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Semileptonic decays $B \to D^{(*)} l\nu$ at nonzero recoil

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We have analyzed the semileptonic decays $B \to D\ell\nu$ and $B \to D^*\ell\nu$ on the full suite of MILC (2+1)-flavor asqtad ensembles with lattice spacings as small as 0.045 fm and light-to-strange-quark mass ratios as low as 1/20. We use the Fermilab interpretation of the clover action for heavy valence quarks and the asqtad action for light valence quarks. We compute the hadronic form factors for $B \to D$ at both zero and nonzero recoil and for $B \to D^*$ at zero recoil. We report our results for $|V_{cb}|$.

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1. Introduction

The CKM matrix element $|V_{cb}|$ plays a prominent role in tests of the Standard Model. It normalizes the legs of the unitarity triangle. The dominant uncertainty in $|V_{cb}|$ has come from theoretical determinations of the decay rates for $B \to c\ell\nu + \ldots$. The purpose of this work is to study the exclusive processes $B \to D\ell\nu$ and $B \to D^*\ell\nu$ in lattice gauge theory in order to reduce the uncertainty in $|V_{cb}|$ determined from these decays.

The Fermilab Lattice and MILC collaborations are completing a B- and D-physics program based on fourteen large ensembles of gauge configurations, generated in the presence of (2+1)-flavors of improved staggered (asqtad) sea quarks. The strange-quark mass m_s is kept at approximately its physical value and the degenerate light (up and down) quark mass $\hat{m}'$ takes on values from $\hat{m}'/m_s = 0.05$ to 0.4. The lattice spacings in these ensembles range from approximately 0.045 fm to 0.15 fm, as discussed in [1]. Clover (Fermilab) fermions are employed for the bottom and charm quarks and staggered (asqtad) fermions, for the light valence quarks with masses set equal to the sea quarks. Among the quantities calculated are the hadronic form factors for $B \to D\ell\nu$ at nonzero recoil and for $B \to D^*\ell\nu$ at zero recoil

Our methodology for $B \to D\ell\nu$ has been outlined in previous conferences in this series [1], and for $B \to D^*\ell\nu$, we update our previously published results [3] with data from the same full set of asqtad ensembles. For both decays, papers with details are in preparation [4, 5]. Here we focus on details of the $B \to D\ell\nu$ analysis that have not been reported previously.

To avoid biases, the analysis described here was blind, following now common practice in experimental high energy physics. The vector current renormalization constants were determined by two members of the collaboration and reported to the rest of the collaboration, multiplied by a common, secret blinding factor. Only after the value of $|V_{cb}|$ was obtained was the blinding factor revealed and removed. That is the value reported here.

2. Notation

We start by reviewing our notation. The differential decay rate for the exclusive process $B \to D\ell\nu$ is given by

$$\frac{d\Gamma}{dw}(B \to D\ell\nu) = |\bar{\eta}_{\mathrm{EW}}|^2 \frac{G_F^2 |V_{cb}^2| M_B^5}{48\pi^3} (w^2-1)^{3/2} r^3 (1+r)^2 \mathcal{G}(w)^2 \,. \tag{2.1}$$

in the approximation that the masses of the leptons $\ell = e, \mu, \nu_e, \nu_\mu$ are much smaller than the B and D mass difference $M_B - M_D$. The recoil parameter, $w = v \cdot v'$, is the dot product of the four-velocities of the B and D mesons, $v = p_B/M_B$ and $v' = p_D/M_D$, respectively, and $r = M_D/M_B$. The factor $|\bar{\eta}_{\mathrm{EW}}|^2$ accounts for electroweak corrections. The hadronic form factor $\mathcal{G}(w)$ is proportional to the vector form factor $f_+(w) = (1+r)\mathcal{G}(w)/(4r)$. That form factor is obtained from a tensor decomposition of the hadronic vector-current matrix element for the transition,

$$\langle D(p_D)|\mathcal{V}^\mu|B(p_B)\rangle = f_+(q^2)\left[(p_B+p_D)^\mu - \frac{M_B^2-M_D^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_B^2-M_D^2}{q^2}q^\mu \,, \tag{2.2}$$

where the four-momentum transfer is $q = p_B - p_D$. Here $\mathcal{V}^\mu = \bar{b}\gamma^\mu c$ is the $b \to c$ vector current and f_+ and f_0 are the vector and scalar form factors, respectively.

