmonic | 12.203 |
| | before | 3.405 | -336.888 | exact | 10.950 |
| $Ca^{40}$ | after | 3.448 | -341.031 | Hartree | 8.340 |
| | Exp. | 3.485 | -342.052 | Harmonic | 8.991 |
| | before | 3.450 | -417.200 | exact | 11.357 |
| $Ca^{48}$ | after | 3.491 | -415.018 | Hartree | 8.452 |
| | Exp. | 3.484 | -415.990 | Harmonic | 8.461 |
| | before | 3.684 | -478.588 | exact | 11.566 |
| $Ni^{56}$ | after | 3.740 | -481.665 | Hartree | 8.530 |
| | Exp. | | -483.992 | Harmonic | 8.037 |
| | before | 3.857 | -591.474 | exact | 10.135 |
| $Ni^{68}$ | after | 3.895 | -589.278 | Hartree | 7.469 |
| | Exp. | | -590.408 | Harmonic | 7.534 |
| | before | 4.431 | -820.937 | exact | 10.071 |
| $Sn^{100}$ | after | 4.494 | -828.274 | Hartree | 7.231 |
| | Exp. | | -824.794 | Harmonic | 6.625 |
| | before | 4.701 | -1129.109 | exact | 9.168 |
| $Sn^{132}$ | after | 4.751 | -1104.652 | Hartree | 6.536 |
| | Exp. | 4.709 | -1102.851 | Harmonic | 6.039 |
| | before | 5.501 | -1666.890 | exact | 8.597 |
| $Pb^{208}$ | after | 5.556 | -1642.091 | Hartree | 5.732 |
| | Exp. | 5.505 | -1636.430 | Harmonic | 5.190 |