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On calculation of the interaction current of the deuteron in the Bethe-Salpeter approach. Analytical structure

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This article is a next step continuing the basic work "On calculation of the interaction current of the deuteron in the Bethe-Salpeter approach. General formulae". Therefore consideration in this work is completely based on the original one. The work is concentrated principally on investigation of the analytical structure of matrix elements of the interaction part of the Mandelstam current of deuteron in the reaction of elastic electron-deuteron scattering. Description of the complete set of all singularities that appear in this model is given. Also the method of the calculation of moving singularities has been discussed in the work.

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1. Analytical structure

While we are going to take the integration over p, p' and t first of all we have to investigate the analytical structure of the model. As it was said in [1], there is only one structure responsible for the singularities that appear in the matrix elements, it is function R:

$$R(p'_{(0)0}, |\vec{p}'_{(0)}, p_0, |\vec{p}|; s)_{L_1L_2L_3L_4} = \phi_{L_1}(p'_{(0)0}, |\vec{p}'_{(0)}|) \\ \times \mathrm{v}_{L_2L_3}(k'^{(l)}_{(0)0}, |\vec{k}'^{(l)}_{(0)}|; k^{(l)}_{(0)0}, |\vec{k}^{(l)}_{(0)}|; s)\phi_{L_4}(p_0, |\vec{p}|), \tag{1.1}$$

where radial parts of the Bethe-Salpeter amplitude can be expressed via radial parts of the vertex function as:

$$\phi_\alpha(p_0, |\vec{p}|) = \sum_\beta S_{\alpha\beta}(p_0, |\vec{p}|; s)\, g_\beta(p_0, |\vec{p}|) \tag{1.2}$$

In this work we consider only positive-energy waves, thus there is only one function $S_{++}(p_0, |\vec{p}|; s)$ in the sum above. The function is:

$$S_{++} = (\frac{\sqrt{s}}{2} + p_0 - E_{\mathbf{p}})^{-1}(\frac{\sqrt{s}}{2} - p_0 - E_{\mathbf{p}})^{-1}$$

Still we were working regardless to the type of NN interaction, but at the stage of investigation of the analytical structure of the model it's important to emphasize the separable character of kernel of interaction. So-called separable *Ansatz* for the kernel has the following form:

$$\mathrm{v}_{ab}(p'_0, |\vec{p}'|; p_0, |\vec{p}|; s) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{(a)}(p'_0, |\vec{p}'|) g_j^{(b)}(p_0, |\vec{p}|) \tag{1.3}$$

In this case radial functions of the Bethe-Salpeter amplitude are also expressed through the functions of separable kernel:

$$g_a(p_0, |\vec{p}|) = \sum_{i,j=1}^{N} \lambda_{ij}(s) g_i^{(a)}(p_0, |\vec{p}|) c_j(s) \tag{1.4}$$

$$\tag{1.5}$$

In the framework of this consideration the functions of separable kernel have the common general relativistic form $g_i^{(a)}(p_0, |\vec{p}|) = 1/(p_0^2 - |\vec{p}|^2 - \beta^2)$.

Now we can rewrite the function R in separable form:

$$R(p'_{(0)0}, |\vec{p}'_{(0)}, p_0, |\vec{p}|; s)_{L_1L_2L_3L_4} = \sum \lambda_{ij}(s) c_j(s) \lambda_{kd}(s) \lambda_{mn}(s) c_n(s) \times \\ \times S_{++}(p'_{(0)0}, |\vec{p}'_{(0)}|; s) g_i^{(L_1)}(p'_{(0)0}, |\vec{p}'_{(0)}|) g_k^{(L_2)}(k'^{(l)}_{(0)0}, |\vec{k}'^{(l)}_{(0)}|) g_d^{(L_3)}(k^{(l)}_{(0)0}, |\vec{k}^{(l)}_{(0)}|) \times \\ \times S_{++}(p_0, |\vec{p}|; s) g_m^{(L_4)}(p_0, |\vec{p}|) \tag{1.6}$$