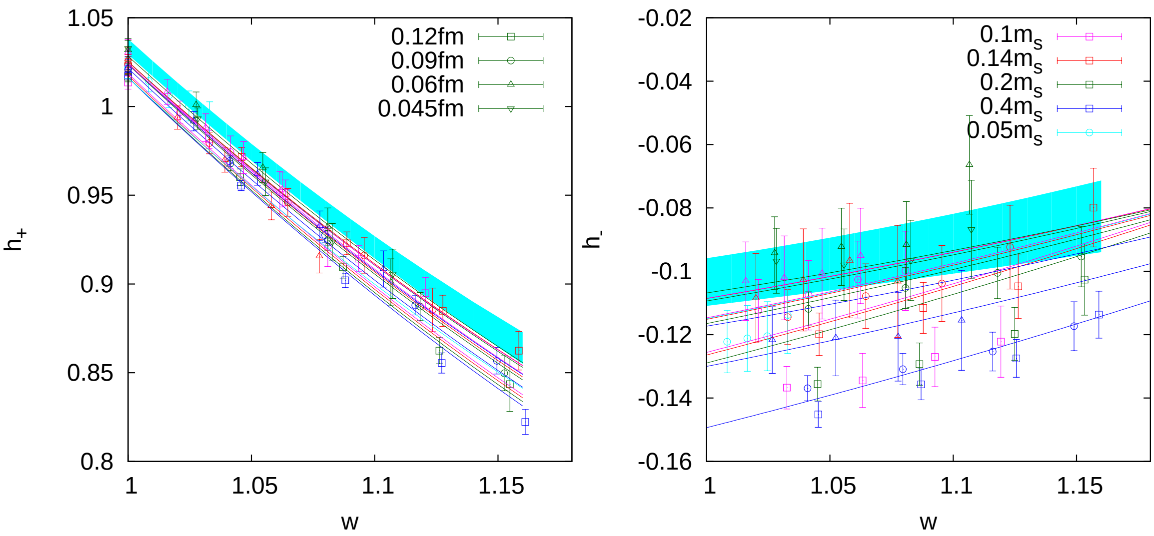

Figure 1: Global fit of all data for the form factors h_+ (left) and h_- (right) vs w, the recoil parameter. The simultaneous fit gives $p = 0.53$. The blue band shows the physical continuum prediction. In both plots the symbol shapes distinguish the lattice spacing as indicated in the left legend and the colors distinguish sea quark mass ratios, as indicated in the right legend. Errors are statistical only.

The alternative form factors h_+ and h_- are convenient for lattice simulations:

$$\frac{\langle D(p_D)|\mathscr{V}^\mu|B(p_B)\rangle}{\sqrt{M_B M_D}} = h_+(w)(v+v')^\mu + h_-(w)(v-v')^\mu \,, \tag{2.3}$$

They are related to f_+ and f_0 through

$$f_+(q^2) = \frac{1}{2\sqrt{r}}\left[(1+r)h_+(w) - (1-r)h_-(w)\right];\; f_0(q^2) = \sqrt{r}\left[\frac{w+1}{1+r}h_+(w) - \frac{w-1}{1-r}h_-(w)\right] \tag{2.4}$$

Lattice calculations at zero recoil ($w = 1$) typically have the smallest errors. However, because of the phase space suppression near zero recoil in $B \to D\ell\nu$, evident from the factor $(w^2-1)^{3/2}$ in Eq. (2.1), experimental errors are largest there. Thus, we aim to work at nonzero recoil where the combined experimental and theoretical error is minimized.

3. Determination of the form factors

Using procedures outlined in previous reports [1], the hadronic form factors $h_+(w)$ and $h_-(w)$ were determined from fits to appropriate three-point and two-point correlation functions. Some w-dependent adjustment was necessary because the simulation values of the charm and bottom quark masses (κ_b and κ_c) were slightly different from our final, preferred, tuned values of these masses. Statistical errors with correlations were propagated through the entire calculation using a single-elimination jackknife.

Results for all ensembles are plotted in Fig. 1. To extrapolate to the physical quark mass and zero lattice spacing, we use the following fit Ansätze for the chiral/continuum extrapolation

$$h_+(a,\hat{m}',w) \;=\; 1 + \frac{X_+(\Lambda_\chi)}{m_c^2} - \rho_+^2(w-1) + k_+(w-1)^2 + c_{1,+}x_l + c_{a,+}x_{a^2} + c_{a,w,+}x_{a^2}(w-1) +$$

Table 1: Systematic error budget (in percent). The total error is obtained by adding the individual errors in quadrature. Not explicitly shown because they are negligible are finite-volume effects, isospin-splitting effects, and light-quark mass tuning.

source	$h_+(\%)$	$h_-(\%)$
κ-tuning adjustment	≤ 0.1	1.4
Lattice scale r_1	0.2	≤ 0.1
Heavy quark discretization	2.0	10.
ρ factor (current matching)	0.4	20.
Total systematic error	2.1	22.

$$
\begin{aligned}
& + \; c_{a,a,+}x_{a^2}^2 + c_{a,m,+}x_l x_{a^2} + c_{2,+}x_l^2 + \frac{g^2_{D^*D\pi}}{16\pi^2 f^2}\mathrm{logs}_{1-\mathrm{loop}}(\Lambda_\chi, w, \hat{m}', a) \qquad (3.1)\\
h_-(a,\hat{m}',w) \; &= \; \frac{X_-}{m_c} - \rho_-^2(w-1) + k_-(w-1)^2 + c_{1,-}x_l + c_{a,-}x_{a^2} + c_{a,w,-}x_{a^2}(w-1)\\
& + \; c_{a,a,-}x_{a^2}^2 + c_{a,m,-}x_l x_{a^2} + c_{2,-}x_l^2 \; .
\end{aligned}
$$

which depend on the light spectator quark mass $x_l = 2B_0\hat{m}'/(8\pi^2 f_\pi^2)$ in the notation of [2], lattice spacing $x_{a^2} = [a/(4\pi f_\pi r_1^2)]^2$, and $w = v \cdot v'$. The chiral "logs" term comes from a staggered fermion version of the one-loop continuum result of Chow and Wise [6] that includes taste-breaking discretization effects [7]. We supplement the next-to-next-leading order (NNLO) heavy-light meson staggered χPT expression with terms analytic in $(w-1)$ to enable an interpolation in w at nonzero recoil. The fit at NLO is already satisfactory ($p = 0.27$). The analytic NNLO terms are then added with priors 0 ± 1 so the final statistical error contains the error of truncation.

The several sources of systematic error in the lattice determination of h_+ and h_- are listed in Table 1 together with estimates of their contributions. Data were adjusted to our best tuned κs, based on a calculation on one ensemble that varied them. The error listed reflects the uncertainty in the adjustment. We used $r_1 = 0.3117(22)$ fm. The uncertainty in this value is systematic. The heavy-quark error in h_+ is estimated from heavy quark effective theory with $\mathscr{O}(\alpha_s(\Lambda/2m_Q)^2)$, and in h_-, with $\mathscr{O}(\alpha_s\Lambda/2m_Q)$. The current renormalization factor ρ^{V_4} is known to one loop. The error shown comes from our estimate of the omitted higher order terms. The corresponding ρ^{V_1} is not known. We assume, very conservatively, that it differs from unity by at most 20%. Because h_- is about one tenth the size of h_+, this choice does not impair the overall precision.

These errors are applied to the result of the chiral/continuum extrapolation in quadrature with the statistical error. The thus-combined errors in h_+ and h_- propagate to f_+ and f_0 according to the linear transformation Eq. (2.4). For a more complete discussion, see [4].