Sources of poles over p_0:	total number of poles	numeration of the poles
$S_{++}(p_0, \|\vec{p}\|, s)$	2	1 , 2
$g_i^{(L_4)}(p_0, \|\vec{p}\|)$	2	3 , 4
$g_i^{(L_3)}(k^{(l)}_{(0)0}, \|\vec{k}^{(l)}_{(0)}\|)$	2+2	5 , 6 , 7 , 8
Sources of poles over p'_0:		
$S_{++}(p'_{(0)0}, \|\vec{p}'_{(0)}\|, s)$	4	9 , 10 , 11 , 12
$g_i^{(L_1)}(p'_{(0)0}, \|\vec{p}'_{(0)}\|)$	2	13 , 14
$g_i^{(L_2)}(k'^{(l)}_{(0)0}, \|\vec{k}'^{(l)}_{(0)}\|)$	2+2	15 , 16, 17, 18

Table 1: Sources of poles

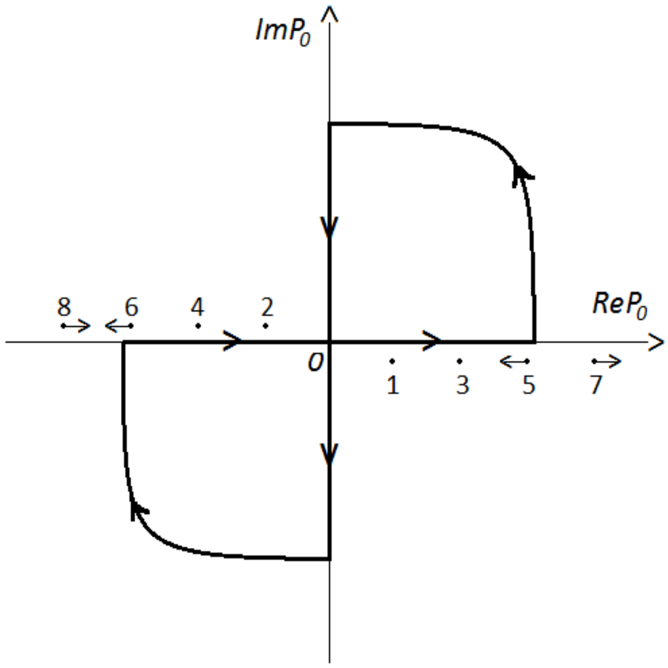

Figure 1: Behavior of the poles in p_0 complex plane

Let's now consider all sources of singularities. There are 18 poles in each matrix element - 8 poles over p_0 and 10 poles over p'_0. In the table 1 all the sources off poles are represented.

The poles can be classified by the one important feature - whether or not a certain pole is able at some $\{\eta, t\}$ to cross axis $Rep_0 = 0$ ($Rep'_0 = 0$) in complex plane, see figures 1 and 2. Those poles which are able to cross the axis are called moving singularities. Hopefully (will be clear further) there are only 6 such poles: over p_0 are 5, 8 (see table 1) and over p'_0 are 9, 12, 16, 17.

For the purpose of calculation of the integrals over p_0 and p'_0 we construct specific contours (see 1 and 2) and use Wick rotation $p_0 \to ip_4$, $p_0 \to ip_4$. In this case integrals over real axis in a complex plane are expressed as:

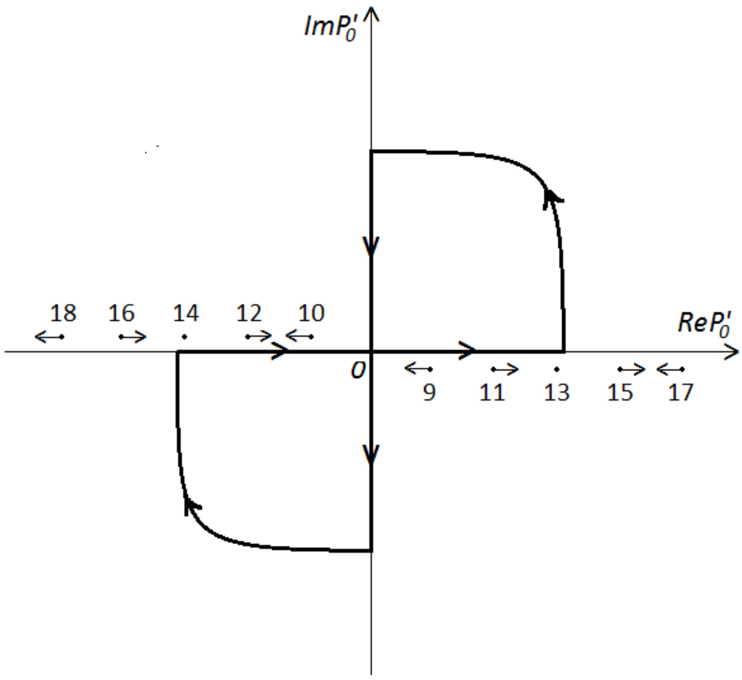

Figure 2: Behavior of the poles in p_0' complex plane

over p_0

$$i\int_{-\infty}^{+\infty} f dp_0 = \int_{-\infty}^{+\infty} f dp_4 - \sum \theta_i (Q^2 - Q^2_{min}) Res(f, p_0 = p_0^{(i)}) \tag{1.7}$$

over p_0'

$$i\int_{-\infty}^{+\infty} f dp_0' = \int_{-\infty}^{+\infty} f dp_4' - \sum \theta_i (Q^2 - Q^2_{min}) Res(f, p_0' = p_0'^{(i)}), \tag{1.8}$$

where Q^2_{min} determines minimal value of transfer momentum squared at which the poles get into the contour of integration. Procedure of the calculation of such values will be discussed further.

Exact expressions for the poles are presented below (here and further $p = |\mathbf{p}|, p' = |\mathbf{p}'|$, $cos\theta = p_z/p$ (or p_z'/p'), $\eta = Q^2/(4M^2)$ and $\gamma = 1 + 4\eta t - 4\eta t^2$):

- for the function $S_{++}(p_0, |\vec{p}|, s)$:

$$p_0^{(1,2)} = \mp\frac{1}{2}M \pm \sqrt{p^2 + m^2} \mp i\varepsilon \tag{1.9}$$

- for the function $S_{++}(p'_{(0)0}, |\vec{p}'_{(0)}|, s)$:

$$p_0^{(9,10)} = -M(\frac{1}{2}+\eta) \pm \sqrt{m^2 + p'^2 + M^2(\eta(1+\eta)) + 2M\sqrt{\eta}\sqrt{1+\eta}p'cos\theta} \mp i\varepsilon \quad (1.10)$$

$$p_0^{(11,12)} = M(\frac{1}{2}+\eta) \pm \sqrt{m^2 + p'^2 + M^2(\eta(1+\eta)) - 2M\sqrt{\eta}\sqrt{1+\eta}p'cos\theta} \mp i\varepsilon \quad (1.11)$$

- for the function $g_i^{(L_4)}(p_0, |\vec{p}|)$ (from the radial function of the initial deuteron):

$$p_0^{(3,4)} = \pm\sqrt{p^2+\beta^2} \mp i\varepsilon \quad (1.12)$$

- for the function $g_i^{(L_1)}(p'_{(0)0}, |\vec{p}'_{(0)}|)$ (from the radial function of the final deuteron):

$$p_0^{(13,14)} = \pm\sqrt{p'^2+\beta^2} \mp i\varepsilon \quad (1.13)$$

- for the function $g_i^{(L_3)}(k^{(l)}_{(0)0}, |\vec{k}^{(l)}_{(0)}|)$ (from the radial function of the kernel of interaction):

  l=1

$$p_0^{(5,6)} = -M\eta t \pm \sqrt{\frac{1}{\gamma}F(p, cos\theta, \eta, t)} \mp i\varepsilon, \quad (1.14)$$

$$F(p, cos\theta, \eta, t) = p^2 + \beta_i^2 + 4\eta t(1-t)p^2\cos^2\theta + \quad (1.15)$$
$$+2M\sqrt{\eta}\sqrt{1+\eta}t\gamma pcos\theta + M^2\eta(1+\eta)t^2\gamma$$

  l=2

$$p_0^{(7,8)} = M\eta t \pm \sqrt{\frac{1}{\gamma}F(p, -cos\theta, \eta, t)} \mp i\varepsilon, \quad (1.16)$$