To compare the lattice and experimental form factors we need to interpolate/extrapolate to a larger w range. We do this using the z-expansion of Boyd, Grinstein and Lebed [8], which provides a model-independent parameterization of the q^2 dependence of f_+ and f_0. This expansion builds in constraints from analyticity and unitarity. It is based on the conformal map

$$
z(w) = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}, \qquad (3.2)
$$

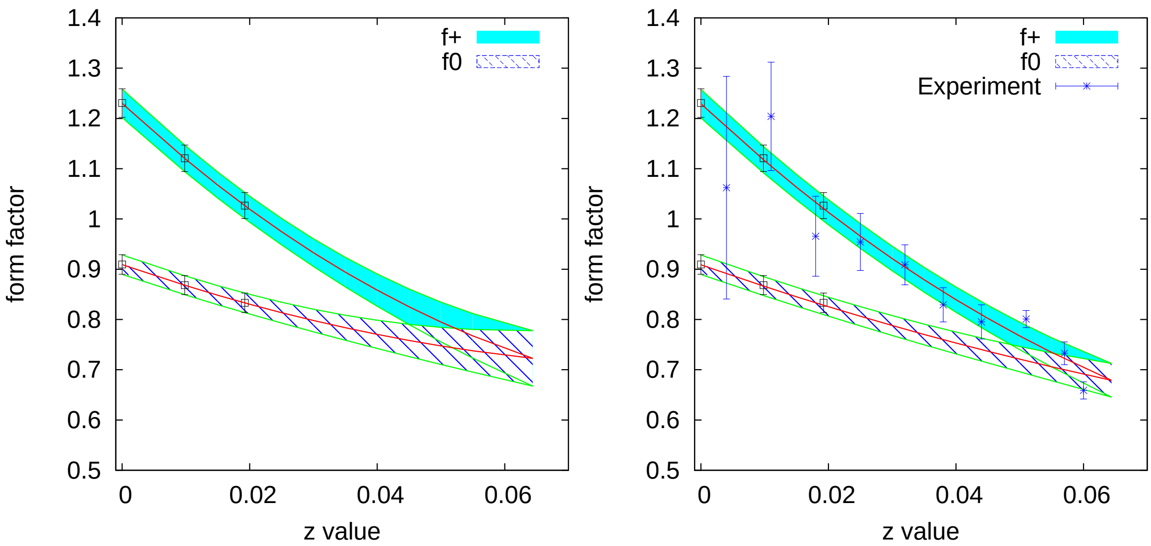

Figure 2: Left: form factors f_+ and f_0 parameterized by the z expansion to cubic order ($p = 0.12$). Right: combined fit result with experimental results from the Babar collaboration [11] ($p = 0.3$). The plotted experimental points have been divided by our best fit value of $\bar{\eta}_{EW}|V_{cb}|$ and converted to f_+.

which for $B \to D\ell\nu$ maps the physical region $w \in [1, 1.59]$ to $z \in [0, 0.0644]$. It pushes poles and branch cuts far away at $|z| \approx 1$. Form factors are then parameterized as

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{N} a_{i,n} z^n , \tag{3.3}$$

where N is the truncation order, $P_i(z)$ are the Blaschke factors and ϕ_i are the "outer functions". The latter are chosen to simplify the unitarity bound: $\sum_n |a_{i,n}|^2 \leq 1$. The constraint on the sum of the coefficients, combined with the small range of z, imply that we need only the first few coefficients in the expansion. We also impose the kinematic constraint $f_+ = f_0$ at $q^2 = 0$ or $z \approx 0.0644$.

To implement the z expansion, we start from the results for f_+ and f_0 at the continuum physical point, as determined from the chiral/continuum fit. We choose three w values, $w = 1.00$, 1.08, and 1.16, and fit the corresponding form factor data to determine the coefficients $a_{i,j}$. These, then, are used to parameterize the form factors over the full kinematic range, as shown in the left panel of Fig. 2. We tested the truncation of the z-expansion by adding higher terms with priors set to 0 ± 1 and found the results were already stable at $N = 2$. We quote results for $N = 3$ so that the fit error incorporates the systematic error of truncation. Details of the fit result, including parameters and correlations will be presented in the forthcoming paper [4].

When fitting to the experimental data it is necessary to take into account electroweak effects still present in the experimental values, but not included in the lattice calculation. These include a Sirlin factor $\eta_{\rm EW} = 1.00662$ for the $W\gamma$ and WZ box diagrams [9] and a further Coulomb correction for final state interactions in B^0 decays. BaBar reports that 37% of the decays were B^0s, which results in a QED correction factor in the amplitude of $1 + 0.37\alpha/(2\pi)$. We have assigned an uncertainty of ± 0.005 to this correction to account for omitted electromagnetic effects at intermedicate distances. The net factor is, then, $\bar{\eta}_{\rm EW} = 1.011(5)$. (We use a bar to denote the EW/QED correction of a sample of neutral and charged Bs.) We prefer to use $\mathcal{G}(w)$ to denote the purely hadronic form factor, so in our notation $\bar{\eta}_{EW}|V_{cb}|\mathscr{G}(w)$ corresponds to the quantity often reported as $|V_{cb}|\mathscr{G}(w)$, and the ratio of experimental to theoretical values must be divided by $\bar{\eta}_{\mathrm{EW}}$ to get $|V_{cb}|$.

The joint fit to our theoretical data and the experimental data from the BaBar collaboration [11] is shown in Fig. 2. The errors here include statistical and systematic errors, combined in quadrature. For the experimental systematic error we assumed, for want of more accurate information, that the quoted percentage value at small w is appropriate over the entire fit range [10].

4. Results and discussion

Our best fit value for $|V_{cb}|$ from the exclusive process $B \to D\ell\nu$ at nonzero recoil is

$$|V_{cb}| = 0.0385(19)_{\mathrm{exp+lat}}(2)_{\mathrm{QED}}\,. \tag{4.1}$$

This value includes the full electroweak/QED correction. The first error combines statistical and systematic errors from both experiment and theory. The second error reflects the uncertainty in the Coulomb correction. To get a sense of the relative importance of the various systematic errors, we repeated the fit with only statistical errors from both theory and experiment with the result 0.0388(11)(2). With all errors, except the theoretical systematic errors, the result is 0.0383(17)(2). With all errors, except the experimental systematic errors, it is 0.0388(14)(2). Thus we conclude that the experimental systematic error contributes more to the resulting error than the theoretical one.

To quantify the improvement due to working at nonzero recoil, we also use the standard method for extracting $|V_{cb}|$ based on extrapolating the experimental data to zero recoil and comparing with the theoretical form factor at this point. If we use the BaBar collaboration result from the same BaBar data as in the nonzero recoil analysis, namely, $\bar{\eta}_{EW}|V_{cb}|\mathscr{G}(1) = 0.0430(19)_{\mathrm{stat}}(14)_{\mathrm{sys}}$ [11], and our extrapolated value $\mathscr{G}(1) = 1.081(25)$, we obtain $|V_{cb}| = 0.0393(22)_{\mathrm{exp+lat}}(2)_{\mathrm{QED}}$. So although the result is consistent with our nonzero recoil determination, the error is 25% larger.

A still better approach would be to fit the lattice results together with a world average of the measured values of $\bar{\eta}_{EW}|V_{cb}|\mathscr{G}(w)$ as a function of recoil parameter w (suitably binned), rather than with data from a single experiment, as we have done. To our knowledge, such a compilation has not been done [12], but it would be welcome.

In our companion study of $B \to D^* l\nu$ at zero recoil, [5] we obtain $\mathscr{F}(1) = 0.906(4)_{\mathrm{stat}}(12)_{\mathrm{sys}}$ for the hadronic form factor. Here, in keeping with our new convention for $B \to D\ell\nu$ we reserve the notation $\mathscr{F}(1)$ for the purely hadronic form factor. The values in the HFAG world average reported in their notation as $|V_{cb}|\mathscr{F}(1)$ are, in ours, $\bar{\eta}_{EW}|V_{cb}|\mathscr{F}(1)$ [12]. Because of the different proportion of neutral B decays in this average, we use $\bar{\eta}_{EW} = 1.015$. The result is

$$|V_{cb}| = 0.0390(5)_{\mathrm{exp}}(5)_{\mathrm{lat}}(2)_{\mathrm{QED}}\,. \tag{4.2}$$

A recent analysis of inclusive decays quotes $|V_{cb}| = 0.0424(9)_{\mathrm{exp+thy}}$ [13]. It disagrees at 1.8σ from ours for the exclusive D final state and by nearly 3σ from our more precise result from the exclusive D^* final state.

The error in our determination of $|V_{cb}|$ from $B \to D\ell\nu$ could be improved by repeating the analysis with a world average of experimental form factors, suitably binned in w, and by improving our understanding of the experimental systematic error at larger w. Improvements in lattice results will come from future studies with still better actions, better statistics, smaller lattice spacing, and ensembles with physical light quark masses that eliminate the need for a chiral extrapolation.

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