- for the function $g_i^{(L_2)}(k'^{(l)}_{(0)0}, |\vec{k}'^{(l)}_{(0)}|)$ (from the radial function of the kernel of interaction):

  l=1

$$p_0'^{(15,16)} = (1-t)M\eta \pm \sqrt{\frac{1}{\gamma}F'(p', cos\theta, \eta, t)} \mp i\varepsilon, \quad (1.17)$$

$$F'(p', cos\theta, \eta, t) = \beta^2 + p'^2 + 4\eta t(1-t)p'^2 cos^2\theta + \quad (1.18)$$
$$+2M\sqrt{\eta}\sqrt{1+\eta}(t-1)\gamma p'cos\theta + M^2\eta(1+\eta)(t-1)^2\gamma$$

l=2

$$p_0'^{(17,18)} = -(1-t)M\eta \pm \sqrt{\frac{1}{\gamma}F'(p', -cos\theta, \eta, t)} \mp i\varepsilon, \tag{1.19}$$

Let's consider $p_0^{(5)}$ in more details. In the figure 3 dependence of the pole on the p and $cos\theta$ is shown at $t = 0.9$ and $\beta = 0.23$.

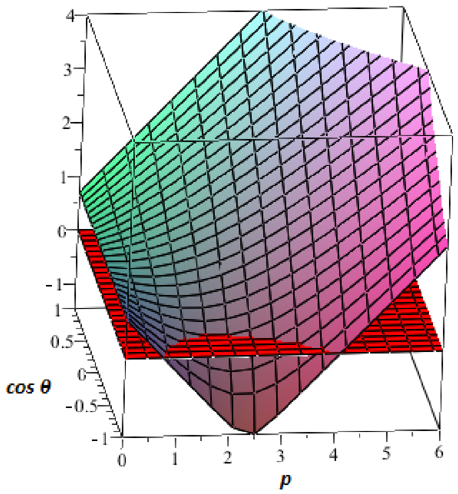

Figure 3: Pole $p_0^{(5)}$

The region where the pole is negative determines the limits of integration over $cos\theta$ and p. As it's seen from the figure 3, the limits for $cos\theta$ are $-1 < cos\theta < cos\theta_{crit}(p, \eta, t)$ and for the p are $p_{crit_1}(\eta, t) < p < p_{crit_2}(\eta, t)$. Expressions for the critical values appear to be too large to consider it in the article. For the purpose of investigation of the region of allowable values over $\{\eta, t\}$ (for the poles 9 and 12 just over η) we have to analyze the conditions of existing of the critical values $p_{crit_1}(\eta, t)$ and $p_{crit_2}(\eta, t)$ by investigation of positiveness of radicand in corresponding expressions. For the poles 5 and 8 the radicand is:

$$Rad(\eta, t) = \eta^2\gamma(\eta^2 M^2 t^2 \gamma - \beta^2), \tag{1.20}$$

see figure 4.

For the poles 16 and 17 the radicand is:

$$Rad'(\eta, t) = \eta^2\gamma(\eta^2 M^2 (1-t)^2 \gamma - \beta^2), \tag{1.21}$$

see figure 5.

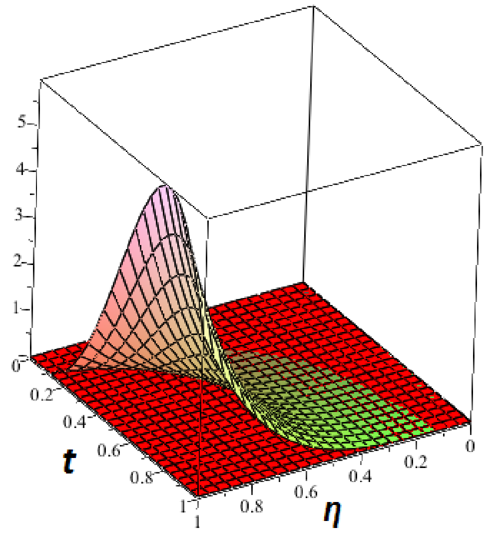

Figure 4: Radicand $Rad(\eta,t)$

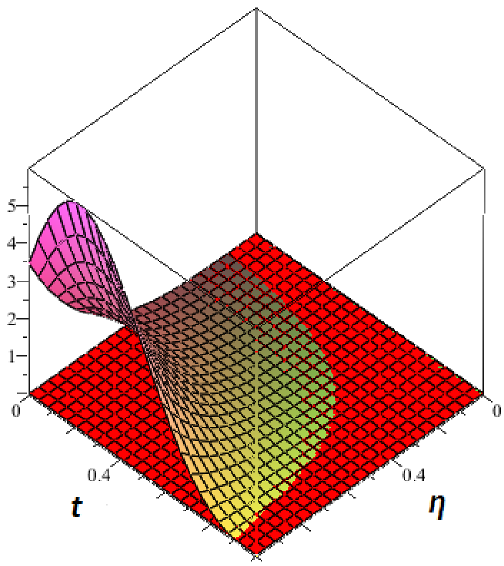

Figure 5: Radicand $Rad'(\eta,t)$

As it's seen from the figures, for the poles 5 and 8 (poles over p_0) minimal η is reached at $t = 1$, but for the poles 16 and 17 (poles over p'_0) minimal η is reached at $t = 0$. While calculations of the integrals over t we have to determine $t_{crit}(\eta)$ at each η greater than η_{min}. To calculate such η_{min} we solve the equations $Rad(\eta,t) = 0$ and $Rad'(\eta,t) = 0$ over t and find critical values $t_{crit}(\eta)$. At the figure 6 the value $Rad(\eta,t)/(\eta^2\gamma)$ is depicted at $\beta = 0.5$ and $\eta = 0.3$. As it's seen from the figure there is only one real solution for the $t_{crit}(\eta)$ which lies in the range of variation of t (from 0 to 1).

From the expressions for $t_{crit}(\eta)$ (they are too huge to show in the article) at $t_{crit} = 1$ for poles

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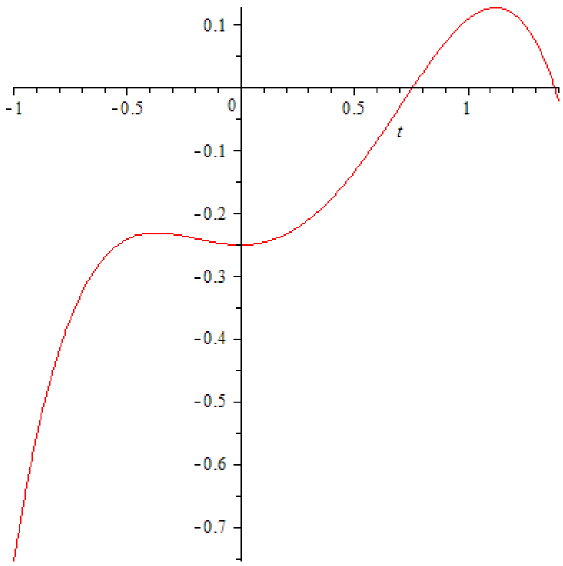

Figure 6: Value $Rad(\eta,t)/(\eta^2\gamma)$

5 and 8 and $t_{crit} = 0$ for poles 16 and 17 we obtain values η_{min}. For both cases they coincide and equal $\eta_{crit} = \beta/M$ ($Q^2_{crit} = 4M\beta$). As for the critical value of the transfer momentum squared for the poles from the function S_{++} it is $Q^2_{min} = 2M(2m-M)$. The critical value of the transfer momentum squared for the poles going from the functions of separable kernel coincides for RIA and for the interaction current but for the poles from the function S_{++} it's different.

2. Conclusions

In the case of the interaction current analytical structure is more complicated than in the case of relativistic impulse approximation. First, there are 6 poles in the IC instead of 2 poles in the RIA, second, we have to consider 2 complex planes - over p_0 and over p'_0. Also the computational situation is complicated by the differential operator D_μ which increases the order of poles. If used separable kernel has functions that have large degrees in denominators the computational task for the IC on typical laptop would be probably a kind of challenge.

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References

[1] Bekzhanov A.V., Bondarenko S.G., Burov V.V., Proceedings of the 12th International Workshop "Relativistic Nuclear Physics from Hundreds of MeV to TeV", June 16-20, 2014., Stará Lesná, Slovak Republic, Editor: I.I. Migulina, Publishing: JINR, Dubna, Russia